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PHYSICS

1967

SovietRxiv

Abstract

Full Text

UDC 536.758

PHYSICS

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CORRECTIONS ΔF, Δn, AND $\Delta\mu$ IN THE METHOD OF ASYMPTOTIC EXPANSIONS IN THE MODIFIED FORMULATION OF THE PROBLEM OF A NONIDEAL BOSE-EINSTEIN SYSTEM

(Presented by Academician N. N. Bogolyubov, 10 XI 1966)

In ([1–3]) a method was developed for finding asymptotic expansions in the small interaction in the problem of a nonideal Bose-Einstein system of many particles. It is based on the problem of the asymptotic solution of a certain system of nonlinear integral equations for $A(p)$, $B(p)$, and $\varepsilon(p)$. In the present note an investigation is given of corrections to the results obtained with the aid of the indicated equations.

The nonideal Bose-Einstein system under study, in the modified formulation, is characterized by a Hamiltonian containing two independent parameters N_0, μ. For this Hamiltonian one must calculate the thermodynamic potential and then set up two equations

$$\frac{\partial}{\partial N_0} F(N_0, \mu) = 0, \qquad \frac{\partial}{\partial \mu} F(N_0, \mu) = -N \tag{1}$$

in order to eliminate from the problem the parameters N_0, μ. The free energy is calculated from the relation

$$\Psi = F(N_0, \mu) + \mu N, \tag{2}$$

where on the right the values obtained from (1) must be substituted for N_0, μ.

The above-mentioned system of nonlinear integral equations arises as a result of carrying out a partial summation of diagrams constructed from pair vertices and proper first-order energy parts. In this case the full $F(N_0, \mu)$ can be represented in the form

$$F(N_0, \mu) = F_0(N_0, \mu) + \Delta F(N_0, \mu), \tag{3}$$

where F_0 corresponds to the diagrams included in the partial summation, and ΔF corresponds to the remaining diagrams ([2]).

Let us give the explicit form of the system of nonlinear integral equations with respect to $A(p)$, $B(p)$, $\varepsilon(p)$, μ, and n_0:

$$A(p) = E(p) + n_0 v(p) + \frac{1}{4\pi^2\hbar^3}\int_0^{+\infty} {p'}^2 dp' \, (v(p,p') -$$

$$- v(p'))\left(\frac{A(p')}{\varepsilon(p')}\operatorname{cth}\frac{\beta\varepsilon(p')}{2} - 1\right) -$$

$$- \frac{1}{4\pi^2\hbar^3}\int_0^{+\infty} {p'}^2 dp' \, v(p')\frac{B(p')}{\varepsilon(p')}\operatorname{cth}\frac{\beta\varepsilon(p')}{2} - \Delta\mu, \tag{4}$$

$$B(p) = n_0 v(p) - \frac{1}{4\pi^2\hbar^3}\int_0^{+\infty} {p'}^2 dp' \, v(p,p')\frac{B(p')}{\varepsilon(p')}\operatorname{cth}\frac{\beta\varepsilon(p')}{2}, \tag{5}$$

$$\varepsilon(p) = \sqrt{A^2(p) - B^2(p)}. \tag{6}$$

$$\mu = n_0 v(0) + \frac{1}{4\pi^2\hbar^3}\int_0^{+\infty} {p'}^2 dp' \, (v(p') + v(0))\left(\frac{A(p')}{\varepsilon(p')}\operatorname{cth}\frac{\beta\varepsilon(p')}{2} - 1\right) +$$

$$+ \frac{1}{4\pi^2\hbar^3}\int_0^{+\infty} {p'}^2 dp' \, \frac{B(p')}{\varepsilon(p')}\operatorname{cth}\frac{\beta\varepsilon(p')}{2} + \Delta\mu, \tag{7}$$

$$n = n_0 + \frac{1}{4\pi^2\hbar^3}\int_0^{+\infty} {p'}^2 dp' \left(\frac{A(p')}{\varepsilon(p')}\operatorname{cth}\frac{\beta\varepsilon(p')}{2} - 1\right) + \Delta n, \tag{8}$$

where

$$\Delta\mu = \Delta\mu(N_0, \mu; A(p), B(p)) = \frac{\partial}{\partial N_0}\Delta F(N_0, \mu, A(p), B(p)), \tag{9}$$

$$\Delta n = \Delta n(N_0, \mu; A(p), B(p)) = -\frac{\partial}{\partial \mu}\Delta F(N_0, \mu, A(p), B(p)), \tag{10}$$

where, in turn, $\Delta F(N_0, \mu; A(p), B(p))$ is a functional of $A(p), B(p)$, composed of the contributions of the corresponding diagrams of perturbation theory; when taking the derivatives in (9), (10), it is not necessary to affect the functional dependence on $A(p), B(p)$.

By fixing n_0 instead of n, we somewhat simplify the problem: instead of solving the system (4)–(10), we need solve only the system (4)–(7), (9). True, afterward it is still necessary to find n_0 as a function of n, proceeding from (8), (10), but this is an independent problem.

Equations (4)–(8) differ from (1)–(5) of [2] only in that now the corresponding terms have been written explicitly instead of ellipses. In [2,3], $\Delta\mu$ was discarded in (4), and then the resulting system of equations (4)–(6) was solved asymptotically. In (7), (8) the correction terms $\Delta\mu$, Δn were also discarded, and from the formulas obtained μ and n were calculated. The discarding of $\Delta\mu, \Delta n$ was justified by the fact that, in comparison with the other terms on the right-hand side of (4), (7), and (8), they are asymptotically less significant.

However, in [2,3] attention was not paid to the following circumstance. After, with the aid of (4)–(7) with $\Delta\mu$ discarded, the terms of the asymptotic expansions for $A(p, n_0)$, $B(p, n_0)$, and $\mu(n_0)$ have been obtained, they can be used to calculate $\Delta\mu$, in order to verify the smallness of this quantity. This was not done in [2,3]; instead, the quantity

$$\Delta F(N_0) = \Delta F(N_0, \mu(n_0); A(p, n_0), B(p, n_0)), \tag{11}$$

or rather its leading asymptotics in the limit of weak interaction, was simply computed, and it was assumed that $\mu = \frac{d}{dN_0}\Delta F/N_0$. However, in order to obtain $\Delta\mu(N_0)$, knowledge of $\Delta F(N_0)$ alone is insufficient; one must also know one more quantity,

$$\Delta n(N_0) = \Delta n(N_0, \mu(n_0); A(p, n_0), B(p, n_0)), \tag{12}$$

or rather its leading asymptotics for weak interaction. Indeed,

$$\Delta\mu(N_0) = \frac{d}{dN_0}\Delta F(N_0) + \Delta N_0 \, \frac{d\mu(n_0)}{dn_0}, \tag{13}$$

so that $\Delta\mu(N_0)$ is not simply equal to the derivative with respect to N_0 of $\Delta F(N_0)$.

Let us note that, for the quantities $\Delta F(N_0)$ and $\Delta n(N_0)$ defined by (11), (12), many perturbation-theory diagrams possible for the more general quantities $\Delta F(N_0, \mu; A(p), B(p))$ and $\Delta n(N_0, \mu; A(p), B(p))$ mutually compensate: there is no need to consider diagrams containing pair vertices or proper first-order self-energy parts.

The leading asymptotic term $\Delta F(N_0)$ comes from the second-order diagrams schematically shown in Fig. 1. The calculation shows that

$$\frac{\Delta F(N_0)}{V} = -\frac{1}{3\pi^2\hbar^3} m^{3/2} n_0^{5/2} v^{3/2}(0) \frac{1}{4\pi^2\hbar^3} \int_0^{+\infty} p^2\, dp\, \frac{v^2(p)}{E(p)} +$$

$$+\frac{1}{16\pi^2\hbar^6} m^3 n_0^3 v^4(0) \left(\frac{4}{3} - \frac{\sqrt{3}}{\pi}\right) \ln v(0) + \text{terms } v^4 \text{ and higher}, \qquad (14)$$

in the case of zero temperature $\theta = 0$, and

$$\frac{\Delta F(N_0)}{V} = \frac{n_0}{2\pi^2\hbar^6} m^3 v^2(0) \theta^2 \ln \frac{n_0 v(0)}{\theta} + \text{terms } v^2 \text{ and higher} \qquad (15)$$

in the general case $\theta \neq 0$*. The calculation of the leading asymptotic term $\Delta n(N_0)$ is more complicated. It is now necessary to take into account an entire subsequence of diagrams, the first of which are shown in Fig. 2.

Fig. 1

Fig. 2

In the case $\theta \neq 0$ we obtain

$$\Delta n = -\frac{1}{(2\pi^2\hbar^3)^{3/2}} m^{3/4} n_0^{1/4} v^{3/4}(0) \theta^{3/2} + \text{terms higher than } v^{3/4}, \qquad (16)$$

where exactly one half of the result comes from the first two second-order diagrams. Thus we find that the correction Δn exactly compensates the correction term $v^{3/4}$ in formula (9) of (2). The remaining formulas in (2) require no corrections.

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Received
19 IX 1966

CITED LITERATURE

[1] V. V. Tolmachev, DAN, **134**, 1324 (1960).
[2] V. V. Tolmachev, DAN, **135**, 41 (1960).
[3] V. V. Tolmachev, DAN, **135**, 825 (1960).

* In (2), the coefficient in formula (15) is incorrect. In addition, in (2) there is an inaccuracy in Fig. 2.

Note: Figure translations are in progress. See original paper for figures.