ON THE FATIGUE OF METALS

THEORY OF ELASTICITY

1967

SovietRxiv

Abstract

Full Text

UDC 539.43

THEORY OF ELASTICITY

T. B. LARINA

ON THE FATIGUE OF METALS

(Presented by Academician L. I. Sedov on 12 I 1967)

In paper (1), to describe the behavior of materials with complex properties it is proposed to use nonholonomic models of continuous media. As the thermodynamic parameters of such media for a one-dimensional problem one may choose $\varepsilon, \sigma, T, \chi$. Here ε is strain, σ is stress, T is absolute temperature, χ is some additional parameter depending not only on ε, σ, T, but also on their derivatives. From the nonholonomic relation

$$d\sigma = A\,d\chi + B\,dT + C\,d\varepsilon,$$

putting $A = g(\sigma, \varepsilon, T)/\chi, \quad B = 0, \quad C = f(\chi)$, we obtain an equation of state of the form

$$\partial\sigma/\partial t = f(\chi)\,\partial\varepsilon/\partial t + g(\sigma, \varepsilon, T).$$

Let $\chi = \sigma f_1(T)/f_1(T_0) - a$; here a is a quantity constant along the characteristic;

$$a = M|\dot{\varepsilon}| \left|\frac{\sigma_\mathrm{d} - \sigma}{\sigma - \varphi(\varepsilon)}\right|_{\chi=0};$$

x is the coordinate of a semi-infinite rod;

$$f(\chi) = \begin{cases} E, & \chi \leqslant \varepsilon_0, \\ \varphi'(\chi), & \chi > \varepsilon_0; \end{cases}$$

$\varphi(\varepsilon)$ is the static diagram; $\varepsilon_0, \sigma_0, \varepsilon_*, \sigma_*$ are the static and dynamic yield limits, respectively; $\sigma_\mathrm{d}(\varepsilon) = \varphi[\varepsilon - (\varepsilon_* - \varepsilon)] + \sigma_* - \sigma_0$ is the limiting dynamic diagram; T_0 is the ambient temperature.

In fatigue fracture the decisive role is played by heating of the specimen, since in the case when the temperature of the specimen was maintained constant, fracture did not occur (2). The rise in the temperature of the specimen is the result of competition (3) between hysteretic heat generation and heat exchange. The amount of heat released is proportional to the work of deformation irreversibly expended owing to hysteresis: $q_+ \sim \sigma_1 \varepsilon_1 \sin\delta$; here σ_1, ε_1 are respectively the amplitudes of stress and strain, and δ is the phase-shift angle between them. The width of the hysteresis loop changes noticeably during the first thousands of cycles, and then remains constant ([2]). In terms of strain this width has the following form: $\Delta\varepsilon = 2\varepsilon_1 \sin\delta$; on the other hand, its dependence on the number of cycles is $\Delta\varepsilon = A_1/n^\beta + 2a$ ([2]). Consequently, as $n \to \infty$, $q_+ \sim \sigma_1 a$, and per unit time, owing to the work of hysteresis, $Q_+ = B_1 \sigma_1 a \omega$ will be released, and the temperature

$$T_k = T_0 + \frac{B_1}{\varkappa} \omega \sigma_1 a$$

will be established; here T_0 is the ambient temperature, $\varkappa$ is the heat-transfer coefficient, and ω is the frequency of the applied load. As long as $\sigma_1 f_1(T)/f_1(T_0) \leqslant \varepsilon_0$, by definition $f(\chi) = E$, and our equation describes an elastic medium.

Let us find the limiting value σ_1^* at which the medium will still remain elastic. Let $f_1(T) = e^{(T-T_*)^2}$; then from the equation

$$\sigma_1^* \exp[(T_k - T_*)^2]/\exp[(T_0 - T_*)^2] = \varepsilon_0$$

we obtain

$$\sigma_1^* \exp\left[2(T_0 - T_*)\frac{B_1}{\varkappa}\omega\alpha\sigma_1^*\right] \exp\left[\left(\frac{B_1}{\varkappa}\omega\alpha\sigma_1^*\right)^2\right] = \varepsilon_0.$$

Taking the logarithm of this expression and solving it, for example graphically, it is easy to see that when $T_* = 0$, σ_1^* decreases with increasing ambient temperature T_0, which corresponds to the behavior of the fatigue limit for copper, while when $T_* \neq 0$, σ_1^* first increases with increasing T_0 and then decreases, which corresponds to the behavior of the fatigue limit for steel and aluminum ([2]). With increasing frequency, σ_1^* decreases. In our case σ_1^* is not the true value of the fatigue limit in the usual sense, but for $\sigma < \sigma_1^*$ fracture does not occur. The true value of the fatigue limit may prove to be higher than σ_1^*; it is determined by the end of the process.

In the case when $\sigma_1 f_1(T)/f_1(T_0) = \sigma_2 > \varepsilon_0$, for values

$$\chi = \sigma_2 - M|\dot{\varepsilon}| \left|\frac{\sigma_\text{d} - \sigma}{\sigma - \varphi(\varepsilon)}\right| > \varepsilon_0$$

$f(\chi) \neq E$, and plastic deformation takes place. On the decreasing part of the cycle the ratio $|\dot{\varepsilon}|/|\sigma - \varphi(\varepsilon)|$ increases, the argument χ decreases and, since

$\varphi'(\chi)$ is a monotonically decreasing function, the characteristics will begin to catch up with one another and will combine into a shock wave.

In the second law of thermodynamics $T\,dS = dQ + dQ'$, let us put the irreversible part $dQ' = \psi(\varepsilon^P)\,d\varepsilon^P$, where ε^P is the plastic component of the deformation ε, and $\psi(\dot{\varepsilon}^P)$ must have the same sign as $\dot{\varepsilon}^P$ and for $\dot{\varepsilon}^P = \infty$ must not tend to zero or infinity. These conditions are satisfied, for example, by the function $\psi(\dot{\varepsilon}^P) = C \operatorname{arctg} D \cdot \dot{\varepsilon}^P$. Then $dQ' \geq 0$, and as shock waves pass through the material, irreversible changes accumulate which, upon reaching some value Q'_*, lead to the formation of microcracks. Because the intensity of a shock wave changes along its front, and we consider all shock waves identical, the first microcrack arises where the wave intensity is maximal. Hence it becomes clear why, other conditions being equal, large parts fail faster than small ones. In large parts the shock wave has time to reach a greater intensity. The number of cycles required for the formation of microdefects can be determined from the relation

$$Q'_* = n\psi(\infty)\Delta\varepsilon^P$$

(where $\Delta\varepsilon^P$ is the jump of ε^P on the shock wave).

This point of view has an experimental basis. It is known ([2]) that the accumulation of an excess number of dislocations is related to the rate of deformation by $\dot{\varepsilon} \sim (C - C_0)/C_0$, where C_0 is the initial concentration of micropores. Hence it is seen that, first, a shock wave is a place of accumulation of micropores and, second, the number of micropores along the shock wave can be counted, since from the microscopic point of view a shock wave has transverse dimensions and therefore $\dot{\varepsilon} \neq \infty$. In the formation of cracks, increasing the frequency promotes an increase in the longevity of the body. As $\omega \to \infty$, the cyclic loading tends to a point application of the load. In this case, instead of shock waves, fans of characteristics are formed, and the formation of cracks is delayed.

For a final solution of the problem it is necessary to consider the problem of fracture of a body permeated by microcracks of different sizes.

Under loading with a nonzero mean stress, creep is also observed in fatigue; therefore it is convenient to choose the function $g(\sigma, \varepsilon, T)$ as was done in the description of creep ([4]):

$$g(\sigma, \varepsilon, T) = C_1 e^{-Q/RT} \left\{ (\sigma - C_2) \left[\frac{\varepsilon - A}{B} \right]^2 + C_2 \frac{A^2}{B^2} \right\}.$$

The author expresses his gratitude to V. A. Iselevich and A. M. Khazen for their constant attention to the work.

Research Institute of Mechanics
Moscow State University

named after M. V. Lomonosov

Received
4 I 1967

REFERENCES

1. L. I. Sedov, M. E. Eglit, *DAN*, **142**, No. 1 (1962).

2. A. J. Kennedy, *Creep and Fatigue in Metals*, 1965.

3. S. B. Ratner, V. I. Korobov, *DAN*, **161**, No. 4 (1965).

4. T. B. Larina, *DAN*, **175**, No. 5 (1967).

Note: Figure translations are in progress. See original paper for figures.