OPTIMAL CONTROL OF COMPOSITE SYSTEMS

MATHEMATICS

1967

SovietRxiv

Abstract

Full Text

UDC 517.933

MATHEMATICS

V. V. VELICHENKO

OPTIMAL CONTROL OF COMPOSITE SYSTEMS

(Presented by Academician L. S. Pontryagin, 10 XII 1966)

1°. Statement of the problem. We shall call a dynamic system *composite* if its motion is described by an m-dimensional system of equations

$$\dot{x} = f(x, u, t) \tag{1}$$

on the time interval $T_0 \le t \le T_1$, and by an n-dimensional system of equations

$$\dot{y} = \varphi(y, v, t) \tag{2}$$

on the time interval $T_1 \le t \le T$. The instant of time T_1 is defined as the instant at which the trajectory of system (1) reaches the surface

$$C_0(x, t) = 0, \tag{3}$$

with respect to which it is assumed that

$$dC_0(x, t)/dt\big|_{t=T_1-0} \ne 0.$$

Continuity between systems (1) and (2) is ensured by fulfillment of the conditions

$$C_\gamma[x(T_1), y(T_1), T_1] = 0, \quad \gamma = 1, \dots, c < m + n. \tag{4}$$

For the composite system (1)–(4) we pose the following optimal-control problem. It is required to find initial conditions $x(T_0)$, T_0, satisfying the conditions

$$A_\alpha[x(T_0), T_0] = 0, \quad \alpha = 0, 1, \dots, a \le m, \tag{5}$$

and an r-dimensional vector function $u(t) \in U$, $T_0 \leq t \leq T_1$, and an s-dimensional vector function $v(t) \in V$, $T_1 \leq t \leq T$, which deliver a minimum to the functional

$$I(x, y, u, v) = \Psi[y(T), T] \tag{6}$$

subject to the conditions

$$B_\beta[y(T), T] = 0, \quad \beta = 0, 1, \dots, b < n. \tag{7}$$

The vector functions $f(x, u, t)$, $\varphi(y, v, t)$ and the functions $C_0(x, t)$, $C_\gamma(x, y, t)$, $A_\alpha(x, t)$, $\Psi(y, t)$, and $B_\beta(y, t)$ are assumed to be continuous together with their first-order partial derivatives and to have bounded second-order partial derivatives with respect to their arguments. As the class of admissible controls we take the class of piecewise continuous vector functions $u(t)$, $v(t)$. The prescribed fixed domains U and V will be considered closed.

A number of optimal-control problems arising in engineering and mathematical economics reduce to the formulation indicated above. In the present note, optimality conditions are obtained for the stated problem, having the form of the maximum principle (1).

2°. Reduction to a problem with intermediate conditions.

Assuming that

$$dA_0(x, t)/dt|_{t=T_0} \neq 0, \qquad dB_0(y, t)/dt|_{t=T} \neq 0,$$

we shall use the conditions

$$A_0[x(T_0), T_0] = 0, \qquad B_0[y(T), T] = 0$$

to determine the time instants T_0 and T.

We continue the right-hand side of system (1) and the control $u(t)$ to the interval $[T_1, T]$, and the right-hand side of system (2) and the control $v(t)$ to the interval $[T_0, T_1]$, in an arbitrary continuous manner. Taking $x(T)$ and $y(T_0)$ to be free, it is easy to reduce the problem posed to the problem with intermediate conditions (4) for the system of equations combining systems (1), (2). The indicated device makes it possible to carry over directly to the problem under consideration the results of [2].

3°. Coupled composite systems.

We shall call the composite system (1)–(4) **coupled** if in (4) $c = n$ and

$$\det \left\| \partial C_\gamma(x, y, t) / \partial y_j \right\|_{j=1}^{\gamma=1,\ldots,n} \Big|_{t=T_1} \neq 0.$$

Introduce the notation

$$A(x,t) = \sum_{\alpha=1}^{a} \lambda_\alpha^A A_\alpha(x,t), \qquad B(y,t) = \sum_{\beta=1}^{b} \lambda_\beta^B B_\beta(y,t) + \lambda^\Psi \Psi(y,t).$$

Using [2], we obtain the following assertion.

Theorem 1. If the controls $u(t)$, $v(t)$ and the trajectories $x(t)$, $y(t)$ are optimal in problem (1)–(7) for a coupled composite system, then there exist numbers $\lambda_\alpha^A, \lambda_\beta^B$ and λ^Ψ satisfying the condition

$$\sum_{\alpha=1}^{a} (\lambda_\alpha^A)^2 + \sum_{\beta=1}^{b} (\lambda_\beta^B)^2 + (\lambda^\Psi)^2 = 1, \qquad \lambda^\Psi \geq 0,$$

and also an m-dimensional vector-function $p(t)$ and an n-dimensional vector-function $q(t)$, satisfying, respectively, the systems of equations

$$\dot{p} = -\operatorname{grad}_x H(x, p, u, t), \qquad T_0 \leq t \leq T_1,$$

$$\dot{q} = -\operatorname{grad}_y H(y, q, v, t), \qquad T_1 \leq t \leq T, \tag{8}$$

where

$$H(x, p, u, t) \equiv (p, f(x, u, t)), \qquad H(y, q, v, t) \equiv (q, \varphi(y, v, t)),$$

the boundary conditions

$$p(T_0) = \left[\operatorname{grad}_x A(x,t) - \left(\frac{dA(x,t)}{dt} \Big/ \frac{dA_0(x,t)}{dt} \right) \operatorname{grad}_x A_0(x,t) \right]_{t=T_0}, \tag{9}$$

$$q(T) = \left[-\operatorname{grad}_y B(y,t) + \left(\frac{dB(y,t)}{dt} \Big/ \frac{dB_0(y,t)}{dt} \right) \operatorname{grad}_y B_0(y,t) \right]_{t=T},$$

and the coupling condition

$$p(T_1) = p^*(T_1) + \mu \operatorname{grad}_x C_0(x,t)\big|_{t=T_1}, \tag{10}$$

where

$$p^*(T_1) = \left[-\left\| \frac{\partial C_k(x,y,t)}{\partial x_i} \right\| \cdot \left\| \frac{\partial C_\gamma(x,y,t)}{\partial y_j} \right\|^{-1} \right]_{t=T_1} q(T_1),$$

$$\mu = \left[\left(H(y,q,v,t) - H(x,p^*,u,t) + \left\| \frac{\partial C_l(x,y,t)}{\partial t} \right\| \cdot \left\| \frac{\partial C_\gamma(x,y,t)}{\partial y_j} \right\|^{-1} q \right) \times \left(\frac{dC_0(x,t)}{dt} \right)^{-1} \right]_{t=T_1}$$

(the indices $i = 1, \dots, m$, $j = 1, \dots, n$ are row indices, $k, \gamma, l = 1, \dots, n$ are column indices), such that the maximum condition is fulfilled

$$H(x,p,u,t) = \sup_{\tilde{u} \in U} H(x,p,\tilde{u},t), \qquad T_0 < t < T_1,$$

$$H(y,q,v,t) = \sup_{\tilde{v} \in V} H(y,q,\tilde{v},t), \qquad T_1 < t < T. \tag{11}$$

Let the composite system (1)–(4) be linear in the phase coordinates

$$\begin{gathered} \dot{x} = F(t)x + \varkappa(u,t), \\ \dot{y} = \Phi(t)y + \chi(v,t), \\ C_\gamma[x(T_1), y(T_1), T_1] \equiv (l_\gamma^x, x(T_1)) + (l_\gamma^y, y(T_1)) + d_\gamma = 0, \end{gathered} \tag{12}$$

the times T_1, T_0, and T are fixed, and conditions (5), (7) and the functional (6) are given in the form

$$\begin{gathered} A_\alpha[x(T_0), T_0] \equiv (l_\alpha^A, x(T_0)) + d_\alpha^A = 0, \qquad \alpha = 1, \dots, a, \\ B_\beta[y(T), T] \equiv (l_\beta^B, y(T)) + d_\beta^B = 0, \qquad \beta = 1, \dots, b, \\ \Psi[y(T), T] \equiv (l^\Psi, y(T)), \end{gathered}$$

where the vectors l and the numbers d are constant.

Theorem 2. *For the optimality of the controls $u(t)$, $v(t)$ and the trajectories $x(t)$, $y(t)$ in problem (12), (13) for a coupled composite system, it is sufficient that there exist numbers λ_α^A, λ_β^B and λ^Ψ, $\lambda^\Psi > 0$, and such vector functions $p(t)$ and $q(t)$, satisfying conditions (8)–(10), that the maximum condition (11) be fulfilled.*

Using (2), it is easy to single out a class of linear composite coupled systems for which the conditions of Theorem 2 are simultaneously sufficient and necessary.

Moscow Institute of Physics and Technology

Received
6 XII 1966

REFERENCES

[1] L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamkrelidze, E. F. Mishchenko, *The Mathematical Theory of Optimal Processes*, Moscow, 1961. [2] V. V. Velichenko, DAN, 174, No. 5 (1967).

Note: Figure translations are in progress. See original paper for figures.

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.