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Abstract

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ON THE CONNECTION BETWEEN VOLUMETRIC AND SHEAR PLASTIC DEFORMATIONS AND SHOCK WAVES IN SOFT SOILS

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1. In the mathematical models ([1–5]) used for the analysis of plastic waves in soft soils, the assumption is made that volumetric and shear deformations are independent. Under hydrostatic compression of sandy soil, the irreversible deformation e is related to the pressure p by the relation $e = -\varphi(p)$, where $\varphi = d\varphi/dp$ is a monotonically decreasing function ($\varphi < 0$) of the pressure for $dp > 0$, and $\varphi = 0$ for $dp \leq 0$. Under pure shear, plastic deformation takes place when Coulomb' s law is satisfied ($|\tau| = p \tan\omega + c$, where τ is the tangential stress on the shear plane, ω is the angle of internal friction of the soil, and c is cohesion). According to the assumption of independence of deformations, volumetric and shear deformations do not influence one another and develop according to the specified laws. However, for soils (see, for example, ([6])), under significant changes in the state of stress, a dilatational state is typical (described already by Reynolds), in which shear causes changes in volume.

2. Experimental data on the dilatational properties of soil are usually obtained in triaxial tests. A cylindrical specimen of the medium is hydrostatically compressed to a pressure p_0, after which, while the radial stress $\sigma_2 = \sigma_3 = -p_0$ is kept unchanged, the axial stress σ_1 is gradually increased and the axial deformation e_1 and the volumetric deformation e are measured. The published data for sand ([6–9]) and lead shot ([10]) admit the following interpretation.

A. On the graph of the dependence (Fig. 1) of the shear deformation $\gamma = e_1 - e_2$ on the maximum tangential stress, and also on the graph of the dependence (Fig. 2) of the volumetric deformation $e = e_1 + 2e_2$ on the pressure $p = -(\sigma_1 + 2\sigma_2)/3$, a horizontal segment is distinguished, characteristic of plastic flow.

Fig. 1 and Fig. 2

Figure 1: Fig. 1 and Fig. 2

Fig. 3 and Fig. 4

Figure 2: Fig. 3 and Fig. 4

B. The connection between shear and volumetric deformation is such (Fig. 2) that the rate of dilatancy $\Lambda = de/d|\gamma|$ (the rate of change of volume with respect to shear) gradually decreases and tends to zero (this is noticeable in loose sands and in shot) as the critical density value is approached. The latter is characterized by the absence of any influence of shear on volume. The rate of dilatancy increases ([6–8]) after preliminary compaction of the medium (for example, according to the data of Fig. 2 of article ([7]), on the segments of plastic flow the estimates $\Lambda \approx 2/5$ for dense sand and $\Lambda \approx 1/22$ for loose sand are valid). In triaxial tests $((^9), p.157)$ carried out under conditions $p =$ const, the value of Λ was practically constant: $\Lambda = 1/5.25;\ 1/5;\ 1/2;\ 1/2.75$, respectively for $p = 4;\ 3;\ 2;\ 1$ atm and for nearly the same initial porosity m.

C. In the plane $|\sigma_1 - \sigma_2|$, p, the points corresponding to the soil states under discussion occupy a region bounded by the limiting (straight) line of the maximum (cf. Fig. 1) tangential stresses. The critical density value corresponds to a straight line (Coulomb condition). In the experiments mentioned in ([9]), p. 157, the results obtained at different p, in the coordinates $|\sigma_1 - \sigma_2|/p,\ e$, lay on a single curve (Fig. 3).

The existence of a complex surface $\Phi(\tau, p, m) = 0$, corresponding to dilatant states of clayey soils, is indicated by Roscoe (see, for example, ([9]), p. 78). In ([8]) data are given on the initial stage of deformation (before the transition to horizontal segments). Here the angle of internal friction ω of sand, as well as the rate of dilatancy Λ, increases with

Fig. 1

Fig. 2

initial density and decreases with increasing pressure $p_0 = -\sigma_2$ (even direct proportionality of the quantities Λ and ω is noted).

G. When the tangential stresses are reduced, unloading occurs (dashed line in Figs. 1 and 2), and the shear strain turns out to be irreversible, while the volumetric deformation of loosening is, at least partially, reversible ([7,10]).

Fig. 3

Fig. 4

3. Let us now take into account that in triaxial tests the initial compression of the specimen (up to pressure p_0) occurred along the curve $e = -\varphi(p)$,

see Fig. 4. By the latter we shall understand the possibility of representing the increment of volumetric deformation as the sum of two components, one of which is determined by the growth of pressure, and the other by the presence of plastic shear, the effect of dilatancy. Accordingly, we shall specify the defining equation of the proposed idealized "rigid-dilatant" model in the form

$$de = -\varphi(p)\, dp + \Lambda(p)|d\gamma|. \tag{1}$$

The function $e = -\varphi(p)$ may be interpreted (for $d\gamma = 0$) as the curve of hydrostatic compression. Then it has the property of concavity toward the pressure axis and the presence of unloading branches ($\varphi = 0,\ dp \leqslant 0$), i.e., for $dp \leqslant 0$ the increments of volumetric deformation are directly proportional to the increments of shear deformation (which agrees with the data of ([9]), p. 157). Relations of type (1) were introduced by D. D. Ivlev and T. N. Martynova for describing an ideally plastic compressible material ([14]).

We shall assume that shear deformation always occurs according to the law of plastic flow, i.e.

$$d\gamma/dt = \lambda(\sigma_1 - \sigma_2), \tag{2}$$

where λ is a scalar positive definite function of the coordinates and time, different from zero when the condition of the dilatational state of the medium is satisfied:

$$\Phi(p, |\sigma_1 - \sigma_2|) = 0 \tag{3}$$

($\lambda = 0$ for $\Phi < 0$; states with $\Phi > 0$ are excluded). The dilatancy rate Λ will be regarded as a determined function; in particular, $\Lambda = \Lambda(p)$, a decreasing function of the pressure for $dp > 0$, and $\Lambda = 0$ for $dp \leqslant 0$.

Thus, for simplicity, we attribute the decrease in the dilatancy rate on the interval $dp > 0$ to the influence of the first term in relation (3), while on the interval $dp \leqslant 0$ we neglect the decrease in the rate Λ as the medium loosens. Judging from the experiments [(9)], p. 157, at least in the region $dp \leqslant 0$ it is admissible to set $\Phi(p, |\sigma_1 - \sigma_2|) = |\sigma_1 - \sigma_2| - kp = 0$—the dashed line in Fig. 3. In the model the condition (3) does not depend on any additional parameter. Nevertheless, for completeness of a preliminary assessment of the nature of the waves, we shall assume that its role is played by the density of the medium; i.e., for spherically symmetric motion we shall use relations of a somewhat more general type than the following consequences of condition (3):

$$\sigma_r - \sigma_\varphi = f_1(p, \rho), \qquad \partial\sigma_r/\partial p = f_2(p, \rho), \qquad \partial\sigma_r/\partial\rho = f_3(p, \rho), \tag{4}$$

where $\sigma_r \equiv \sigma_1$, $\sigma_\varphi \equiv \sigma_2 = \sigma_3$ are the radial and tangential stresses.

4. If one seeks to preserve the basic proposition of plasticity theory concerning the existence of a yield surface in stress space, then the surface of dilatational states must depend on an additional parameter χ, i.e.,

$$\Phi(p, |\sigma_1 - \sigma_2|, \chi) = 0, \tag{5}$$

and under hydrostatic compression of the medium ($\sigma_1 = \sigma_2$) condition (5) gives a certain relation $\chi = \chi(p)$. The parameter χ plays the role of a thermodynamic state parameter [11], i.e. $\varepsilon = \varepsilon(p, \tau, \chi)$, where ε is the internal energy of the medium, and the relation $\mu = (\partial\varepsilon/\partial\chi)_{p,\tau} = F(p, \tau, \chi)$ holds.

Neglecting the elastic components of deformation, one may introduce the kinetic equations

$$de/dt = \xi p + M\mu; \qquad d\gamma/dt = \lambda\tau; \qquad d\chi/dt = Mp + N\mu. \tag{6}$$

Relations (6) determine the rates of plastic deformation, if the kinetic coefficients ξ, λ are regarded as new unknown functions and the system of equations is supplemented by the conditions of shear and volumetric plasticity [12]. In dilatational media these conditions coincide and give one relation (5), which corresponds to the independence of only one of the kinetic coefficients, for example λ [12].

If now (5) is resolved in the form $p/|\tau| = f(p, \chi)$ and a linear dependence of the kinetic coefficients on the function λ is assumed, then relations (6) take the form

$$de = \Lambda_1(\chi, p)|d\gamma| + \Lambda_2(\chi, p)d\chi; \qquad d\chi = \psi(\chi, p)d\gamma. \tag{7}$$

The first of equations (7) may be regarded as a nonholonomic defining equation of the medium. For $\chi = \chi(p)$ this model passes into that considered above, if $\Lambda_1(\chi(p), p) = \Lambda(p)$, $\Lambda_2(\chi(p), p)\chi(p) \neq -\varphi(p)$.

5. For the analysis of spherically symmetric wave motions, equations (1), (4) are supplemented by the continuity and momentum equations

$$\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + 2\frac{u}{r}\right) = 0, \tag{8}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r}\right) = f_2\frac{\partial p}{\partial r} + f_3\frac{\partial\rho}{\partial r} + 2\frac{f_1}{r}, \tag{9}$$

where u is the velocity of motion of a particle, and equation (1) is transformed to the form

$$\frac{\partial u}{\partial r}+2\frac{u}{r}=-\varphi\left(\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}\right)+\Lambda\left|\frac{\partial u}{\partial r}-\frac{u}{r}\right|. \tag{10}$$

The characteristics of the system of equations (8)–(10) for $u/r > \partial y/\partial r$ have the form

$$\left(\frac{dr}{dt}\right)_1=u,\qquad \rho\dot{\varphi}\frac{dp}{dt}-(1+\Lambda)\frac{d\rho}{dt}=3\rho\frac{u\Lambda}{r}\quad (f_2\neq 0); \tag{11}$$

$$\left(\frac{dr}{dt}\right)_{2,3}=u\pm c,\qquad c^2=-\left(f_3+f_2\frac{1+\Lambda}{\rho\dot{\varphi}}\right),$$

$$f_2\frac{dp}{dt}+f_3\frac{d\rho}{dt}\pm\rho c\frac{du}{dt}=\pm 2c\frac{f_1}{r}-\left(2\rho f_3-\frac{f_2(2-\Lambda)}{\dot{\varphi}}\right)\frac{u}{r}. \tag{12}$$

From experimental data for dry sand ([4]) it follows that the function $\dot{\varphi}(p)$ decreases relatively slowly with increasing pressure. A complete analysis requires a preliminary determination of the experimental functions f_2, f_3, and $\Lambda(p,\rho)$. One may suppose that f_2, f_3 also vary only weakly (under the Coulomb condition $f_2=\text{const}, f_3=0$). Hence the velocity c of propagation of disturbances relative to the particles of the medium (despite the concavity of the curve $e=-\varphi(p)$) decreases with increasing pressure p, provided only that the dilatancy rate $\Lambda(p)$ decreases sufficiently rapidly with pressure. In this case, in a shock wave whose velocity is less than the velocity of propagation of the dilatancy region,* loading occurs not discontinuously, but continuously. The absence of a shock front in loading waves in soils has been noted earlier (see, for example, ([13]), p. 72, where this effect is explained by the condition $\varphi > 0$ while preserving the usual concepts) and was convincingly demonstrated in the experiments of I. L. Zelmanov, O. S. Kolkov, A. M. Tikhomirov, and A. F. Shapukevich, carried out at the Institute of Physics of the Earth.

In considering the problem of a camouflet explosion, one should seek a continuous solution for the loading region $R(t) < r \leq r_*(t)$, where $r_*(t)$ is the outer boundary of the region in which condition (3) is satisfied, and $R(t)$ is the coordinate of the maximum pressure in the soil (when comparing calculations and experiments, R, as a rule, is taken to be the coordinate of the shock front). In the region $a(t) \leq r \leq R(t)$, where $a(t)$ is the radius of the cavity, unloading occurs ($dp \leq 0$). Here $\varphi = 0$, and in zones where $\Lambda(r,t) \approx \text{const}$, equation (10) gives, approximately,

$$u \approx C/r^{\alpha}=u(R)(R/r)^{\alpha},\qquad \alpha=(2-\Lambda)/(1+\Lambda). \tag{13}$$

Satisfaction of equality (13) was recorded in experiments on a camouflet explosion ([2]), as well as in the aforementioned experiments of I. L. Zelmanov et al. The value $\alpha = 1.5$ obtained in the experiments ([2]) corresponds to the quantity $\Lambda = 1/5$.

Considerations of large shear strains as the cause of loosening of sand in the unloading region during a camouflet explosion were expressed in ([2]). The author expresses his gratitude to V. N. Rodionov for suggesting that this effect be considered in greater detail.

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* Outside this region the material should be regarded as elastic.

Note: Figure translations are in progress. See original paper for figures.