ON THE EQUATIONS OF RADIATION DYNAMICS IN OPTICAL QUANTUM GENERATORS

PHYSICS

1967

SovietRxiv

Abstract

Full Text

UDC 535.14

PHYSICS

Yu. A. TARASOV, T. N. ZUBAREV

ON THE EQUATIONS OF RADIATION DYNAMICS IN OPTICAL QUANTUM GENERATORS

(Presented by Academician A. P. Aleksandrov on 12 VIII 1966)

For most solid-state optical quantum generators, the emergence in the resonator of many types of oscillations in the form of standing waves is characteristic; moreover, owing to the nonlinear interaction with the active medium, these oscillations turn out to be coupled. One of the interesting features of such generators is the appearance, under certain conditions, of undamped oscillations of the intensity of the induced radiation ([1,2]). In the present work, on the basis of the quantum-mechanical Hamiltonian for two-level systems interacting with the radiation field, a system of equations for the field amplitudes and level populations is obtained as applied to a solid-state quantum generator, with allowance for many types of oscillations. Using as an example the interaction of two types of oscillations close in intensity, it is shown that the system of equations may contain solutions corresponding to undamped oscillations of the radiation intensity. In considering equations of radiation dynamics in which the radiation field is described by traveling waves ([3]), solutions of this kind were not found.

Let us consider a system of N_0 two-level atoms interacting with the radiation field inside a resonator with perfectly reflecting walls. For simplicity we shall assume that the resonator is entirely filled with an active medium. The Hamiltonian of such a system will be written in the form ([4,5])

$$H = \sum_{j=1}^{N_0} \frac{\hbar\omega_0}{2}\sigma_z^j + \sum_\lambda \hbar\omega_\lambda a_\lambda^+ a_\lambda + \sum_{\lambda j} B_{\lambda j}\hbar\left(a_\lambda^+\sigma_-^j + a_\lambda\sigma_+^j\right).$$

Here the terms represent, respectively, the operators of the proper energy of the atoms, of the free radiation field, and of the interaction of the atoms with the radiation field. The operators a_λ and σ^j satisfy the commutation relations:

$$[a_\lambda, a_{\lambda'}^+] = \delta_{\lambda\lambda'}, \quad [a_\lambda, a_{\lambda'}] = 0, \quad [\sigma_z^j, \sigma_\pm^{j'}] = \pm 2\sigma_\pm^j\delta_{jj'}, \quad [\sigma_\pm^j, \sigma_\mp^{j'}] = \sigma_z^j\delta_{jj'}.$$

In what follows we shall pass to continuous coordinates, singling out a volume element $\Delta V(\mathbf{x})$ around the point $\mathbf{x}_j$, in which there are sufficiently many active particles (it is assumed that the wavelength of light considerably exceeds the distance between active particles). Then the third term in H takes the form:

$$\sum_\lambda \int dx\, B_\lambda(\mathbf{x})\hbar\left[a_\lambda^+\sigma_-(\mathbf{x}) + a_\lambda\sigma_+(\mathbf{x})\right],$$

where

$$B_\lambda(\mathbf{x}) = \sqrt{\frac{2\pi}{\hbar\omega_\lambda}}\,[\mathbf{M}(\mathbf{x})\mathbf{e}_\lambda]\varphi_\lambda(\mathbf{x}) \equiv p_\lambda(\mathbf{x})\varphi_\lambda(\mathbf{x});$$

$\mathbf{M}(\mathbf{x})$ is the mean dipole moment of the volume element, and $\varphi_\lambda(\mathbf{x})$ are the proper functions of the resonator.

Using the rules of differentiation of operators, it is not difficult to obtain a system of equations for the quantities $a_\lambda, \sigma(\mathbf{x})$. Then we pass to equations for the quantum-mechanical mean quantities $\langle a_\lambda\rangle, \langle\sigma(\mathbf{x})\rangle$, etc.,

where, to simplify the problem, we shall assume that their factorization is admissible (4): $\langle a_\lambda\sigma_-(\mathbf{x})\rangle = \langle a_\lambda\rangle\langle\sigma_-(\mathbf{x})\rangle$, etc.

This corresponds to the assumption that the coupling between atoms by means of the radiation field is sufficiently weak. In order to pass to real resonators, relaxation parameters γ_1, W_0, and γ_2 should be introduced into such equations; these describe, respectively, the leakage of radiation from the resonator, the pumping of the crystal by an external source, and the linewidth of the radiation of the substance. As a result, the system of equations takes the form:

$$i\left(\dot{A}_\lambda + \frac{\gamma_1}{2}A_\lambda\right) - \omega_\lambda A_\lambda = \int dx\, B_\lambda(\mathbf{x})\, r(\mathbf{x}),$$

$$i\left[\dot{r}(\mathbf{x}) + \frac{\gamma_2}{2}r(\mathbf{x}) - \omega_0 r(\mathbf{x})\right] = -\sum_{\lambda'} B_{\lambda'}(\mathbf{x})A_{\lambda'}N_-(\mathbf{x}), \qquad (1)$$

$$i\left\{\dot{N}_-(\mathbf{x}) - W_0\left[N_0V^{-1} - N_-(\mathbf{x})\right] + \frac{N_0V^{-1} + N_-(\mathbf{x})}{\tau}\right\} = 2\sum_{\lambda'}\left[B_{\lambda'}(\mathbf{x})A_{\lambda'}r^*(\mathbf{x}) - B^*_{\lambda'}(\mathbf{x})A^*_{\lambda'}r(\mathbf{x})\right].$$

Here the following notation has been introduced: $\langle a_\lambda\rangle = A_\lambda$; $\langle a_\lambda^+\rangle = A_\lambda^*$; $\langle\sigma_-(\mathbf{x})\rangle = r(\mathbf{x})$; $\langle\sigma_z(\mathbf{x})\rangle = N_-(\mathbf{x}) = N_2(\mathbf{x}) - N_1(\mathbf{x})$, the population difference of the levels; $1/\tau \equiv W$, the probability of spontaneous emission of an atom. The mean value of the quantity $p_\lambda(\mathbf{x})p^*_\lambda(\mathbf{x})$, averaged over all directions of the vector $\mathbf{M}(\mathbf{x})$, will have the form $1/4\tau_0^2$, where $1/\tau_0^2 = 2\pi c^3 W/\omega_0^2 V$. Next, we expand the quantities $r(\mathbf{x})$ in the eigenfunctions of the resonator: $r(\mathbf{x}) = \sum_\lambda \varphi_\lambda(\mathbf{x}) y_\lambda(t)$. After multiplying the second equation (1) by φ_λ, and the third by $\varphi_\lambda \varphi_{\lambda'}$, and integrating over the volume, we obtain the system

$$i\left(\dot{A}_\lambda + \frac{\gamma_1}{2} A_\lambda\right) - \omega_\lambda A_\lambda = \bar{p}_\lambda y_\lambda,$$

$$i\left(\dot{y}_\lambda + \frac{\gamma_2}{2} y_\lambda\right) - \omega_0 y_\lambda = -\sum_{\lambda'} \bar{p}_{\lambda'} N_{\lambda\lambda'} A_{\lambda'},$$

$$\dot{N}_{\lambda\lambda'} - W_0(N_0\delta_{\lambda\lambda'} - N_{\lambda\lambda'}) + \frac{N_0\delta_{\lambda\lambda'} + N_{\lambda\lambda'}}{\tau} = -2i \sum_{\lambda''\lambda'''} \mu_{\lambda\lambda'\lambda''\lambda'''} \bar{p}_{\lambda''} \left[y_{\lambda''} A_{\lambda'''} - y_{\lambda'''} A^*_{\lambda''}\right], \tag{2}$$

where

$$N_{\lambda\lambda'} = \int \varphi_\lambda N_-(\mathbf{x}) \varphi_{\lambda'}\, dx, \qquad \mu_{\lambda\lambda'\lambda''\lambda'''} = \int \varphi_\lambda \varphi_{\lambda'} \varphi_{\lambda''} \varphi_{\lambda'''}\, dx.$$

From (2) one can obtain equations for slow motions by setting $\dot{y}_\lambda = 0$. These equations, if one passes from the variables A_λ to amplitudes and phases $A_\lambda = a_\lambda e^{i\varphi_\lambda}$, will have the form

$$\dot{a}_\lambda + \frac{\gamma_1}{2} a_\lambda = \frac{\alpha_\lambda}{2} \sum_{\lambda'} N_{\lambda\lambda'} a_{\lambda'} \left(\cos \Phi_{\lambda'\lambda} + s_\lambda \sin \Phi_{\lambda'\lambda}\right),$$

$$a_\lambda(\dot{\Phi}_{\lambda\lambda'} + \Delta_{\lambda\lambda'}) = \frac{\alpha_\lambda}{2} \sum_{\lambda''} N_{\lambda\lambda''} a_{\lambda''} \left(\sin \Phi_{\lambda''\lambda} - s_\lambda \cos \Phi_{\lambda''\lambda}\right), \tag{3}$$

$$\dot{N}_{\lambda\lambda'} + W_0 N_{\lambda\lambda'} + \frac{N_{\lambda\lambda'}}{\tau} =$$

$$= -2\left[\mu_{\lambda\lambda'} \sum_{\lambda''} N_{\lambda'\lambda''} a_\lambda a_{\lambda''} \left(a_{\lambda'} \cos \Phi_{\lambda\lambda''} - 2a_{\lambda'} S_{\lambda'} \sin \Phi_{\lambda\lambda''}\right) + \right.$$

$$\left. + \mu_{\lambda\lambda'} \sum_{\lambda''} N_{\lambda\lambda''} a_{\lambda'} a_{\lambda''} \left(a_\lambda \cos \Phi_{\lambda'\lambda''} - a_\lambda S_\lambda \sin \Phi_{\lambda'\lambda''}\right)\right] \quad \text{for } \lambda \neq \lambda',$$

$$\dot{N}_{\lambda\lambda} - W_0(N_0 - N_{\lambda\lambda}) + \frac{N_0 + N_{\lambda\lambda}}{\tau} =$$

$$= -2 \sum_{\lambda'\lambda''} \mu_{\lambda\lambda'} N_{\lambda\lambda''} a_{\lambda'} a_{\lambda''} \left(a_{\lambda'} \cos \Phi_{\lambda\lambda''} - a_{\lambda'} s_{\lambda'} \sin \Phi_{\lambda\lambda''}\right),$$

where $\alpha_\lambda = \gamma_2^{-1}\tau_0^{-2}(1+s_\lambda^2)^{-1}$, $s_\lambda = -2\gamma_2^{-1}(\omega_\lambda - \omega_0)$, $\Phi_{\lambda\lambda''} = \varphi_{\lambda'} - \varphi_{\lambda''}$, $\mu_{\lambda\lambda'} \equiv \mu_{\lambda\lambda'\lambda\lambda'}$, $\Delta_{\lambda\lambda'} = \omega_\lambda - \omega_{\lambda'}$.

The principal parameters in equations (3), when ruby is used as the active medium, are, in order of magnitude: $\gamma_2 \sim 10^{11}$ sec^{-1}, $\gamma_1 \sim 10^9$ sec^{-1}, $W_0 \sim 10^3 - 10^4$ sec^{-1}, $W \simeq 3 \cdot 10^2$ sec^{-1}.

Let us investigate the simplest case of interaction of two types of oscillations close in intensity. In this case the equations have the form

$$\begin{gathered}
\dot{I} + \gamma_1 I = \beta I[N_{11} + N_{12}(\cos\Phi - s\sin\Phi)], \\
\dot{\Phi} + \Delta = -\beta[sN_{11} + N_{12}(\sin\Phi + s\cos\Phi)], \\
\dot{N}_{11} - \beta_0 + \gamma_0 N_{11} = -2\beta\nu I[N_{11} + N_{12}(\cos\Phi - s\sin\Phi)], \\
\dot{N}_{12} + \gamma_0 N_{12} = -4\beta\mu_{12} I[N_{12} + N_{11}(\cos\Phi + s\sin\Phi)].
\end{gathered} \tag{4}$$

Here $\beta = aN_0$, $I = a_1^2 \simeq a_2^2$, $\nu = \mu_{11} + \mu_{12}$, $\gamma_0 = W_0 + 1/\tau$, $\beta_0 = W_0 - 1/\tau$. As $N_{12} \to 0$, equations (4) pass into the kinetic equations (7).

Let us investigate the stability of the stationary states of system (4). One of the stationary states has the form

$$(N_{12})_0 = -\gamma\delta, \quad (N_{11})_0 = \gamma + \delta^2\gamma^2[\gamma^{-1} + \gamma_0\nu/2\mu(\beta_0 - \gamma\gamma_0)], \tag{5}$$

$$I_0 = [\beta_0 - \gamma_0(N_{11})_0]/2\beta\nu\gamma, \quad \Phi_0 = \pi/2 - \delta\gamma[\gamma^{-1} + \gamma_0\nu/2\mu(\beta_0 - \gamma\gamma_0)].$$

Here $\delta = \Delta(\beta\gamma)^{-1} \ll 1$, $\gamma = \gamma_1\beta^{-1}$, $\mu = \mu_{12}$. For ruby $\beta \sim 10^9$ sec^{-1}. This stationary state corresponds to the minimum interaction energy of the oscillation types. If the stability of this state is investigated, then the solutions of the corresponding linearized system of equations have the form $\sim \exp\lambda_i t$, where

$$\lambda_{1,2} = -\frac{1}{2}(\gamma_0 + 2\beta\nu I_0) - \frac{\delta^2\gamma_0\gamma}{4I_0}(\nu - 2\mu) \pm i\omega_{\text{kin}},$$

$$\lambda_{3,4} = -\frac{1}{2}(\gamma_0 + 4\beta\mu I_0) + \frac{\delta^2\gamma_0\gamma}{4I_0}(\nu - 2\mu) + i\sqrt{2\mu/\nu}\,\omega_{\text{kin}}. \tag{6}$$

Here $\omega_{\text{kin}} = (\beta_0 - \gamma_0\gamma)\gamma_1\gamma^{-1}$ is the kinetic frequency. Since $\nu > 2\mu$ (for a parallelepiped $\nu = 35/8$, $\mu = 1$), there is a growing solution $\lambda_{3,4}$ oscillating with a frequency close to the kinetic one. The growth increment increases as the pumping is decreased, which is in qualitative agreement with experiment ([2]). An instability of this type was found earlier in work ([6]), in which a system of equations for the electric-field intensity and the polarization of the active medium was used.

The other stationary state has a phase difference $\Phi_0 = \delta(2\mu+\nu)\gamma_0\gamma/2\mu(\beta_0-\gamma\gamma_0)$ and corresponds to the maximum interaction energy of the oscillation types. The solutions of the linearized system of equations have the form $\sim \exp\lambda_i t$, where

$$\lambda_{1,2} = \gamma_0(\nu-\mu)/2\mu - \beta I_0(\nu+2\mu) \pm i\sqrt{2\mu/\nu}\,\omega_{\text{kin}},$$

$$\lambda_3 = 4\mu\beta\gamma_1 I_0\gamma_0^{-1}, \qquad \lambda_4 = -\gamma_0(\nu+2\mu)/2\mu. \tag{7}$$

Since $\lambda_3 > 0$, this stationary state is always unstable, and the system will pass into a stationary state of the first type.

If several types of oscillations are excited in the resonator, then a similar instability may arise in the interaction of oscillation types with identical longitudinal but different transverse indices.

Received
3 VIII 1966

CITED LITERATURE

1. B. Davis, D. Keller, *Appl. Phys. Lett.*, **5**, 80 (1964).
2. T. N. Zubarev, A. K. Sokolov, *DAN*, **159**, 539 (1964).
3. Yu. A. Tarasov, *DAN*, **165**, 537 (1965).
4. E. Buley, F. Cummings, *Phys. Rev.*, AB **4**, 1454 (1964).
5. A. I. Alekseev, Yu. A. Vdovin, V. M. Galitskii, *ZhETF*, **46**, 320 (1964).
6. N. G. Basov, V. N. Morozov, A. N. Oraevskii, *ZhETF*, **49**, 895 (1965).
7. A. K. Sokolov, T. N. Zubarev, *FTT*, **6**, 2590 (1964).

Note: Figure translations are in progress. See original paper for figures.