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# The structure of a solution of the equation $u^{'} = U_0(\nu)u + \sum_{n=1}^{\infty}(\nu)u^{1+\alpha_n} \equiv U(u,\nu)$ in a small neighborhood of the origin

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**Date:** 1967-01-01T00:00:00+00:00

## Abstract

The article constructs a solution to the equation

$$u' = U_0(\nu)u + \sum_{n=1}^{\infty}(\nu)u^{1+\alpha_n} \equiv U(u,\nu) \tag{1}$$

where $u$ and $\nu$ are polar coordinates. At the same time, there is one case where the solution to equation (1) cannot be obtained in the specified form.

In this case, the equation to which equation (1) is reduced is specified, the investigation of which is the subject of a subsequent article.

## Full Text

## Introduction

This section examines a class of differential equations and their asymptotic properties, as presented in the work submitted to Moscow State University in May 1967. We consider a system of the form:

$$u' = U_0(v)u + \sum U_n(v)u^{1+\alpha_n} = U(u,v)$$

where $u$ and $v$ are variables, and the coefficients $U_s(v)$ are defined by the series:

$$U_s(v) = \sum U_{s,j}e^{p_s v}$$

In this context, $p$ is a constant, and the functions $U(u,v)$ and $U_s(v)$ are assumed to be analytic. We further assume the conditions $p_{s,j} < p_{s,j+1}$ and $p_{s,0} = 0$ for

$s \geq 1$ and $j \geq 1$. For the initial values $u \approx u_0$ and $v \approx v_0$, we define the characteristic term $U_{0,0}(v) \approx v_0$. The expansion coefficients are given by:

$$U_n(v) = \sum u_{n,s}(v) e^{b_{n,s} v}$$

where $b_n$ and $b_{n,s}$ are constants satisfying $b_n < b_{n+1}$ and $b_{n,s} < b_{n,s+1}$. It is established that $u_{n,0} = 0$ for $n > 1$ and $s > 0$.

By applying a transformation to the original system (1), we can reduce it to the following form:

$$z' = 1 + \sum L_n(v) z^n$$

where the functions $L_n(v)$ are related to the original coefficients $U_n(v)$ through the integral:

$$L_n(v) = U_n(v) \exp\left(\alpha_n \int U_0(\eta) d\eta\right)$$

Based on this formulation, we consider two primary cases. First, we analyze the behavior of the system when the constant $c'$ is fixed. Second, we examine the conditions under which the solution $u(v)$ can be represented as a series:

$$u = c + \sum u_n(v) u^n$$

where $c$ and $b_n$ are positive constants. For the system described in (1), we assume the sequence of exponents satisfies $b_n < b_{n+1}$ for $n \geq -1$. The functions $U_n(v)$ are defined according to the properties established in (2). This analytical framework was formally presented at Moscow State University by M. V. [Name truncated] on May 16, 1967.

*Note: Figure translations are in progress. See original paper for figures.*

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