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Abstract

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DETERMINATION OF THE CONTINUOUS USEFUL COMPONENT OF QUASI-CONTINUOUS PULSE SYSTEMS

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In a very broad class of pulse systems, quantization of actions in time is only an intermediate stage in the transformation of a continuous input action into a continuous action at the output, or into such a pulse action whose working function is performed by its continuous "useful" component. For example, in information circuits quantization may serve as a means of reducing the influence of interference, of multiplexing channels, etc.; in power circuits, as a method for regulating a constant energy parameter, and so forth.

The quality of such quasi-continuous pulse systems is characterized by the minimum of the distortions introduced by quantization into the reproduction of the output action. For quasi-continuous pulse systems it is necessary to know the continuous useful component of the output action, and it is sufficient to be able to estimate the distortion component of this useful component—the quantization noise.

However, in the theory of pulse systems ((*1-4*) and others), complete expressions for actions are sought by means of the special apparatus of the discrete Laplace transform and the z-transform, while the question of determining continuous useful functions and estimating distortions remains open.

The action at the output of a pulse element $x_2(t)$ may be regarded as a periodic function discretely modulated by the input action. Therefore an adequate description of pulse systems is given by step Fourier series (*5*). The constant component, amplitudes, phases, and frequencies of the harmonics of such a series are step functions of time.

The output action $x_2(t)$ is written as a step series:

$$x_2(t) = \sum_{n=1}^{\infty} \overline{x}_{2)n} + \sum_{n=1}^{\infty}\sum_{k=1}^{\infty} \overline{A}_{nk} \sin(k\omega_n t - \varphi_{kn}). \tag{1}$$

Here n is the number of the interval of the step function; k is the number of the harmonic; ω_n is the pulse repetition frequency.

The bar above signifies that the quantity beneath it is equal to zero over the entire time axis except the given interval,

$$\overline{f}(t) = \begin{cases} f_n(t), & \text{for } nT < t < (n+1)T, \\ 0, & \text{for } (n+1)T < t < nT. \end{cases} \tag{2}$$

The frequency of the series is determined by the pulse repetition period, i.e., by the intervals T_n:

$$\omega_n = 2\pi/T_n, \tag{3}$$

and in the general case the frequency of the modulated series is also a step function of time.

The coefficients of the step series are found by expanding the nonperiodic pulse function $x_2(t)$ in Fourier series on each finite interval T_n.

Let us assume that, for all intervals, there exist single-valued analytic dependences of the constant component, as well as of all amplitudes, phases, and the fundamental frequency (or repetition intervals T_n) on the discrete value x_{1n} of the continuous input function $x_1(t)$ in this interval, taken, for definiteness, at the left end of the interval:

$$x_{20n} = Q_0(x_{1n}), \tag{4}$$

$$A_{kn} = Q_{Ak}(x_{1n}), \qquad \varphi_{kn} = Q_{\varphi k}(x_{1n}), \tag{5}$$

$$\omega_n = Q_\omega(x_{1n}) \quad \text{or} \quad T_n = Q_T(x_{1n}).$$

These dependences are the characteristics of the pulse element, respectively for the constant component (4) or for the harmonics (5).

Then the useful action will be defined as a continuous function of time depending on the continuous input action according to the corresponding equation (4) or (5) of the characteristic of the pulse element, but lagging behind it in time by one half of the interval.

Namely, if the modulated constant component is useful, then it will be defined by the formula

$$x_{20}(t) = Q_0[x_1(t - \tfrac{1}{2}T)], \tag{6}$$

whereas if one of the harmonics is useful, then its amplitude and phase will be continuous functions of time

$$A_{k0}(t) = Q_{A0}[x_1(t - \tfrac{1}{2}T)],$$

$$\varphi_{k0}(t) = Q_{\varphi 0}[x_1(t - \tfrac{1}{2}T)]. \qquad (7)$$

In exactly the same way, under frequency modulation,

$$\omega_{0n}(t) = Q_{\omega 0}[x_1(t - \tfrac{1}{2}T)]$$

or

$$T_n(t) = Q_{T0}[x_1(t - \tfrac{1}{2}T)]. \qquad (8)$$

In many practically important cases the characteristic of the pulse element is linear in the operating region, and then

$$x_{20}(t) = Q_0 x_1(t - \tfrac{1}{2}T). \qquad (9)$$

and the pulse element has the transfer function

$$Q_{20}(p) = Q_0 e^{-\frac{1}{2}Tp}. \qquad (10)$$

The distortion components are determined by the same formulas.

When a continuous linear circuit, which usually has or is endowed with filtering properties, is connected after the pulse element, the useful action at its output, by virtue of the superposition principle, is determined directly by the characteristic of the pulse element, while the distortions are estimated from the pulsations of the steady-state regime in the worst zone.

As a result, it becomes possible to carry out the analysis of quasi-continuous pulse systems by the ordinary apparatus of continuous circuits. The class of pulse systems that can be analyzed is substantially expanded. Their physical content is revealed.

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Note: Figure translations are in progress. See original paper for figures.