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Abstract

Full Text

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AN ALGORITHM FOR REFINING THE PARAMETERS OF A PARTICLE MOVING IN A HYDROGEN CHAMBER, WITH ALLOWANCE FOR THE INHOMOGENEITY OF THE MAGNETIC FIELD AND FOR SLOWING DOWN

(Presented by Academician A. I. Alikhanov on 25 III 1967)

A method is described for refining the parameters of a particle moving in a liquid-hydrogen bubble chamber in a variable magnetic field. The method also takes into account the slowing down of the particle as a function of its mass. The advantage of this method is the simplicity of its implementation as a computer program and the small amount of computing time required. At the same time, the accuracy it provides is sufficient for real chambers. The algorithm described is used in the program for processing observations in hydrogen chambers at the Institute of Theoretical and Experimental Physics and proves itself well, in any case, in the processing of photographs obtained with a 50-centimeter chamber with a field of the order of 20,000 oersteds.

1°. Initial data. The initial data are: first, the coordinates of a certain number of points on the track (X_i, Y_i, Z_i), $i = 1, 2, \dots, n$; second, an initial approximation to the particle trajectory, found in the form of a helix whose axis is parallel to the Z axis. The points (X_i, Y_i, Z_i) are assumed to have been obtained with errors which, for different points, are independent of one another. In addition, for a given point the error of the Z coordinate is independent of the errors of the X and Y coordinates. The variances of the errors of Z are the same for all points, and, for the projection onto the XY plane, the errors in the direction of the normal to the trajectory also have identical variances.

We denote by ρ_0, x_0, y_0, respectively, the radius and the coordinates of the center of the helix, and by θ_0 the angle of the helix with the XY plane.

2°. Equation of motion and slowing-down function. Let the particle have mass m. Its motion in the chamber is described by the equations

$$\ddot{\mathbf{r}}(s) = [\dot{\mathbf{r}}(s), \mathbf{H}(s)]/p(s), \tag{1}$$

$$\dot{p}(s) = f(p(s)/m), \tag{2}$$

where s is arc length; a dot denotes differentiation with respect to s; p is the magnitude of the momentum with the sign of the particle charge; $\mathbf{H}$ is the magnetic-field vector.

Put

$$f\left(\frac{p}{m}\right) = -\frac{\alpha_1(m^2+p^2)^{3/2}}{p^3}\left[\ln\frac{\alpha_2 p^2}{m^2} - \frac{p^2}{m^2+p^2}\right],$$

where $\alpha_1 = 0.001769839$; $\alpha_2 = 0.676 \cdot 10^5$; p is measured in MeV/c; m in MeV; s in millimeters, and the unit of measurement of H is $\frac{1}{3} \cdot 10^5$ oersteds (see ([1])).

To find the solution of equation (2), we introduce, as usual, the function $s(p)$, equal to the range length in hydrogen (up to stopping) of a proton with initial momentum p. For computing the function $s(p)$, and also its inverse function $p(s)$, the following approximate formulas are proposed:

$$s = (p/67.8)^{1/0.2937} \qquad \text{for} \qquad p < 650,$$

$$s = 1.562 \cdot 10^{-5} p^{1/0.3454} \qquad \text{for} \qquad 650 \leq p < 1000,$$

$$s = 47\,060 - \sqrt{22.15 \cdot 10^8 - 1.724 \cdot 10^6 (p - 621.3)} \qquad \text{for} \qquad 1000 \leq p \leq 1500;$$

$$p = 67.8 s^{0.2937}$$
$$p = (10^5 s/1.562)^{0.3454}$$
$$p = -58 \cdot 10^{-6} s^2 + 5.46 \cdot 10^{-2} s + 621.3$$

for $s < 2138$,

for $2138 \leq s < 7575$,

for $7575 \leq s \leq 20\,528$.

For values $p > 1500$ the formulas are not given, since in real chambers a proton with such an initial momentum does not undergo noticeable losses. Tables have been prepared at ITEP that make it possible to compare the proposed approximate formulas with the results of direct integration of equation (2). For the approximate formulas, see also [2].

3°. The described method of calculation assumes that the error in measuring the magnetic field reaches 1%, and therefore, when determining the magnitude of the momentum from the curvature, there is no need to seek greater accuracy. It is also assumed that the magnetic field is directed approximately along the Z-axis: the horizontal component of the vector $\mathbf{H}$ does not exceed 10% of the magnitude of this vector.

4°. Simplification of equations (1). A cylindrical coordinate system ρ, φ, Z is introduced with its pole at the center (x_0, y_0) of the helix and with the polar axis directed toward the point (X_1, Y_1). Let (ρ_i, φ_i, Z_i) be the cylindrical coordinates of the point (X_i, Y_i, Z_i), $i = 1, 2, \dots, n$. In this coordinate system, equation (1) is replaced by the system

$$(\Delta\rho)'' = \rho_0 - \frac{\rho_0^2}{p(s)\cos\theta} H_z - \frac{\rho_0}{p(s)\cos\theta}(H_y \cos\varphi - H_x \sin\varphi); \qquad (3)$$

$$Z'' = -\frac{\rho_0^2}{p(s)\cos\theta}(H_x \cos\varphi + H_y \sin\varphi), \qquad (4)$$

where $\Delta\rho = \rho - \rho_0$, $1/\cos\theta = \sqrt{1 + 1/\rho_0^2 \cdot (dz/d\varphi)^2}$. The terms neglected here lead to errors in determining the magnitude of the momentum not exceeding 0.5%.

5°. Integration of equations (3) and (4). First equation (4) is integrated, and then equation (3), using the value $dz/d\varphi$ obtained in the integration of (4).

5.1. Integration of equation (4). When integrating this equation, the value of the magnitude of the momentum is considered constant and equal to the value obtained by fitting a helix. The values $f_1, f_2, \dots, f_n$ of the right-hand side of the equation are computed at all available points. A polynomial $B(\varphi) = b_2 + b_3\varphi + b_4\varphi^2$ is sought such that

$$\sum_{i=1}^{n} [f_i - B(\varphi_i)]^2 = \min .$$

As the integral of the equation, the polynomial

$$Z(\varphi) = b_0 + b_1\varphi + b_2\varphi^2/2 + b_3\varphi^3/6 + b_4\varphi^4/12$$

is taken. It contains two arbitrary constants, b_0 and b_1, which are found from the condition

$$\sum_{i=1}^{n} [Z_i - Z(\varphi_i)]^2 = \min .$$

5.2. After integration of equation (4), equation (3) is integrated. The integration of this equation is carried out in the following two modes: 1) the value of the momentum magnitude at the initial point $p(0)$ is known, and 2) the value of the momentum magnitude at the initial point is unknown. In case 2), the magnitude of the momentum is regarded as unchanged along the track.

5.2.1. The value of the momentum magnitude $p(0)$ at the initial point is known. The values $g_1, g_2, \dots, g_n$ of the right-hand side of the equation are computed at all points; in doing so it is assumed that $dz/d\varphi = b_1+b_2\varphi+b_3\varphi^2/2+$ $+b_4\varphi^3/3$. A polynomial $A(\varphi) = a_2 + a_3\varphi + a_4\varphi^2 + a_5\varphi^3 + a_6\varphi^4$ is sought such that

$$\sum_{i=1}^{n} \left[g_i - A(\varphi_i)\right]^2 = \min .$$

As the integral of equation (3), the polynomial

$$\Delta\rho(\varphi) = a_0 + a_1\varphi + \frac{a_2\varphi^2}{2} + \frac{a_3\varphi^3}{6} + \frac{a_4\varphi^4}{12} + \frac{a_5\varphi^5}{20} + \frac{a_6\varphi^6}{30}$$

is taken. It contains two arbitrary constants a_0 and a_1. These constants are found from the condition

$$\sum_{i=1}^{n} \left[\Delta\rho(\varphi_i) - (\rho_i - \rho_0)\right]^2 = \min . \tag{5}$$

5.2.2. The value of the momentum modulus at the initial point is unknown, but the momentum modulus is assumed constant along the track. The values $h_1, h_2, \dots, h_n$ of the expression

$$\frac{\rho_0^2}{\cos\theta} H_z + \frac{\rho_0}{\cos\theta}(H_y \cos\varphi - H_x \sin\varphi)$$

are computed at the points (X_i, Y_i, Z_i), $i = 1, 2, \dots, n$. A polynomial $D(\varphi) = d_2 + d_3\varphi + d_4\varphi^2 + d_5\varphi^3 + d_6\varphi^4$ is sought such that

$$\sum_{i=1}^{n} \left[h_i - D(\varphi_i)\right]^2 = \min .$$

As the integral of equation (3), the polynomial

$$\Delta\rho(\varphi) = d_0 + d_1\varphi - \frac{1}{p}\left[\frac{d_2\varphi^3}{2} + \frac{d_3\varphi^3}{6} + \frac{d_4\varphi^4}{12} + \frac{d_5\varphi^5}{20} + \frac{d_6\varphi^6}{30}\right] + \frac{\rho_0\varphi^2}{2}$$

is taken.

It contains three arbitrary constants: d_0, d_1, and $1/p$. These constants are found from condition (5).

6°. Refinement of the particle parameters. The quantity

$$\frac{\Delta p}{p} = \frac{1}{p_0}\left\{p_0 - \frac{m}{938}p\left[s\left(p_0\frac{938}{m}\right) - \frac{938}{m}l_0\right]\right\},$$

is computed, where p_0 and l_0 are approximate values of the momentum modulus and track length obtained by fitting a helical line; m is the mass assigned to the particle. This quantity characterizes the relative decrease of the momentum modulus along the track. Depending on whether it is less than the prescribed standard* or not, one should proceed in one of the following ways.

6.1. $\Delta p/p$ is less than the standard. In this case the momentum modulus is assumed constant along the track. Two subcases are possible:

6.1.1. The momentum modulus p is known in advance (for an incoming particle). Integration of equations (3) and (4) is carried out according to §§ 5.1 and 5.2.1.

6.1.2. The momentum modulus is not known in advance. Integration of equations (3) and (4) is carried out according to §§ 5.1 and 5.2.2.

On the basis of the results of integrating equations (3) and (4), the particle parameters are then refined.

6.2. The quantity $\Delta p/p$ is greater than or equal to the standard. The initial value of the momentum modulus is unknown, and it changes along the track according to the formulas of § 2. Refinement of the values of the particle parameters is carried out by means of a trial process: such a value $p(0)$ —the momentum modulus at the initial point—is selected so that, as a result of integra-

of the solution of equations (3) and (4), according to Secs. 5.1 and 5.2.2, the minimum (5) was the smallest.

Let us note that this extremum is expressed sufficiently well. For a 50-centimeter chamber with a field of the order of 20,000 oersted, quadratic interpolation over points separated by a distance of the order of $0.1\,p(0)$ gives it with sufficient accuracy. Therefore the search can be made short.

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* For a 50-centimeter chamber with a field of the order of 20,000 oersteds, we take the standard for $\Delta p/p$ to be equal to 0.02.

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CITED LITERATURE

1. *Experimental Nuclear Physics*, ed. E. Segrè, 1, Moscow, 1955, p. 144.

2. *High-Energy Particle Data*, 2, Berkeley, California, June, 1957.

Note: Figure translations are in progress. See original paper for figures.

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