CALCULATION OF EDDY CURRENTS IN A SEA WITH A COMPLEX BOTTOM RELIEF

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Abstract

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CALCULATION OF EDDY CURRENTS IN A SEA WITH A COMPLEX BOTTOM RELIEF

In the article ([1]) we attempted to solve analytically the problem of eddy currents induced in a circular sea of constant depth by oscillations of the vertical component Z of the geomagnetic field. Owing to the very large value of the sea radius R, the skin effect in it should arise at frequencies ω and at electrical conductivities σ that are negligibly small in comparison with those encountered, for example, in radio engineering. The qualitative results of the analysis agreed quite well with observations in nature.

However, despite the large absolute values of the sea depth, there are grounds to believe that the solution of Maxwell' s equations in ([1]) may prove too crude for the actual structure of the electromagnetic field. Special experiments by V. B. Fedoseev with induced currents in thin copper and brass disks revealed errors arising when the problem is solved according to the scheme of ([1]). At the same time we were unable to find any similarity criterion for passing from laboratory experiments with such disks to recalculation for natural conditions.

Therefore it is necessary to return to the numerical method of solving the problem, which we used even before work ([1]). For the time being we shall consider the sea depth ζ to be the same everywhere, and the form of the shoreline to be a circle with radius $R = 355$ km. Let us separate, within the thickness of sea water, cylindrical rings of equal width $\Delta = 10$ km. If the specific electrical conductivity of sea water is $\sigma\ (\text{ohm} \cdot \text{cm})^{-1}$, then the electrical conductivity of each linear centimeter of the arc of a ring will be $\sigma_1 = 10^{11}\zeta \cdot \Delta\sigma$ ohm^{-1}.

In view of the expected strong skin effect, it will be sufficient to investigate the field of 20 such rings, beginning from the mean radius $R_0 = 350$ km to a mean radius of 160 km; naturally, we shall neglect the current fields in rings of smaller radii, where the currents will fall to zero toward the center of the sea according to a deliberately linear law. In any case, this can be done for oscillations of the variable part z of the vertical component of the geomagnetic field with periods of several minutes.

For other parameters, an analogous problem was posed by the Egyptian mathematician A. Ashur ([2]), who reduced it to the solution of Fredholm integral equations. At present it is more expedient to apply the more elegant solution of a system of linear algebraic equations by means of an electronic computer; below it will be shown that such a route makes it possible, without any difficulty, to proceed to calculating currents in a sea with an arbitrarily complex bottom relief, provided only that it is modeled by a surface of revolution.

Let us denote the strengths of the currents in our 20 rings by letters without subscripts (which will be needed below): $a, b, c, \dots, u, v$. Then, for example, in the 2nd ring from the shore the current b will be related to the oscillations of z and to the currents in the various rings by the equation

$$b = -\frac{1}{2}R_b\sigma_1\frac{dz}{dt}10^{-8}+\frac{\mathcal{M}_{ba}}{2\pi R_b}\sigma_1\frac{da}{dt}10^{-6}+\frac{\mathcal{L}_{bb}}{2\pi R_b}\sigma_1\frac{db}{dt}10^{-6}-\frac{\mathcal{M}_{bc}}{2\pi R_b}\sigma_1\frac{dc}{dt}10^{-6}-\dots-\frac{\mathcal{M}_{bv}}{2\pi R_b}\sigma_1\frac{dv}{dt}10^{-6}\text{ amp.} \tag{1}$$

Here R_b is the mean radius of the 2nd ring; $\mathcal{M}$ and $\mathcal{L}$ are the coefficients of mutual inductance and self-inductance (expressed in μH), which we borrow from the handbook ([3]). There the quantities $\mathcal{M}$ are expressed by the formula

$$\mathcal{M}_{ba} = \mathcal{N}_{ba}\sqrt{R_{bR}a}, \tag{2}$$

in which the coefficients $\mathcal{N}$ depend on the corresponding ratios $R_a/R_b,\ R_c/R_b, \dots$ and are tabulated over a sufficiently wide range of arguments.

In equation (1) one may substitute the simpler expressions:

$$M_{ba} = \mathcal{M}_{ba}/R_b = \mathcal{N}_{ba}\sqrt{R_a/R_b}, \qquad L_{bb} = \mathcal{L}_{bb}/R_b,$$

$$M_{bc} = \mathcal{N}_{bc}\sqrt{R_c/R_b} \quad \text{etc.} \tag{3}$$

For the greatest simplification, multiply both sides of equations of type (1) by $2\pi/\sigma_1 \cdot 10^8$. After regrouping the terms we obtain

$$-\frac{2\pi}{\sigma_1}10^8 b + 100M_{ba}\frac{da}{dt} + 100L_{bb}\frac{db}{dt} - 100M_{bc}\frac{dc}{dt} - \dots = \pi R_b\frac{dz}{dt}. \tag{4}$$

The remaining 19 equations for the currents in the rings are written analogously. But, besides these unknowns, the equations contain another 20 unknowns: the time derivatives of the current strengths t. The missing 20 equations are easily obtained from equations of type (4), by differentiating both sides of each of them with respect to time. Then the second derivatives of the currents and of z that arise can be replaced by the functions themselves with the opposite sign, multiplied by ω^2. As a result, after dividing both sides of all the additional equations by ω^2, we obtain them in the form

$$-100M_{ba}a - 100L_{bb}b + 100M_{bc}c + \cdots - \frac{2\pi}{\sigma_1\omega^2}\frac{db}{dt}10^8 = -\pi R_b z. \qquad (5)$$

To solve the resulting 40 equations with 40 unknowns, let us compose a matrix from all the coefficients of the unknowns:

$t\backslash k$	1	2	3	$\cdots$	19	20	21	22	23	$\cdots$	39	40
1	$-\frac{2\pi}{\sigma_1}10^8$	0	0	$\cdots$	0	0	$100L_{aa}$	$-100M_{ab}$	$-100M_{ac}$	$\cdots$	$-100M_{au}$	$-100M_{av}$
2	0	$-\frac{2\pi}{\sigma_1}10^8$	0	$\cdots$	0	0	$100M_{ba}$	$100L_{bb}$	$-100M_{bc}$	$\cdots$	$-100M_{bu}$	$-100M_{bv}$
3	0	0	$-\frac{2\pi}{\sigma_1}10^8$	$\cdots$	0	0	$100M_{ca}$	$100M_{cb}$	$100L_{cc}$	$\cdots$	$-100M_{cu}$	$-100M_{cv}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
19	0	0	0	$\cdots$	$-\frac{2\pi}{\sigma_1}10^8$	0	$100M_{ua}$	$101M_{ub}$	$100M_{uc}$	$\cdots$	$100L_{uu}$	$-100M_{uv}$
20	0	0	0	$\cdots$	0	$-\frac{2\pi}{\sigma_1}10^8$	$100M_{va}$	$100M_{vb}$	$100M_{vc}$	$\cdots$	$100M_{vu}$	$100L_{vv}$
21	$-100L_{aa}$	$100M_{ab}$	$100M_{ac}$	$\cdots$	$100M_{au}$	$100M_{av}$	$-\frac{2\pi}{\sigma_1\omega^2}10^8$	0	0	$\cdots$	0	0
22	$-100M_{ba}$	$-100L_{bb}$	$100M_{bc}$	$\cdots$	$100M_{bu}$	$100M_{bv}$	0	$-\frac{2\pi}{\sigma_1\omega^2}10^8$	0	$\cdots$	0	0
23	$-100M_{ca}$	$-100M_{cb}$	$-100L_{cc}$	$\cdots$	$100M_{cu}$	$100M_{cv}$	0	0	$-\frac{2\pi}{\sigma_1\omega^2}10^8$	$\cdots$	0	0
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
39	$-100M_{ua}$	$-100M_{ub}$	$-100M_{uc}$	$\cdots$	$-100L_{uu}$	$100M_{uv}$	0	0	0	$\cdots$	$-\frac{2\pi}{\sigma_1\omega^2}10^8$	0
40	$-100M_{va}$	$-100M_{vb}$	$-100M_{vc}$	$\cdots$	$-100M_{vu}$	$-100L_{vv}$	0	0	0	$\cdots$	0	$-\frac{2\pi}{\sigma_1\omega^2}10^8$

In addition, let us write out the vertical columns that will contain the free terms, calculated for various phases of the oscillations z according to the law

$$\pi R_i \frac{dz}{dt} = \pi R_i z_0 \omega \sin \omega t, \qquad -\pi R_i z = \pi R_i z_0 \cos \omega t. \qquad (6)$$

The matrix that has been composed contains 4 parts possessing characteristic properties. a) In the upper left part, along the diagonal, are located the coeff–

coefficients–$2\pi/\sigma_1 \cdot 10^8$ for the unknown current intensities in 20 rings. They depend on σ_1, i.e., on the specific electrical conductivity σ, the sea depth ζ, and the diameter of the sea, if the width of each of the rings Δ is a definite fraction of the mean radius R_0 of the ring nearest the shore. We took $\Delta/R_0 = 1/35$. b) In the lower right part, also along the diagonal, are placed the coefficients at the derivatives of the currents, which are of no independent interest, but serve only to close the system of equations. c) In the upper right and lower left parts of the matrix are grouped quantities that can be retained in solving many problems, since they depend only on how many rings the circular sea is divided into and on the value of the ratio Δ/R_0.

Fig. 1 Fig. 2

Fig. 1 **Fig. 2**

Let us specify a typical amplitude of oscillations $z_0 = 10\gamma = 10^{-4}$ oersted and consider three variants of the problem, relating to a sea for which $R_0 = 7 \cdot 10^7$ cm.

1. The sea depth is constant, $\zeta = 10^5$ cm, the cyclic frequency $\omega = 2 \cdot 10^{-2}$ sec^{-1}.
2. The sea depth is constant, $\zeta = 5 \cdot 10^4$ cm, $\omega = 10^{-2}$ sec^{-1}.
3. The sea depth ζ varies according to a law typical for an inland sea, shown in Fig. 2 under the abscissa axis; $\omega = 2 \cdot 10^{-2}$ sec^{-1}.

For the given matrix and columns of free terms, the equations were solved on the electronic computer of Moscow University by B. I. Volkov for the three variants listed above.

Figure 1 shows the oscillations of the current intensity in various rings, based on the solution of the problem in the 1st variant. The current oscillations in the ring nearest the shore reach an amplitude of 2000 amp. and lag behind the oscillations of the derivative dz/dt by an angle of 146°. The oscillations in the remaining rings rapidly decrease with distance from the shore, and at the same time the phase lag relative to the oscillations of dz/dt also decreases. Consequently, an electromagnetic wave propagates from the center of the sea toward the shore; its amplitude is negligible far from the shore and increases rapidly near the shore, where the propagation velocity of this wave is 300 m/sec; it is of the same order as that obtained in work (1).

Figure 2 shows a summary of the results of the calculations in all three variants. Along the abscissa axis are marked the distances from the center of the sea to the middle of each ring. Along the ordinate axis are given the values of the amplitudes of the electric-field intensity in seawater at the corresponding distance, which are directly measurable in the sea. These quantities are related to the current strength in the rings by the simple relation

$$E_i = \frac{(I_{\text{ampl}})_i}{\Delta\zeta\sigma}. \tag{7}$$

In the 1st variant, at the very shore $E = 60$ mV/km, and with distance from the shore E decreases rapidly—already at a distance of 60 km it falls to 4 mV/km. In the 2nd variant, at the very shore $E = 39$ mV/km; at a distance of 25 km E reaches the maximum value of 41 mV/km and decreases in the open sea considerably more slowly than in the 1st variant: at a distance of 200 km from the shore, $E = 7$ mV/km.

The greatest interest is presented by the 3rd variant. To prepare the numerical values σ_1 entering the corresponding cells of the matrix, the bottom profile, shown by the solid curve below the abscissa axis in Fig. 2, had to be replaced by a stepped line, which is plotted with a dashed line (ζ denotes depths in meters). As we see, the maximum electric-field intensity $E = 60$ mV/km corresponds to that point of curve 3 in Fig. 2 which lies at the edge of the continental shelf (farther on the depths increase rapidly). The electric-field intensity decreases to 51 mV/km at the very shore and falls sharply in the open sea. For example, at a distance of 60 km from the edge of the continental shelf $E = 4.5$ mV/km, and 10 km farther $E = 1$ mV/km.

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Note: Figure translations are in progress. See original paper for figures.