SOME FEATURES OF THE BEHAVIOR OF ULTRALONG WAVES IN THE ATMOSPHERE

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Abstract

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SOME FEATURES OF THE BEHAVIOR OF ULTRALONG WAVES IN THE ATMOSPHERE

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The longest quasi-stationary waves in the atmosphere $m = 1, 2, 3$ ($a_m \cos m\lambda + b_m \sin m\lambda$, λ is longitude) have attracted much attention from climatologists, apparently in connection with the first failures of numerical forecasts on the hemisphere ([1,2]). In modern forecasting schemes they are artificially stabilized. Nevertheless, a more refined analysis has shown that these waves constantly oscillate about some fixed position, while their amplitudes vary greatly. The maximum displacement of ultralong waves to the west and east relative to the normal position reaches 1/4 of the wavelength. At present there are no numerical schemes that would forecast their changes and displacements. However, these waves play a role in the formation of large-scale weather and in the overall energetics of the atmosphere. It is they that effect the principal transfer of kinetic energy from the lower layers of the atmosphere to the upper ones; they are also most effective in transporting heat northward ([3,4]).

In the present paper an attempt is made to reproduce the behavior of the longest atmospheric waves in the simplest idealized model, describing the motion of an incompressible fluid with an interface surface $h(x, y, t)$, whose density is $\rho = \rho + \rho'(x, y)$. We assume that $\frac{\rho'}{\rho} \ll 1$ and

$$\frac{\rho - \rho_0}{\rho} \approx 1 - \frac{\rho_0}{\rho}\left(1 - \frac{\rho'}{\rho}\right),$$

where $\rho_0 = \text{const}$ is the density of the air layer lying above the interface surface. We write the equations of motion

$$\begin{gathered}\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - lv = -\left(\tilde{g} + g\frac{\rho_0}{\rho^2}\rho'\right)\frac{\partial h}{\partial x},\\ \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + lu = -\left(\tilde{g} + g\frac{\rho_0}{\rho^2}\rho'\right)\frac{\partial h}{\partial y},\\ \frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y} + h\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0, \qquad \tilde{g} = g\left(1 - \frac{\rho_0}{\rho}\right);\end{gathered} \tag{1}$$

u and v are the velocity components along the axes x and y; l is the Coriolis parameter; g is the acceleration of gravity.

Instead of the first two equations (1), we shall henceforth take the equations of vorticity and divergence, linearize them, as well as the third equation of (1), with respect to the basic westerly current

$$U = -\frac{\tilde{g}}{l}\frac{dH}{dy} = \text{const}$$

and simplify by assuming that all perturbations are independent of y. Neglecting squares of perturbations, we obtain

$$\frac{\partial v_x}{\partial t} + U\frac{\partial v_x}{\partial x} + \beta v + l u_x = f(x),$$

$$\frac{\partial u_x}{\partial t} + U\frac{\partial u_x}{\partial x} + \beta u - l v_x + \tilde{g} h_{xx} = 0,$$

$$\frac{\partial h}{\partial t} + U\frac{\partial h}{\partial x} - \frac{lU}{\tilde{g}} v + H u_x = 0. \tag{2}$$

($\beta = dl/dy$). On the right-hand side of the first equation

$$f(x) = g\frac{\rho_0}{\rho^2}\frac{lU}{\tilde{g}}\frac{\partial \rho}{\partial x}$$

represents a certain external stationary source, analogous to the horizontal baroclinicity in the atmosphere, which is generated by the climatic inhomogeneity of the Earth' s surface (by continents and oceans).

The system (2), with initial data $h^0 = a \sin kx,\ v^0 = \frac{\tilde{g}}{l} ka \cos kx,\ u^0 = 0$, and if $f(x) = -A\frac{k^3\tilde{g}}{l}\cos kx\ \left(\sigma = U - \frac{\beta}{k^2}\right)$, has the solution

$$h = \sum_{j=1,2,3} A_j \sin k(x - c_{jt}) + A \sin kx,$$

$$v = \sum_{j=1,2,3} \delta_{jA} j \cos k(x - c_{jt}) + Ak\frac{\tilde{g}}{l}\cos kx,$$

$$u = \sum_{j=1,2,3} \alpha_{jA} j \sin k(x - c_{jt}), \tag{3}$$

where c_j are the roots of the characteristic equation of system (2). Over a broad range of real parameter values these roots are real. In solution (3) α_j and δ_j are equal to

$$\alpha_j = \frac{k^2\tilde{g}(\beta - k^2 d_j)}{(\beta - k^2 d_j)^2 - k^2 l^2}, \qquad \delta_j = -\frac{kl\alpha_j}{\beta - k^2 d_j}, \qquad d_j = U - c_j;$$

the A_j are determined from the initial conditions. Of course, solution (3) is valid only when $\sigma \neq 0$.

Let us note that if $a = A$, system (2) will have the stationary solution

$$h = A \sin kx, \qquad v = Ak\frac{\tilde{g}}{l}\cos kx, \qquad u = 0.$$

We take the following parameter values: $a = 100$ m, $H = 3 \cdot 10^6$ m, $\tilde{g} = 2.3$ m $\cdot$ sec^{-1}, $U = 7.5$ m $\cdot$ sec^{-1}, $l = 10^{-4}$ sec^{-1}, $\beta = 1.6 \cdot 10^{-11}$ m$^{-1} \cdot$ sec^{-1}, $k = 2\pi/L$, $L = 1.5 \cdot 10^7$ m. We draw attention to the fact that here we are considering a very long wave and assume that the interface is located approximately at the 700-mb level.

From (3), as a result of calculations we obtain

$$h = (a - A)\,[0.9658 \sin k(x - c_1 t) - 0.0078 \sin k(x - c_2 t) + 0.0420 \sin k(x - c_3 t)] + A \sin kx,$$

$$v = (a - A)\,[0.0103 \cos k(x - c_1 t) + 0.0011 \cos k(x - c_2 t) - 0.0018 \cos k(x - c_3 t)] + 0.0096 A \cos kx,$$

$$u = (a - A)\,[-0.0032 \sin k(x - c_1 t) + 0.0012 \sin k(x - c_2 t) + 0.0020 \sin k(x - c_3 t)]$$

(h is expressed in meters, v and u in m $\cdot$ sec^{-1}), $c_1 = -10.55$ m $\cdot$ sec^{-1}, $c_2 = -330.19$ m $\cdot$ sec^{-1}, $c_3 = 180.96$ m $\cdot$ sec^{-1}.

To an accuracy of up to 8.4% (in amplitude), an approximate solution for the function h is

$$h \approx (a - A) \sin k(x - c_1 t) + A \sin kx = \tilde{a}(t) \sin[kx - \psi(t)], \tag{4}$$

where

$$\tilde{a}(t) = \sqrt{a^2 - 2A(a - A)[1 - \cos kc_1 t]},$$

$$\psi(t) = \operatorname{arc\,tg}(a - A) \sin kc_1 t/[(a - A) \cos kc_1 t + A]. \tag{5}$$

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

It is now clear that system (2) will describe the oscillation of a wave about a certain fixed position if $(a-A)\cos kc_1t + A$ in the expression for $\psi(t)$ does not vanish, i.e., if

$$\frac{1}{2} < A/a < 1 \quad \text{or} \quad A/a > 1. \tag{6}$$

Apparently, for the real atmosphere the fulfillment of the second condition in (6) is unlikely. Of greatest interest is the case $\frac{1}{2} < A/a < 1$. Considering only this case, one may say that oscillations of the wave being studied are possible here if a (a is the amplitude of the "specific" ultralong wave at the moment when its phase coincides with the "climatic" wave) does not reach and does not exceed twice the amplitude of the corresponding stationary "climatic" wave.

Fig. 1

Undoubtedly, the longest waves in the atmosphere are generated by such climatic factors as the thermal inhomogeneity of the Earth's surface and large mountains. However, arising under the influence of these causes, long waves are apparently amplified at the expense of certain other sources of energy. In a number of works (for example, ([5])) it has been shown that mobile cyclonic waves of smaller scale $m > 4$ continually supply kinetic energy to the longest quasi-stationary waves. The possibility has also been found of increasing the kinetic energy of these waves through the direct release of their potential and internal energy ([3]).

In the work of Eliassen ([6]), the phase angles of individual geopotential waves on the 500 mb surface were calculated as functions of time for the period from 21 X to 30 XI 1950. In Fig. 1 the graph from ([6]) is reproduced for $m = 1$ (50° N). If one disregards oscillations with periods of 5-6 days, we see that the behavior of the wave $m = 1$ is characterized by the following features. The wave oscillates about its normal position with a period of approximately one month. It slowly moves westward, and when the deviation from the normal position reaches 1/4 of its length, a very rapid displacement eastward occurs, by approximately half a wavelength; then it again moves slowly westward.

In Fig. 2 the phase angles $\psi(t)$ are shown, and in Fig. 3 the amplitudes $\tilde{a}(t)$, calculated from formulas (5) for various values of A/a. The period of oscillation in this case is $T = 16.46$ days.

Fig. 2

Fig. 3

Figure 3: Fig. 3

From Figs. 1 and 2 it is clear that the ratio A/a in the real atmosphere is greater than 1/2, but rather close to this limiting value. The latter circumstance apparently is of definite interest and must be explained in the future.

In Figs. 2 and 3 examples are given for which $0 \leqslant A/a \leqslant 1/2$. In these cases the solution (4) is a wave running westward.

Calculations show that a stationary source of the horizontal-baroclinicity type in equations (2), whatever its intensity, leads to a decrease in the westward propagation speed of ultralong waves (in the case of a "weak" source, $0 \leqslant A/a \leqslant 1/2$, this is true for the time intervals when $\tilde{a}(t)$ is maximal). The influence of large mountain systems and of the stratosphere is analogous. In this connection it appears expedient to include quasi-stationary sources even in forecast models for 3–5 days. We note that the influence of horizontal baroclinicity on the stabilization of ultralong waves has been pointed out more than once by E. N. Blinova (for example, (7)).

Fig. 3

From a scale analysis of the equations of hydrothermodynamics, and also from observations, it is known that the meridional dimensions of the longest atmospheric waves themselves are considerably smaller than their zonal wavelengths. Let us give estimates of the phase velocity of propagation of long two-dimensional waves. When 1/4 of the wavelength in the direction of the y-axis is $W = 1.5 \cdot 10^6$ m (the remaining parameters as before), $c_1 = -2.38$ m sec^{-1}, $c_2 = -318.42$ m sec^{-1}, $c_3 = 335.79$ m sec^{-1}, $T_1 = 72.95$ days; if $W = 3 \cdot 10^6$ m, $c_1 = -6.67$ m sec^{-1}, $c_2 = -262.28$ m sec^{-1}, $c_3 = 283.95$ m sec^{-1}, $T_1 = 26.03$ days (T_1 is the period of oscillation of the Rossby wave).

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Note: Figure translations are in progress. See original paper for figures.