

The field of a horizontal magnetic dipole in the presence of an ellipsoid

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Date: 1967-01-01T00:00:00+00:00

Abstract

The problem of electromagnetic field diffraction by a spheroid is solved. The source of the primary field is a magnetic dipole located on the polar axis of the spheroid. The dipole moment is directed at a right angle to the axis; the spheroid is perfectly conducting. The solution to the problem is reduced to finding two auxiliary scalar functions (Hertz potentials) that satisfy the Helmholtz wave equation in spheroidal coordinates, the Sommerfeld radiation condition at infinity, and certain boundary conditions on the surface of the spheroid (the potentials are not separated in the boundary conditions). The required potentials are sought in the form of series in terms of spheroidal wave functions, for the coefficients of which an infinite system of linear equations is obtained from the boundary conditions, suitable for the numerical solution of the problem using a computer. 1 illustration. 11 references.

Full Text

Preamble

1967, Vol. III, K 517.946 : 9; 538.56 E. A. I. § 1. The problem of diffraction by a disk has been considered in [1], and for a sphere in [2, 3] and [4]. In this paper, we consider the case of a spheroid. In previous works [3, 5], solutions were obtained for various conditions (scalar, acoustic, and electromagnetic [6]), including the case of a perfectly conducting spheroid [6] and [7]. In [6], the solution (for the case of a dipole) was obtained (expressed in terms of spheroidal functions, for the case $a = 0$). However, it is found that in the center, the solution is not unique. From the condition at the boundary $\xi = \xi_0$, we obtain $\xi = \text{const}$. In [8], the problem is reduced to a system of equations for the coefficients $\Phi(\xi, \eta)$ as in [4], where $\eta = 1$ (at $\xi = \xi_0$), and the solution is sought in the form of a series [6] of functions $Y(\xi, \eta)$. We introduce a coordinate system related to the spheroid. Let $x = f[(\xi^2 - 1)(1 - \eta^2)]^{1/2} \cos \phi$, $y = f[(\xi^2 - 1)(1 - \eta^2)]^{1/2} \sin \phi$, $z = f\xi\eta$, where $1 < \xi < \infty$, $-1 < \eta < 1$, $0 < \phi < 2\pi$. The surface of the spheroid

is defined by $\xi = \xi_0$. Let the incident field be a plane wave $\Pi^* = \{1, 0, 0\}$, where the potential is $R = (x^2 + y^2 + z^2)^{1/2}$. As $r \rightarrow \infty$, the field behaves as $\Pi = \Pi^* + \Pi^s$, where Π^s is the scattered field. We represent the potentials as $\Phi = \Phi^0(\xi, \eta) + \Phi^1(\xi, \eta) \cos \phi$. The boundary conditions at $\xi = \xi_0$ are $\Pi_x = 0$ and $\Pi_y = 0$. The electromagnetic field components are given by $E = ik\mu \text{rot } \Pi$ and $H = \text{rot rot } \Pi = \text{grad div } \Pi + k^2\Pi$. Here $\Phi^0(\xi, \eta)$ corresponds to the primary field. The time dependence is assumed to be $e^{-i\omega t}$. The potential u satisfies the wave equation $\Delta u + k^2 u = 0$. In spheroidal coordinates (ξ, η, ϕ) , the solution can be expanded in terms of spheroidal wave functions.

From the boundary conditions (4), we find that at $\xi = \xi_0$:

$$\begin{aligned} (A_n, \Pi_n) &= \dots \\ \Phi \cos \phi + \Psi \cos \phi &= \dots \end{aligned}$$

The coefficients are determined by the orthogonality of the functions $S_{mn}(c, \eta)$. Following the method of E. A. Ivanov [9], we obtain a system of algebraic equations for the unknown coefficients.

§ 3. Solution of the Problem

In this section, we expand the primary potential Φ^0 as:

$$\Phi^0 = 2ik \sum_{n=0}^{\infty} \frac{S_{0n}(c, \eta)}{N_{0n}(c)} R_{0n}^{(1)}(c, \xi)$$

where $c = fk$, $S_{mn}(c, \eta)$ are the angular spheroidal functions, and $R_{mn}^{(3)}(c, \xi)$ are the radial spheroidal functions of the third kind. The normalization constants are $N_{mn}(c)$. The scattered field potential $\Phi^s(\xi, \eta)$ is represented as:

$$\Phi^s(\xi, \eta) = 2ik \sum_{n=0}^{\infty} a_n R_{0n}^{(3)}(c, \xi) S_{0n}(c, \eta)$$

For the case $m = 0$, the boundary conditions (5) lead to a system of equations for the coefficients a_n . Similarly, for the components proportional to $\cos \phi$, we have:

$$W(\xi, \eta) = 2ik \sum_{n=1}^{\infty} b_n R_{1n}^{(3)}(c, \xi) S_{1n}(c, \eta)$$

Substituting these into the boundary conditions (9), we obtain a coupled system for a_n and b_n . Using the properties of spheroidal functions:

$$\int_{-1}^1 S_{mn}(c, \eta) S_{mN}(c, \eta) d\eta = \delta_{nN} N_{mN}(c)$$

we can reduce the problem to an infinite system of linear algebraic equations. Let $C_{Nn}, D_{Nn}, V_{Nn}, W_{Nn}$ be the matrix elements derived from the boundary

conditions. The system takes the form:

$$\sum_n (C_{Nn}a_n + D_{Nn}b_n) = B_N$$

where B_N are coefficients related to the incident field.

7. $R_{mn}(c, \xi)$

Using the results from [6] and the expressions for the coefficients (14), we can analyze the convergence of the solution. For $N = 0, 1, \dots$, the terms with $n = N$ dominate the series. The system (14) can be solved numerically by truncation. For a perfectly conducting spheroid, the coefficients a_n and b_n are determined by the physical parameters of the problem.

The far-field radiation pattern is obtained by taking the asymptotic limit $r \rightarrow \infty$. In this limit, the spheroidal coordinates transition to spherical coordinates: $\xi \rightarrow r/f$ and $\eta \rightarrow \cos \theta$. The components of the electric field in the far zone are:

$$\begin{aligned} E_\theta &= ik\Phi(\theta) \sin \phi \\ E_\phi &= -ik\Psi(\theta) \cos \phi \end{aligned}$$

where $\Phi(\theta)$ and $\Psi(\theta)$ are the scattering amplitudes. For the case of a small spheroid ($c \ll 1$), the results coincide with the known solutions for dipole scattering.

The case of a disk is recovered by taking the limit $\xi_0 \rightarrow 0$. In this limit, the spheroidal functions degenerate into Bessel functions and Legendre polynomials. The results obtained here are consistent with the classical solutions for diffraction by a circular aperture or disk [10, 11].

For a prolate spheroid, the coordinates are defined as: $x = f[(\xi^2 - 1)(1 - \eta^2)]^{1/2} \cos \phi$, $y = f[(\xi^2 - 1)(1 - \eta^2)]^{1/2} \sin \phi$, $z = f\xi\eta$. For an oblate spheroid, the transformation $\xi \rightarrow i\xi$ and $f \rightarrow -if$ is applied. The boundary conditions and the general form of the solution (9)-(23) remain valid under this transformation.

In conclusion, we have derived a rigorous solution for the diffraction of an electromagnetic wave by a spheroid using the method of separation of variables in spheroidal coordinates. The resulting infinite system of equations is suitable for numerical computation.

References

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Submitted December 18, 1966.

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