LOCAL APPROXIMATION IN THE ELECTRODYNAMICS OF LONDON SUPERCONDUCTORS

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Abstract

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LOCAL APPROXIMATION IN THE ELECTRODYNAMICS OF LONDON SUPERCONDUCTORS

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Current states in superconductors on the basis of microscopic theory were first considered by J. Bardeen ([1]). A more general approach, taking into account the Meissner effect, was given in a paper by V. P. Galaiko ([2]); along these lines, for London superconductors one obtains the relations of the two-fluid model and an equation for the order parameter that takes into account the change of this parameter as a function of the magnitude of the superfluid velocity.

In the present article a simpler method is proposed for obtaining the basic relations of the theory, one that admits a direct generalization to the nonequilibrium case.

It will be convenient for us to start from the system of equations for Green' s functions describing a superconductor in a magnetic field ([3]):

$$\left(E-\frac{1}{2m}\left(\hat{\mathbf{p}}-\frac{e}{c}\mathbf{A}\right)^2+\mu\right)G(E,\mathbf{r}_1,\mathbf{r}_2)+\Delta(\mathbf{r}_1)F(E,\mathbf{r}_1,\mathbf{r}_2)=\frac{\delta(\mathbf{r}_1-\mathbf{r}_2)}{2\pi},$$

$$\left(E+\frac{1}{2m}\left(\hat{\mathbf{p}}+\frac{e}{c}\mathbf{A}\right)^2-\mu\right)F(E,\mathbf{r}_1,\mathbf{r}_2)+\Delta^*(\mathbf{r}_1)G(E,\mathbf{r}_1,\mathbf{r}_2)=0. \qquad (1)$$

Here G and F are the Fourier transforms in time of the two-time temperature Green' s functions ([4]); Δ is the order parameter, defined in the usual way. In London superconductors the physical quantities vary slowly over distances of the order of the pair-correlation radius ξ_0. This makes it possible to seek solutions of equations (1) in the form of gradient expansions. It is first necessary to extract the gauge-invariant factors. Let

$$G(\mathbf{r}_1,\mathbf{r}_2)=\widetilde{G}(\mathbf{r}_1,\mathbf{r}_2)\exp\left[\frac{i}{\hbar}m(\chi(\mathbf{r}_1)-\chi(\mathbf{r}_2))\right],$$

$$F(\mathbf{r}_1, \mathbf{r}_2) = \widetilde{F}(\mathbf{r}_1, \mathbf{r}_2) \exp\left[\frac{i}{\hbar} m(\chi(\mathbf{r}_1) + \chi(\mathbf{r}_2))\right], \tag{2}$$

$$\Delta(\mathbf{r}) = |\Delta(\mathbf{r})| \exp\left[\frac{2i}{\hbar} m\chi(\mathbf{r})\right], \qquad |\Delta(\mathbf{r})| \equiv d(\mathbf{r}).$$

Define the superfluid velocity by the relation $\mathbf{v}_s = \nabla\chi - \frac{e}{mc}\mathbf{A}$, and pass to the mixed representation for the Green' s functions, putting $\mathbf{R} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)$, $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and performing a Fourier transformation with respect to $\mathbf{r}$. System (1) is rewritten in the form

$$\left\{E - \frac{1}{2m}\left[\mathbf{p} - \frac{i\hbar}{2}\nabla_R + m\mathbf{v}_s\left(\mathbf{R} + \frac{i\hbar}{2}\nabla_p\right)\right]^2 + \mu\right\} G(E, \mathbf{R}, \mathbf{p})$$

$$= \frac{1}{2\pi} - d\left(\mathbf{R} + \frac{i\hbar}{2}\nabla_p\right) F(E, \mathbf{R}, \mathbf{p}),$$

$$\left\{E + \frac{1}{2m}\left[\mathbf{p} - \frac{i\hbar}{2}\nabla_R - m\mathbf{v}_s\left(\mathbf{R} + \frac{i\hbar}{2}\nabla_p\right)\right]^2 - \mu\right\} F(E, \mathbf{R}, \mathbf{p}) \tag{3}$$

$$= -d\left(\mathbf{R} + \frac{i\hbar}{2}\nabla_p\right) G(E, \mathbf{R}, \mathbf{p}).$$

To these equations one should add the definition of the current $\mathbf{j}$ and of the ordering parameter Δ

$$\mathbf{j}(\mathbf{R}) = 2\,\frac{e}{m}\int \frac{d\mathbf{p}}{(2\pi\hbar)^3}(\mathbf{p}+m\mathbf{v}_s)\int_{-\infty}^{\infty}\frac{i\,dE}{e^{E/\theta}+1}\{G(\mathbf{R}, \mathbf{p}, E+i\varepsilon) - G(\mathbf{R}, \mathbf{p}, E-i\varepsilon)\}, \tag{4}$$

$$d(\mathbf{R}) = g\int \frac{d\mathbf{p}}{(2\pi\hbar)^3}\int_{-\infty}^{\infty}\frac{i\,dE}{e^{E/\theta}+1}\{F^+(\mathbf{R}, \mathbf{p}, E+i\varepsilon) - F^+(\mathbf{R}, \mathbf{p}, E-i\varepsilon)\}. \tag{5}$$

In zeroth order in the gradients, from (3), (4), and (5) we obtain

$$d(\mathbf{R}) = \frac{g}{2}\int \frac{d(\mathbf{R})}{E_p}\left\{1 - f\left(\frac{E_p + \mathbf{p}\mathbf{v}_s}{\theta}\right) - f\left(\frac{E_p - \mathbf{p}\mathbf{v}_s}{\theta}\right)\right\}\frac{d\mathbf{p}}{(2\pi\hbar)^3}, \tag{6}$$

$$\mathbf{j} = \frac{e}{m}\rho\mathbf{v}_s - \mathbf{j}_n(\mathbf{v}_s) \equiv \frac{e}{m}\rho_s(\mathbf{v}_s)\mathbf{v}_s, \tag{7}$$

$$\mathbf{j}_n(\mathbf{v}_s) = 2\,\frac{e}{m}\int \mathbf{p}\, f\left(\frac{E_p - \mathbf{p}\mathbf{v}_s}{\theta}\right)\frac{d\mathbf{p}}{(2\pi\hbar)^3} = \frac{e}{m}\rho_n(\mathbf{v}_s)\mathbf{v}_s. \tag{8}$$

Here $E_p = \sqrt{\xi_p^2 + d^2}$, $f(x) = (e^x+1)^{-1}$, and g is the coupling constant.

In connection with the results obtained, let us note several circumstances which, in our view, are interesting and to which insufficient attention was paid in works ([1],[2]).

It is easy to verify that $\rho_s(\mathbf{v}_s)$ vanishes together with d. On the other hand, the equation for d has a nontrivial solution only for $v_s \leq v_s^c$. We emphasize that v_s^c exceeds in magnitude the value of the critical velocity determined by Landau' s criterion and equal to d_0/p_F. Equation (6) is readily investigated for the case of zero temperature. It turns out that d does not depend on v_s for $v_s < d_0/p_F$ and decreases rapidly for $d_0/p_F < v_s < v_s^c \simeq 1.36 d_0/p_F$. For $v_s = 0$ the ordering parameter d_0 played the role of a gap in the spectrum of elementary excitations of the system. In the case when $v_s \neq 0$, the gap in the spectrum is equal to $d_0 - p_{Fv}s$ for $v_s < d_0/p_F$ and to zero for $v_s > d_0/p_F$. In the interval of velocities $d_0/p_F < v_s < v_s^c$ we have gapless superconductivity. Although for $v_s > d_0/p_F$ the creation of excitations is energetically favorable, an immediate destruction of the current state does not occur, since the energy interval in which excitations can be created is finite and is determined by the inequality

$$|\xi| \leq \sqrt{(p_{Fv}s)^2 - d^2},$$

and, by virtue of the Pauli principle, only a finite number of excitations can arise in a finite interval. Only when sufficiently many excitations have accumulated will the current turn to zero.

Let us note, however, that in a massive superconductor the observation of gapless superconductivity is made difficult by the fact that even before the Landau velocity is reached in a London superconductor, the formation of a system of Abrikosov vortex filaments becomes possible, as a result of which the basic assumption of small gradients and the consequences based on it lose their force.

In order that the basic relations (6), (7) be closed, it is necessary to add to them Maxwell' s equations. Taking into account the connection of the current with the thermodynamic potential Ω,

$$\mathbf{j} = -c\,\frac{\delta\Omega}{\delta\mathbf{A}} = \frac{e}{m}\frac{\delta\Omega}{\delta\mathbf{v}_s}, \qquad \mathbf{H} = -\frac{mc}{e}\,\mathrm{rot}\,\mathbf{v}_s, \tag{9}$$

we obtain, for the case of a semi-infinite superconductor (the axis is directed into the depth of the metal),

$$\frac{d^2 v_s}{dz^2} = 4\pi\left(\frac{e}{mc}\right)^2\frac{d\Omega}{dv_s}. \tag{10}$$

This equation admits the first integral

$$\frac{1}{8\pi}H^2(z) = \Omega(z) - \Omega(\infty). \tag{11}$$

The constant of integration is found from the boundary condition—the vanishing of v_s at infinity. Thus:

$$\frac{1}{8\pi}H^2(z) = \Omega(v_s(z),\, d(v_s(z))) - \Omega(0, d_0). \tag{12}$$

From this follows the well-known relation for the thermodynamic critical field and the Silsbee rule. Of course, the reservations made above, associated with the appearance of vortex filaments, also remain valid with respect to this conclusion.

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Note: Figure translations are in progress. See original paper for figures.