ON NONLINEAR DRIFT WAVES IN A PLASMA

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Abstract

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V. I. PETVIASHVILI

ON NONLINEAR DRIFT WAVES IN A PLASMA

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In the presence of an inhomogeneity in a plasma transverse to the magnetic field, the propagation of drift waves is possible ($^{1-2}$). We shall show that, with the passage of time, the slope of the leading front of these waves becomes steeper and steeper in comparison with the initial state, until the mean frequency of the wave becomes so large that it becomes necessary to take into account the inertia of the ions across the magnetic field. The wave then passes into a stationary state (provided the amplitude is not too large). This may lead to strong turbulization of the plasma in the presence of an instability with respect to low-frequency drift waves, since the correlation between oscillations at any two points of the plasma is strongly weakened as the amplitude of the oscillations grows.

In the rest frame of the plasma let us direct the z-axis along the magnetic field and the x-axis along the gradient of the particle density. For simplicity we shall take the ion temperature to be zero. When the plasma pressure is small, the oscillation of the electric field may be regarded as potential and, at a low frequency of oscillations, the hydrodynamic velocity of the ions across the magnetic field has the character of a drift:

$$\mathbf{v}_\perp = \frac{e}{m\Omega^2}\,[\vec{\Omega}\nabla\varphi], \tag{1}$$

where $\vec{\Omega}$ is the vector of the ion cyclotron frequency and φ is the electric potential. Taking into account that the divergence of (1) is zero, we write the continuity equation for the ions in the form

$$\frac{\partial}{\partial t}\ln n_i + (\mathbf{v}\vec{\nabla})\ln n_i + \frac{\partial v_z}{\partial z} = 0. \tag{2}$$

Assuming the plasma to be quasineutral, in (2) one may substitute the electron density for the ion density n_i. In a strong magnetic field the components of the hydrodynamic velocity of the electrons across the magnetic field may be taken to be zero, and the distribution of the electron density n_e as a function of the potential φ is Boltzmannian:

$$n_e = n_0(x)\exp\{e\varphi/T\}, \tag{3}$$

where $n_0(x)$ is the mean density and T is the electron temperature. Substituting (3) into (2), and taking (1) into account, we obtain

$$\frac{\partial\psi}{\partial t} + v_{\mathrm{dr}}\frac{\partial\psi}{\partial y} + v_z\frac{\partial\psi}{\partial z} + c_s^2\frac{\partial v_z}{\partial z} = 0, \tag{4}$$

$$\psi = \frac{e\varphi}{m}; \qquad c_s^2 = \frac{T}{m}; \qquad v_{\mathrm{dr}} = -\frac{c_s^2}{\Omega}\frac{\partial}{\partial x}\ln n_0.$$

The ion velocity along z satisfies the equation

$$\partial v_z/\partial t + (\vec{v}\vec{\nabla})v_z = -\partial\psi/\partial z. \tag{5}$$

Assuming v_{dr} to be constant, which is valid when the wavelength across the magnetic field is much smaller than the characteristic scale of the plasma inhomogeneity, we shall seek a solution of the system (4), (5) in the form of a one-dimensional simple Riemann wave (^{3}), i.e., we set

$$v_z = v_z(\psi); \qquad \psi = \psi(t,\xi); \qquad \xi = \mathbf{kr} = k_{xx} + k_{yy} + k_{zz}. \tag{6}$$

Substituting (6) into (4), (5), and taking (1) into account, we obtain:

$$\partial\psi/\partial t + \left(k_{yv_{\mathrm{dr}}} + k_{zv}z + k_{zc}s^2\partial v_z/\partial\psi\right)\partial\psi/\partial\xi = 0; \tag{7}$$

$$\frac{\partial v_z}{\partial\psi}\frac{\partial\psi}{\partial t} + \left(k_{zv}z\frac{\partial v_z}{\partial\psi} + k_z\right)\frac{\partial\psi}{\partial\xi} = 0. \tag{8}$$

Dividing (7), (8) by $\partial\psi/\partial\xi$ and denoting

$$\frac{\partial\psi}{\partial t}\bigg/\frac{\partial\psi}{\partial\xi} = -\left.\frac{\partial\xi}{\partial t}\right|_{\psi,\,v_z} = -V, \tag{9}$$

we obtain

$$V = k_{yv_{\mathrm{dr}}} + k_{zv}z + k_{zc}s^2\partial v_z/\partial\psi; \tag{10}$$

$$\frac{\partial v_z}{\partial \psi} V = k_z + k_{zv} z \frac{\partial v_z}{\partial \psi}. \tag{11}$$

From (10), (11) we eliminate $\partial v_z / \partial \psi$ and obtain

$$V = \omega_{\mathrm{dr}} + k_{zv} z, \tag{12}$$

where ω_{dr} is the frequency of linear drift oscillations with wave vector $\mathbf{k}$,

$$\omega_{\mathrm{dr}} = \frac{1}{2} k_{y v_{\mathrm{dr}}} \pm \sqrt{\frac{1}{4} k_y^2 v_{\mathrm{dr}}^2 + c_s^2 k_z^2}. \tag{13}$$

Substituting (9) into (12) and integrating, we obtain

$$\xi \equiv \mathbf{kr} = \omega_{\mathrm{dr}} t + k_{z v_{zt}} + f(v_z), \tag{14}$$

where f is an arbitrary function giving the distribution of the ion velocity at the instant $t = 0$. For example, if at the initial instant the wave was sinusoidal with amplitude v_0, then $f = \arcsin(v_z/v_0)$. (14) defines v_z implicitly as a function of time and coordinates. Eliminating V from (10), (11) and integrating, we find that ψ is proportional to v_z.

Let us consider how the solution (14) behaves with time. It follows from (12) that the phase velocity of the wave at a given point V is not constant and depends on v_z. At those points where v_z is larger, the wave moves faster and catches up with regions where v_z is small. The nonuniformity of the speed of displacement of the points of the wave profile leads to a change in its shape with time (^{3}). The leading front of the wave becomes ever steeper, until over a time $t \sim 1/k_{zv}0$ the function $v_z(t, \xi)$ becomes discontinuous in both variables, turning into a shock wave. As is seen from (1), $\mathbf{v}_\perp$ is proportional to the derivative of ψ, while ψ, as was noted, is proportional to v_z. Therefore $\mathbf{v}_\perp(t, \xi)$ tends to a discontinuous function much faster than v_z.

In reality, a shock wave is not formed for the following reasons. If the ion temperature is sufficiently high, then as the steepness of the wave front increases, the rate of absorption of the wave energy due to ion viscosity and Landau damping on ions increases. As a result, the wave is absorbed before a discontinuity is formed. If, however, the ion temperature is low, the mean time frequency of the wave, owing to the increase in the steepness of the wave front, may become so large that it becomes necessary to take the inertia of the ions into account.

We shall show that at frequencies for which the ion inertia cannot be neglected, and at least for small amplitudes of the oscillations, a stationary periodic wave can exist in an inhomogeneous plasma. From this we shall conclude that with time the drift wave passes into such a wave or into a superposition of such waves.

We shall take the inertial motion of the ions into account only in the first nonvanishing approximation (1). Then, instead of (1), we have

$$\mathbf{v}_\perp = \frac{1}{\Omega^2}\left([\Omega\vec{\nabla}\psi] + \vec{\nabla}_\perp\frac{\partial\psi}{\partial t}\right), \tag{15}$$

and from the continuity equation and (3), (15), instead of (4) we have

$$\frac{\partial\psi}{\partial t} + v_{\mathrm{dr}}\frac{\partial\psi}{\partial y} + \frac{c_s^2}{\Omega^2}\Delta_\perp\frac{\partial\psi}{\partial t} + v_z\frac{\partial\psi}{\partial z} + c_s^2\frac{\partial v_z}{\partial z} = 0. \tag{16}$$

Since we take the ion inertia into account only in terms linear in the oscillation amplitude, in (16) one may substitute the expression (see (1), (5), (6))

$$\partial\psi/\partial z = -\partial v_z/\partial t - v_z\partial v_z/\partial z. \tag{17}$$

We shall seek a solution of the system (16), (17) in the form of a stationary wave, i.e., we put

$$\psi = \psi(\xi_1); \qquad v_z = v_z(\xi_1); \qquad \xi_1 = \mathbf{kr} - \omega t. \tag{18}$$

Then from (16), (17) we obtain

$$\left(\frac{c_s k_\perp \omega}{\Omega}\right)^2 \frac{\partial^2}{\partial\xi^2}v_z + (\omega^2 - \omega k_y v_{\mathrm{dr}} - k_z^2 c_s^2)v_z - \frac{1}{2}k_z(\omega - k_{y v_{\mathrm{dr}}})v_z^2 + \frac{1}{3}k_z^2 v_z^3 = 0. \tag{19}$$

For large ω, (19) has periodic solutions, since it is the equation of a nonlinear oscillator. We see that without the first term, containing the small parameter Ω^{-2} and the second derivative of v_z, the drift oscillations do not have stationary oscillatory solutions (in agreement with (14)). As follows from (14), with time the time and coordinate derivatives of v_z grow, and the first term of (19) becomes essential. Then further "overturning" of the drift wave ceases, and the drift wave passes into a superposition of periodic solutions of (19). A superposition of periodic waves gives a quasistationary wave described by equations (16), (17).

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Note: Figure translations are in progress. See original paper for figures.