ON THE CALCULATION OF ICE DRIFT AND CURRENTS IN THE ARCTIC BASIN

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Abstract

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ON THE CALCULATION OF ICE DRIFT AND CURRENTS IN THE ARCTIC BASIN

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Let us consider the stationary problem of determining ice drift, current velocity, temperature, and salinity of the water in the Arctic basin. The ice cover, with possible compressions and rarefactions taken into account, will be represented as a continuous medium that behaves as a solid body in the vertical direction and as a fluid in the horizontal directions ([1–6]).

The initial system of equations and boundary conditions is as follows:

$$T_x + R_x + \Omega\rho\, hv + g\rho\, h\frac{\partial\zeta'}{\partial x} - \rho\, h\left(u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y}\right) = 0,$$
$$T_y + R_y - \Omega\rho\, hu + g\rho\, h\frac{\partial\zeta'}{\partial y} - \rho\, h\left(u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y}\right) = 0; \qquad (1)$$

$$A\frac{\partial^2 u}{\partial z^2} + \Omega v = -g\frac{\partial\zeta'}{\partial x} + \frac{g}{\rho_0}\int_0^z \frac{\partial\rho}{\partial x}\,dz + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = 0,$$
$$A\frac{\partial^2 v}{\partial z^2} - \Omega u = -g\frac{\partial\zeta'}{\partial y} + \frac{g}{\rho_0}\int_0^z \frac{\partial\rho}{\partial y}\,dz + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = 0; \qquad (2)$$

$$w = \int_z^H \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\,dz; \qquad (3)$$

$$\zeta' = \zeta - \frac{1}{g\rho}p_a - \frac{\rho}{\rho}h; \qquad (4)$$

$$\iint_\sigma \zeta\,dx\,dy = 0; \qquad (5)$$

$$\frac{\partial}{\partial x}\left(\rho_0 S_x + \rho\, hu\right) + \frac{\partial}{\partial y}\left(\rho_0 S_y + \rho\, hv\right) = 0; \qquad (6)$$

$$u\frac{\partial\tau}{\partial x}+v\frac{\partial\tau}{\partial y}+w\frac{\partial\tau}{\partial z}=\frac{\partial}{\partial z}\left(\varkappa_{\tau z}\frac{\partial\tau}{\partial z}\right)+\frac{\partial}{\partial x}\left(\varkappa_{\tau x}\frac{\partial\tau}{\partial x}\right)+\frac{\partial}{\partial y}\left(\varkappa_{\tau y}\frac{\partial\tau}{\partial y}\right),$$
$$u\frac{\partial s}{\partial x}+v\frac{\partial s}{\partial y}+w\frac{\partial s}{\partial z}=\frac{\partial}{\partial z}\left(\varkappa_{sz}\frac{\partial s}{\partial z}\right)+\frac{\partial}{\partial x}\left(\varkappa_{sx}\frac{\partial s}{\partial x}\right)+\frac{\partial}{\partial y}\left(\varkappa_{sy}\frac{\partial s}{\partial y}\right); \quad (7)$$

$$\rho=f(\tau,s)+\delta z; \quad (8)$$

$$A_a\frac{\partial^2 u_a}{\partial z^2}+\Omega v_a=\frac{1}{\rho_a}\frac{\partial p_a}{\partial x}, \qquad A_a\frac{\partial^2 v_a}{\partial z^2}-\Omega u_a=\frac{1}{\rho_a}\frac{\partial p_a}{\partial y}; \quad (9)$$

for $z=-h+\zeta$

$$u_a=u\,, \qquad v_a=v\,; \quad (10)$$

$$T_x=-\rho_a A_a\frac{\partial u_a}{\partial z}, \qquad T_y=-\rho_a A_a\frac{\partial v_a}{\partial z}; \quad (11)$$

at $z=\zeta$

$$u_1=u, \qquad v_1=v; \quad (12)$$

$$R_x=\rho A\frac{\partial u}{\partial z}, \qquad R_y=\rho A\frac{\partial v}{\partial z}; \quad (13)$$

$$a_\tau^0\frac{\partial\tau}{\partial z}+b_\tau^0\tau=f_\tau^0, \qquad a_s^0\frac{\partial s}{\partial z}+b_s^0 s=f_s^0; \quad (14)$$

at $z=H$

$$u=v=0; \quad (15)$$

$$a_\tau^H\frac{\partial\tau}{\partial z}+b_\tau^H\tau=f_\tau^H, \qquad a_s^H\frac{\partial s}{\partial z}+b_s^{Hs}=f_s^H; \quad (16)$$

at $z\to-\infty$

$$u_a=-\frac{1}{\Omega\rho_a}\frac{\partial p_a}{\partial y}, \qquad v_a=\frac{1}{\Omega\rho_a}\frac{\partial p_a}{\partial x}; \quad (17)$$

on the contour L

$$\rho_0 S_n + \rho_1 h v_{1n} = Q; \tag{18}$$

$$a_\tau^L \frac{\partial \tau}{\partial n} + b_\tau^L \tau = f_\tau^L, \qquad a_s^L \frac{\partial s}{\partial n} + b_s^{Ls} = f_s^L. \tag{19}$$

In (1)–(19), u, v, w are the components of the current velocity along the Cartesian coordinate axes X, Y, Z; T_x, T_y and R_x, R_y are the components of the tangential stresses on the upper and lower surfaces of the ice; p is pressure; ρ is density; τ is temperature; s is salinity; δ is the coefficient of compressibility; g is the acceleration due to gravity; Ω is the Coriolis parameter; A is the kinematic coefficient of vertical exchange of momentum; $\varkappa_\tau$ and $\varkappa_s$ are the coefficients of turbulent diffusion; h is the ice thickness; H is the depth of the basin; $z = \zeta$ is the equation of the lower surface of the ice*; n is the direction of the normal to the contour of the basin L, bounding the area σ; S_x, S_y are the components of the total transport:

$$S_x = \int_\zeta^H u\, dz, \qquad S_y = \int_\zeta^H v\, dz; \tag{20}$$

the subscripts l and a refer to ice and atmosphere; ρ_0 is the mean density of sea water.

The quantities a, b, f, Q must be chosen so that the condition of stationarity of the problem is not violated: there must be no accumulation of heat, salt, or water mass in the basin.

Condition (18) is formulated, as in the case of an ice-free sea (7), on a contour some distance from the coast. On the part of the contour where the normal meets the shore, $Q = 0$ is assumed. On liquid boundaries the quantity Q is in most cases taken equal to the water discharge, since the first term in the left-hand side of (18) is usually considerably larger than the second.

The right-hand sides in (9) are taken to be independent of z. Taking into account that the velocity of the geostrophic wind is approximately 100 times greater than the ice-drift velocity, from (9), (10), (11) we obtain with sufficient accuracy

$$T_x = -\sqrt{\frac{A_a}{2\Omega}} \left(\frac{\partial p_a}{\partial x} + \frac{\partial p_a}{\partial y} \right), \qquad T_y = \sqrt{\frac{A_a}{2\Omega}} \left(\frac{\partial p_a}{\partial x} - \frac{\partial p_a}{\partial y} \right). \tag{21}$$

From (2), (13), (15) we obtain

$$\begin{aligned} u &= -NR_x - MR_y + \Theta\, \partial\zeta'/\partial x + \Lambda\, \partial\zeta'/\partial y + u^*, \\ v &= MR_x - NR_y - \Lambda\, \partial\zeta'/\partial x + \Theta\, \partial\zeta'/\partial y + v^*, \end{aligned} \tag{22}$$

where the conditions (13) have been approximately referred to $z = 0$; the quantities with asterisks are due to the inhomogeneity of the water and the nonlinearity of equations (2) (7, 8).

* The Z-axis is directed vertically downward. The origin of coordinates is chosen so that condition (5) is satisfied.

Substituting (22) into (20) and taking the lower limit of integration in (20) equal to zero, we obtain, with sufficient accuracy,

$$\begin{aligned} S_x &= -nR_x - mR_y + \vartheta\,\partial\zeta'/\partial x + \lambda\,\partial\zeta'/\partial y + S_x^*, \\ S_y &= mR_x - nR_y - \lambda\,\partial\zeta'/\partial x + \vartheta\,\partial\zeta'/\partial y + S_y^*. \end{aligned} \tag{23}$$

Equation (6) makes it possible to introduce the function $\psi(x, y)$:

$$\rho_0 S_x + \rho_l h u_l = -\partial\psi/\partial y, \qquad \rho_0 S_y + \rho_l h v_l = \partial\psi/\partial x. \tag{24}$$

From conditions (12), referred approximately to $z = 0$ (index 0), and (22), we obtain

$$\begin{aligned} u_l &= -N_0 R_x - M_0 R_y + \Theta_0 \partial\zeta'/\partial x + \Lambda_0 \partial\zeta'/\partial y + u_l^*, \\ v_l &= M_0 R_x - N_0 R_y - \Lambda_0 \partial\zeta'/\partial x + \Theta\,\partial\zeta'/\partial y + v_l^*. \end{aligned} \tag{25}$$

Substitute (25) into (1) and solve the resulting system of equations with respect to R_x, R_y:

$$\begin{aligned} R_x = &-(1 + \Omega\rho_l h M_0) b T_x - \Omega\rho_l h N_0 b T_y + \rho_l h b\big[(\Omega\Lambda_0 - g)(1 + \Omega\rho_l h M_0) + \\ &+ \Omega^2 \rho_l h \Theta_0 N_0\big]\partial\zeta'/\partial x + \rho_l h b\big[-\Omega\Theta_0(1 + \Omega\rho_l h M_0) + (\Omega\Lambda_0 - g)\Omega\rho_l h N_0\big] \times \\ &\times \partial\zeta'/\partial y - (1 + \Omega\rho_l h M_0) b (F_u^* + \Omega\rho_l h v_l^*) - \Omega\rho_l h N_0 b (F_v^* - \Omega\rho_l h u_l^*), \\ R_y = \; &\Omega\rho_l h N_0 b T_x - (1 + \Omega\rho_l h M_0) T_y + \rho_l h b\big[-(\Omega\Lambda_0 - g)\Omega\rho_l h N_0 + \\ &+ \Omega\Theta_0(1 + \Omega\rho_l h M_0)\big]\partial\zeta'/\partial x + \rho_l h b\big[\Omega^2\rho_l h \Theta_0 N_0 + \\ &+ (\Omega\Lambda_0 - g)(1 + \Omega\rho_l h M_0)\big]\partial\zeta'/\partial y + \Omega\rho_l h N_0 b (F_u^* + \Omega\rho_l h v_l^*) - \\ &- (1 + \Omega\rho_l h M_0) b (F_v^* - \Omega\rho_l h u_l^*), \end{aligned} \tag{26}$$

where

$$b = \big[(1 + \Omega\rho_l h M_0)^2 + (\Omega\rho_l h N_0)^2\big]^{-1}; \tag{27}$$

$$F_u^* = -\rho_l h\left(u_l \frac{\partial u_l}{\partial x} + v_l \frac{\partial u_l}{\partial y}\right), \qquad F_v^* = -\rho_l h\left(u_l \frac{\partial v_l}{\partial x} + v_l \frac{\partial v_l}{\partial y}\right). \tag{28}$$

We now substitute (26) into (23), (25), and then the result into (24). We obtain

$$c_1\frac{\partial\zeta'}{\partial x}+c_2\frac{\partial\zeta'}{\partial y}=-\frac{1}{b}\frac{\partial\psi}{\partial y}+F_1+F_1^*,$$
$$c_3\frac{\partial\zeta'}{\partial x}+c_4\frac{\partial\zeta'}{\partial y}=-\frac{1}{b}\frac{\partial\psi}{\partial x}+F_2+F_2^*, \tag{29}$$

where

$$\begin{aligned}
c_1 = {} & (\rho_0\vartheta+\rho_l h\Theta_0)/b-(\rho_l hN_0+\rho_0 n)\rho_l h[(\Omega\Lambda_0-g)(1+\Omega\rho_l hM_0)+\\
&+\Omega^2\rho_l h\Theta_0N_0]+(\rho_l hM_0+\rho_0 m)\rho_l h\Omega[(\Omega\Lambda_0-g)\rho_l hN_0-\\
&-\Theta_0(1+\Omega\rho_l hM_0)],\\
c_2 = {} & (\rho_0\lambda+\rho_l h\Lambda_0)/b+(\rho_0 n+\rho_l hN_0)\rho_l h\Omega[\Theta_0(1+\Omega\rho_l hM_0)-\\
&-(\Omega\Lambda_0-g)\rho_l hN_0]-(\rho_0 m+\rho_l hM_0)\rho_l h[\Omega^2\rho_l h\Theta_0N_0+\\
&+(\Omega\Lambda_0-g)(1+\Omega\rho_0 hM_0)],\\
c_3 = {} & -(\rho_0\lambda+\rho_l h\Lambda_0)/b+(\rho_0 m+\rho_l hM_0)\rho_l h[(\Omega\Lambda_0-g)(1+\Omega\rho_l hM_0)+\\
&+\Omega^2\rho_l h\Theta_0N_0]+(\rho_0 n+\rho_l hN_0)\rho_l h\Omega[(\Omega\Lambda_0-g)\rho_l hN_0-\\
&-\Theta_0(1+\Omega\rho_l hM_0)],\\
c_4 = {} & (\rho_0\vartheta+\rho_l h\Theta_0)/b-(\rho_0 m+\rho_l hM_0)\rho_l h\Omega[\Theta_0(1+\Omega\rho_l hM_0)-\\
&-(\Omega\Lambda_0-g)\rho_l hN_0]-(\rho_0 n+\rho_l hN_0)\rho_l h[\Omega^2\rho_l h\Theta_0N_0+\\
&+(\Omega\Lambda_0-g)(1+\Omega\rho_l hM_0)];
\end{aligned} \tag{30}$$

$$\begin{aligned}
F_1 = {} & [-(\rho_0 n+\rho_l hN_0)(1+\Omega\rho_l hM_0)+(\rho_0 m+\rho_l hM_0)\Omega\rho_l hN_0]T_x-\\
&-[(\rho_0 n+\rho_l hN_0)\Omega\rho_l hN_0+(\rho_0 m+\rho_l hM_0)(1+\Omega\rho_l hM_0)]T_y,
\end{aligned} \tag{31}$$

$$\begin{aligned}
F_2 = {} & [(\rho_0 m+\rho_\ell hM_0)(1+\Omega\rho_\ell hM_0)+(\rho_0 n+\rho_\ell hN_0)\Omega\rho_\ell hN_0]T_x+\\
&+[(\rho_0 m+\rho_\ell hM_0)\Omega\rho_\ell hN_0-(\rho_0 n+\rho_\ell hN_0)(1+\Omega\rho_\ell hM_0)]T_y;
\end{aligned} \tag{31}$$

$$\begin{aligned}
F_1^* = {} & -(\rho_0 S_x^*+\rho_\ell hu_\ell^*)/b-(\rho_0 n+\rho_\ell hN_0)[(F_u^*+\Omega\rho_\ell hv_\ell^*)(1+\Omega\rho_\ell hM_0)+\\
&+(F_v^*-\Omega\rho_\ell hu^*)\Omega\rho_\ell hN_0]+(\rho_0 m+\rho_\ell hM_0)[(F_u^*+\Omega\rho_\ell hv_\ell^*)\Omega\rho_\ell hN_0-\\
&\quad-(F_v^*-\Omega\rho_\ell hu_\ell^*)(1+\Omega\rho_\ell hM_0)],\\
F_2^* = {} & -(\rho_0 S_y^*+\rho_\ell hv_\ell^*)/b+(\rho_0 m+\rho_\ell hM_0)[(F_u^*+\Omega\rho_\ell hv_\ell^*)(1+\Omega\rho_\ell hM_0)+\\
&+(F_v^*-\Omega\rho_\ell hu^*)\Omega\rho_\ell hN_0]+(\rho_0 n+\rho_\ell hN_0)[(F_u^*+\Omega\rho_\ell hv_\ell^*)\Omega\rho_\ell hN_0-\\
&\quad-(F_v^*-\Omega\rho_\ell hu^*)(1+\Omega\rho_\ell hM_0)].
\end{aligned} \tag{32}$$

Solving (29) with respect to $\partial\xi'/\partial x$ and $\partial\xi'/\partial y$,

$$\begin{aligned}\partial\xi'/\partial x &= -c_2'\,\partial\psi/\partial x - c_4'\,\partial\psi/\partial y + bc_4'F_1 - bc_2'F_2 + bc_4'F_1^* - bc_2'F_2^*,\\ \partial\xi'/\partial y &= c_1'\,\partial\psi/\partial x + c_3'\,\partial\psi/\partial y + bc_1'F_2 - bc_3'F_1 + bc_1'F_2^* - bc_3'F_1^*,\end{aligned} \tag{33}$$

where

$$c_i' = c_i/b\,(c_1c_4 - c_2c_3) \qquad (i = 1, 2, 3, 4). \tag{34}$$

Eliminating ξ' from (33), we obtain the basic equation

$$\begin{aligned}\frac{\partial}{\partial x}\left(c_1'\frac{\partial\psi}{\partial x}\right) + \frac{\partial}{\partial y}\left(c_2'\frac{\partial\psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(c_3'\frac{\partial\psi}{\partial y}\right) + \frac{\partial}{\partial x}\left(c_4'\frac{\partial\psi}{\partial y}\right) &=\\ = \frac{\partial}{\partial y}(bc_4'F_1 - bc_2'F_2) + \frac{\partial}{\partial y}(bc_4'F_1^* - bc_2'F_2^*)&\\ - \frac{\partial}{\partial x}(bc_1'F_2 - bc_3'F_1) - \frac{\partial}{\partial x}(bc_1'F_2^* - bc_3'F_1^*).&\end{aligned} \tag{35}$$

The boundary condition (18) can be written in the form

$$\psi = Q'(L), \tag{36}$$

where Q' is a known function.

If the fluid is homogeneous and the problem is linear, then all quantities with an asterisk vanish. In this case equation (35) is first solved under condition (36), and then all the sought functions are calculated from the formulas obtained, analogously to how this is done for an ice-free sea ([9]). The nonlinear problem for a homogeneous fluid is solved by the method of successive approximations ([10]). If the density field is known from observations*, then the problem is solved in an analogous way. Finally, if the temperature and salinity fields are to be found, then a system of equations is solved that includes (7), (8) ([8]). We note that in this case it is important to choose correctly the dependence of χ_{tz} and χ_{sz} on z. It can be determined in advance when solving the problem from the temperature and salinity fields of seawater known from observations, if horizontal diffusion is neglected.

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* A similar problem for an ice-free ocean is considered in ([11]).

Note: Figure translations are in progress. See original paper for figures.