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ON THE QUESTION OF THE RADIATION OF ELEMENTARY PARTICLES

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In quantum field theory, noninteracting elementary particles are regarded as pointlike. Under interactions, a particle is as if "smeared out," and then one may ascribe to it a definite radius of interaction. It is obvious that one can "screen oneself" from all types of interaction with an external field except the gravitational one. Therefore particles will be "smeared out" in the gravitational field and will possess definite dimensions depending on the "constant" of gravitation and Planck' s constant. The gravitational radius

$$r_g = 2Gm/c^2 \tag{1}$$

cannot claim to be the size of a particle in the gravitational field, and $\hbar$ does not enter into it.

From the quantities $\hbar, G, c$, as Planck already showed, one can form a new quantity having the dimension of length ([1–4]),

$$L = \sqrt{G\hbar/c^3} \approx 10^{-33} \text{ cm}. \tag{2}$$

According to the assumption of L. D. Landau ([4]), this quantity L may be the characteristic size of an elementary particle (electron, nucleon) in the gravitational field or, more precisely, characterize the size of the region of interaction with the gravitational field.

It is easy to show that L coincides with the quantity L^*, calculated as the characteristic dimensions of metric (size) fluctuations in the Metagalaxy. In this case fluctuations of momentum and angular momentum will also be observed. Their relative magnitude will likewise be of the order $L/r_0 \approx 10^{-20}$. Indeed, since the mean-square fluctuation of some quantity f (or operator $\hat{f}$) can be written in the form

$$\overline{(\Delta f)^2} = \overline{f^2} - (\bar{f})^2, \tag{3}$$

and for an equilibrium macroscopic system $\bar{f} = 0$, we shall have

$$\overline{(\Delta f)^2} = \overline{f^2}. \tag{4}$$

It follows that the mean-square fluctuation of the velocity of particles, arising through interaction with the gravitational field,

$$\overline{v^2} = 3kT_g/m, \tag{5}$$

where $kT_g = E_g = \hbar c/a$ is the energy of a quantum of the gravitational field; T_g is the temperature of the gravitational field. From (5) we have $\overline{v^2}/c^2 = 3E_g/E_p = 10^{-40}$, whence

$$\sqrt{\overline{\Delta v^2}}/c = v^*/c = 10^{-20}. \tag{6}$$

Similarly, for fluctuations of the angular momentum of particles in the gravitational field we shall have:

$$\overline{M^2} = 2E_g I_p \approx \frac{3kT_g}{m} m^2 r^2 = 3E^2_{gmr} = \hbar^2 \frac{3E_g}{E_p} \approx 10^{-40}\hbar^2; \tag{7}$$

$$\sqrt{\overline{\Delta M^2}} = \overline{M^*} = \hbar \cdot 10^{-20}, \tag{8}$$

where $I_p = \text{const} \cdot mr^2$ is the moment of inertia of the nucleon.

Obviously, all components of the quadratic fluctuations of the coordinates, momenta, and angular momenta will be the same.

The study of the motion of particles with spin $s = 1/2$ in an external field, based only on solving the Dirac equation, is in general not a correct problem. In order to make the problem correct, it is necessary to solve jointly the quantized equation of the field as well. However, for weak fields, such as the gravitational field, a classical or quasiclassical treatment of the problem is quite legitimate.

As was shown by N. N. Bogolyubov and S. V. Tyablikov ([5]), in an electromagnetic (Coulomb) field the dimensions of a particle (electron), characterized by its “trembling” due to interaction with the field, are determined by the expression

$$r_e^* = \sqrt{r_k r_e}, \tag{9}$$

where $r_k = \lambda = \hbar/mc$ is the Compton wavelength; $r_e = e^2/mc^2$ is the classical electromagnetic radius of the particle. Thus, $r_e^2 = \sqrt{\alpha\hbar}/mc$, where $\alpha = e^2/\hbar c = 1/137$.

Analogously, one can establish that the dimensions of a particle in a gravitational field, characterized by its "trembling," are in this field determined by the expression

$$r_g^* = \sqrt{r_g r_k}, \tag{10}$$

where $r_g = 2Gm/c^2$ is the classical gravitational radius of the particle. Thus,

$$r_g^* = \sqrt{G\hbar/c^3} = L, \tag{11}$$

which makes justified the assumptions made above concerning the dimensions of particles in a gravitational field.

For $G = 0$ or for $\hbar = 0$, $L = 0$, and the correspondence principle is fulfilled. It now makes sense to pass from the Dirac representation, which describes the motion of particles with spin $s = 1/2$, to the so-called Foldy–Wouthuysen (F.–W.) representation ([6,7]), where one can introduce the coordinate operator of the particle, which could not be done in the old Dirac representation.

The mean value of the coordinate operator in the F.–W. representation is

$$\langle \hat{x} \rangle = \hat{x} - \frac{\hbar c^2}{2E_0} \frac{[\hat{\sigma} \times \hat{p}]}{E_0 + mc^2} = \hat{x} + \Delta\hat{x}, \tag{12}$$

where $E_0 = c\sqrt{p^2 + m^2c^2}$; σ are the Dirac (Pauli) matrices; p is the momentum, and here the even part of the operator is taken, since upon averaging the odd part $\approx c^2p^2/E_0^2$ gives zero. In the absence of any external field and interaction with it, $p = 0$, $\Delta\hat{x} = 0$.

In the case of an electromagnetic field

$$p \approx mc, \qquad E_0 \approx mc^2, \qquad \Delta\hat{x} \approx \hat{x};$$

in the case of a gravitational field

$$p \approx mcL/r_0, \qquad E_0 \approx mc^2, \qquad \Delta\hat{x} \approx \hat{x}L/r_0, \qquad \langle \hat{x} \rangle = \hat{x}(1 + \text{const} \cdot L/r_0). \tag{13}$$

Thus, the coordinate fluctuations are of order L, the relative fluctuation is of order L/r_0, which agrees with the results obtained above.

In the Dirac representation the sums of the total angular momentum and spin commute with the Hamiltonian; in the F.–W. representation the separate quantities commute,

$$\langle \hat{M} \rangle = [\hat{x} \times \hat{p}], \qquad \langle \hat{s} \rangle = \hat{s} - \frac{i\beta c[\hat{\alpha} \times \hat{p}]}{E_0}. \tag{14}$$

The quantity $\langle \hat{s} \rangle$ is called the mean-spin operator. The quantity $\langle \hat{M} \rangle$ may be called the mean-moment operator. Here again the even part of the operator is taken (the odd part $\simeq c^2p^2/E_0^2$ gives zero upon averaging). Thus, in the F.–V. representation the quantity

$$\langle \hat{s} \rangle = \hat{s} \left(1 + \mathrm{const}\sqrt{G\hbar/c^3 r_0^2}\right) = \hat{s} \left(1 + \mathrm{const}\sqrt{3E_g/E_p}\right) \tag{15}$$

is conserved. In this case, indeed, a quantity of order $\sqrt{G\hbar/c^3 r_0^2} \simeq 10^{-20}$ is added to the total moment.

The velocity operator

$$\langle \overline{v} \rangle = \frac{d\langle \hat{x} \rangle}{dt} = \frac{1}{i\hbar}\,[\hat{x}, \hat{H}_{\Phi.-V.}] = \hat{\beta}\,\frac{c^2 p}{E_0}. \tag{16}$$

The Hamiltonian in the F.–V. representation is

$$\hat{H}_{\Phi.-V.} = \hat{\beta} E_0 = \hat{\beta} c\sqrt{p^2 + m^2c^2}. \tag{17}$$

Hence

$$\langle \overline{v} \rangle / c = \hat{\beta} cp/E_0 \simeq \hat{\beta} 3E_g/E_0 \tag{18}$$

or

$$v_{\Phi.-V.}/c = 3E_g/E_0 = 10^{-40}.$$

It is obvious that $v_{\Phi.-V.}/c = v^{*2}/c^2$, or $v^*/c = \sqrt{v_{\Phi.-V.} v_D/c^2}$, where $d\hat{x}_D/c\,dt = v_D/c = 1$ is the velocity in the Dirac representation.

It follows that the velocity in the Dirac representation always corresponds only to the electromagnetic dimensions of the particle, while in the case of the F.–V. representation the velocity corresponds to the field in which the particle is located. The root-mean-square velocity, like the dimensions of the particle in a given field, corresponds to the geometric mean of the velocities or dimensions of the trembling inherent in the given and electromagnetic fields.

Since in the Dirac representation, for the time derivative of the even part of the coordinate operator, one may also write the expression

$$\langle \hat{x} \rangle = \hat{x} + \frac{i\hbar c\overline{\lambda}}{2E_0} - \frac{i\hbar cp^2}{2E_0^2} = \hat{x} + \frac{ic}{\omega}\left(1 - \frac{cp}{E_0}\right), \tag{19}$$

where

$$\overline{\lambda} = \hat{H}_D/\sqrt{H_D^2} = (c\hat{\alpha}\hat{p} + \hat{\beta}mc^2)/c\sqrt{p^2 + m^2c^2}, \tag{20}$$

it becomes obvious from this that the "trembling" of the particle occurs in any field (at any impulse) with frequency $\omega = 2mc^2/\hbar$; for a nucleon this gives $\omega = 10^{-23}$ sec^{-1}, i.e., with the frequency of strong interactions. In the Dirac representation the trajectory of the particle is oscillatory, but with spin $s = 1/2$; in the F.–V. representation the trajectory is non-oscillatory, but the spin is "smeared" ($s^* \neq 1/2$), which leads to a quantum quadrupole moment for this particle different from zero.

In conclusion it is easy to show that, by considering fluctuations of the metric of the initial quasi-Euclidean space, one easily arrives at a nonfluctuating Riemannian space.

At a distance from the "center" of the nucleon $r = r_0$, the fluctuation magnitude is $L_r = L$, while at a distance $r \geq r_0$ the value is $L_r = L\sqrt{r_0/r}$.

This expression is obvious, since

$$\frac{\overline{v}_r^2}{c^2} = \frac{L_r^2}{r_0^2} = \frac{kT_r}{m_{pc}^2} = \frac{kT_0}{m_{pc}^2}\frac{r_0}{r}, \qquad \frac{T_r}{T_0} = \frac{r_0}{r}, \qquad \frac{L_r^2}{L^2} = \frac{r_0}{r}. \tag{21}$$

The relative magnitude of the fluctuations falling on the dimensions of the nucleon will be L_r/r_0, and the absolute magnitude of the fluctuations themselves falling at the distance r will be

$$\xi \simeq L_r r/r_0, \tag{22}$$

the mean value $\overline{\xi} = 0$, which is obvious; hence it also follows that

$$\xi^2 = r_{gr}0 = Lr/r_0. \tag{23}$$

Now one can write that $\xi^\alpha = n^\alpha \xi$, where n^α are the components of a unit vector.

Further, it is obvious that the interval can be written in the form

$$-ds^2 = -dx_0^2 + g_{0\alpha\beta}(dx^\alpha + d\xi^\alpha)(dx^\beta + d\xi^\beta) = g_{\alpha\beta}dx^\alpha dx^\beta - dx_0^2, \tag{24}$$

which gives

$$g_{lm} = g_{0lm} + \frac{\partial \xi_l}{\partial x^m} + \frac{\partial \xi_m}{\partial x^l} + \frac{\partial \xi_\alpha}{\partial x^l}\frac{\partial \xi^\alpha}{\partial x^m}. \tag{25}$$

Thus, knowing the magnitudes of the fluctuations of the particle "dimensions," one can find the "corrections" to the initial quasi-Euclidean metric, i.e., these fluctuations make the metric Riemannian.

The development of these ideas automatically leads to the fact that the quantity $1/\alpha = hc/e^2 = \ln(a/L) = \ln T_m^{3/2}$; the electron mass decreases with time according to the law $m_e = \dfrac{m_{0e}}{T_m^2 \ln T_m^{3/2}}$; the photon energy $E_\phi = E_{0\phi}/T_m^2$; the ratio

$$\frac{m_p}{m_e} \simeq \frac{1}{\alpha} \ln\left(\frac{1}{\alpha}\frac{m_p}{m_e}\right),$$

whence $m_p/m_e \approx 1800$; the strong-interaction constant

$$\frac{g^2}{\hbar c} = \ln\left(\frac{1}{\alpha}\frac{m_p}{m_e}\right) = 13.5.$$

Here $T_m = a/r_0 = \sqrt{1/Rr_0^2}$ is the world Dirac time, where R is the curvature.

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