ON THE THEORY OF INTERPRETATION OF MAGNETIC AND GRAVITATIONAL ANOMALIES ON THE BASIS OF ANALYTIC CONTINUATION

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Abstract

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V. N. STRAKHOV

ON THE THEORY OF INTERPRETATION OF MAGNETIC AND GRAVITATIONAL ANOMALIES ON THE BASIS OF ANALYTIC CONTINUATION

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In paper ([1]) the author put forward the following concept: from the values observed at $z = 0$ of a certain stationary (magnetic, gravitational, electric) field, only the singular points of the harmonic function describing this field can be determined uniquely. Consequently, the inverse problem for stationary fields consists in determining the indicated singular points.

In principle, three approaches are possible to the problem of determining the singular points of functions describing anomalous fields. All of them are connected with the apparatus of analytic continuation of these functions, since a singular point of a function is uniquely defined as a point to which continuation is impossible. For simplicity we shall restrict ourselves to consideration of the plane (two-dimensional) problem.

The **first approach** consists in determining the position of certain (finite or infinite) lines on which the singular points of the function are contained (at least one) and which are the boundaries of the domains of analyticity of the function. The position of the indicated lines can be determined on the basis of analytic representations of the function inside these domains. For example, the distance H from the Ox axis to the nearest singular point of the function can be determined as the ordinate of convergence of the Fourier integral or Newton series representing the function in the half-plane $z > -H$ ([2], [3]).

The **second approach** consists in using a procedure for constructing the field in an ever-expanding sequence of domains (each subsequent one contains the preceding one as its part), with emergence into the neighborhoods of singular points. After analysis of the character of the field (represented, for example, by a family of isolines) in the neighborhood of singular points, the (approximate) finding of the latter is carried out. For example, in a number of important cases the singularities can be found by extrapolating (by eye) the course of the isolines to their intersection at one point (the sought singularity).

Figure 1

Figure 1: Figure 1

The described method proves effective in the interpretation of anomalies ΔZ and Δg from series of sheet-like perturbing bodies with approximately the same depth of their upper edges ([4], [5]). In this case the sequence of expanding domains is the set of horizontal strips, $0 \geq z \geq z_0 > -H$. An example of the interpretation of one of the magnetic anomalies ΔZ at the KMA is given in Fig. 1 (after V. M. Devitsyn and I. A. Zhavoronkin).

The **third approach** consists in using the method of erasing singularities. The latter is based on the fact that any practically realizable method of analytic continuation is discrete and approximate, and therefore, formally, analytic continuation can be carried out into any domains, including those that knowingly contain field sources (singular points of this field). Its essence, however, is that we first actually carry out a formal analytic continuation into a specified domain knowingly containing sources, and then look for signs

that in this region there exist sources (singular points). In doing so, we naturally assume that the singular points (or at least some of them) are stable—in the sense that the process of their erasure is not complete and that, in the formally constructed "field,"strong traces of their "former"existence are preserved, making it possible to determine (approximately) the location of the erased singularities.

Most often, horizontal strips $0 \geq z \geq z_0 > -H_{\text{calculated}}$, where $H_{\text{calculated}} > +H$, are chosen as the regions in which the formal continuation is carried out. In this variant it is possible reliably to detect singular points from the "upper tier" of singularities—of the pole type or well-localized logarithmic branch points corresponding to sources of "high intensity"(with a large coefficient A in the term $A \ln(s - s_0)$, $s = x + iz$, corresponding to such points in the complex-analytic description of the field).

Fig. 1. Example of interpreting a ΔZ anomaly (KMA).
1—ferruginous quartzites; 2—host rocks; 3—isolines in thousands of gammas; 4—boreholes

Let there be, at the level $z = -H$, a finite number of singular points of the type indicated above. We shall find the function, formally treated as the analytic continuation of the field, at the levels $z = z_k = -k\Delta z$, $\Delta z > 0$, $k = 1, 2, \ldots, n$, $z_n > H_{\text{calculated}}{}^*$. In passing from levels $z_n > -H$ to levels $z_k < -H$, the character of the field changes abruptly—the so-called "field disintegration effect" appears. On graphs of the field for levels $z = \text{const}$, it is manifested by a sharp change in the shape of the graphs when crossing the boundary straight line $z = -H$ (which carries the erased singularities). The smooth, "regular" form familiar to the eye of an exploration geophysicist is replaced by an "irregular" form of "wave packets" with a definite (constant for all levels) "frequency of oscillation" and with decaying amplitude. The projections of the centers of the

Figure 2

Figure 2: Figure 2

"wave packets" onto the straight line $z = -H$ are located near the "erased" singularities. As the depth of the level increases, the amplitudes of the "wave packets" increase.

On maps of isolines of the "field" in the vertical plane, the position of the "boundary" straight line $z = -H$ and of the singularities located on it is recorded most clearly. Namely, upon approaching the boundary straight line, the isolines begin to show a tendency to form bundles of vertically oriented isolines. The most "intense" bundles form near vertical straight lines passing through the singular points. A "striped structure of the field" also forms, consisting of vertical bands of alternating positive and negative field values. The "ends" of these bands are localized near the boundary straight line $z = -H$.

* For levels $z_k > -H$ the formal continuation coincides with the true one; but the value of H is unknown to us and is precisely what is to be determined.

The noted basic regularities of the "field-decay effect" are clearly expressed in the theoretical example of the field Δg from a horizontal circular cylinder of infinite extent, with its center at the point $z = -H,\ x = 0$ (Fig. 2). The theoretical explanation of the "field-decay effect" consists, in its main features, of the following. All linear methods of approximate analytic continuation in which computational formulas of the form

$$U(k\Delta x, z_k) = \sum_{-N}^{+N} C_m^{(N)} U(k\Delta x + m(\Delta x)_{\text{calc}}, 0)$$

are used (where $C_m^{(N)}$ are coefficients, Δx is the spacing at which the field is specified, $(\Delta x)_{\text{calc}}$ is the "step" selected in calculating the field values), and which provide sufficiently high accuracy of continuation for $z \gtrless -H/2$, are, to a first approximation, equivalent to the so-called Rendu method[6]. The essence of the latter is as follows: the function $U(x, 0)$, specified by its values at the points $x_k^* = k(\Delta x)_{\text{calc}},\ k = 0,\ \pm 1,\ \pm 2, ...,$ is replaced by the function interpolating it with a limited spectrum of degree not higher than $\sigma = \pi/(\Delta x)_{\text{calc}}$. The latter is analytically continued over the entire plane of the complex variable $s = x + iz$, except for the infinitely distant straight line $z = -\infty$, as an entire harmonic function of degree not higher than $\sigma = \pi/(\Delta x)_{\text{calc}}$. It can be shown that this entire harmonic function, at the points where the singularities of the original function $U(x, z)$ are located, must have precisely the character revealed by computations on model examples. In this case

Fig. 2. "Field-decay effect" for a horizontal circular cylinder. **a** –graphs of the field for levels $-z = \text{const}$; **b** –map of isolines of the field in a vertical plane.

Fig. 3. "Field-decay effect" in the presence of random errors in the initial data. *a*—continuation without smoothing; *b*—continuation with smoothing

Figure 3: Fig. 3. "Field-decay effect" in the presence of random errors in the initial data. *a*—continuation without smoothing; *b*—continuation with smoothing

the formation of "wave packets" occurs when each singularity is passed; however, in practice only the singularities of the "upper tier" are determinable, since the "field-decay effect" when passing the lower-lying singularities occurs against the background already formed earlier (when passing the singular points closest to the Ox axis) and is therefore imperceptible.

The correctness of this theoretical explanation of the "field-decay effect" is confirmed by calculation. In Fig. 2 the dashed lines show

Fig. 3. "Field-decay effect" in the presence of random errors in the initial data. *a*—continuation without smoothing; *b*—continuation with smoothing

the graphs of an entire harmonic function of finite degree $\sigma = -\pi/(\Delta x)_{\text{grid}}$, interpolating the initial field at $z = 0$ in a system of equidistant points with spacing $(\Delta x)_{\text{grid}}$.

In practice, the application of the method of eliminating singularities and determining the position of singular points from the "field-decay effect" is hindered by the presence of appreciable random errors in the observed field values and by the known (7) instability of the analytic-continuation procedure. As a result, the objective effect of decay (associated with the process of "eliminating" singularities) is obscured by another effect, associated with the growth of the random component in the initial field, which is in no way connected with the singularities. However, by applying a special smoothing operation the process of formal continuation can be made stable. Figure 3 gives the results of calculations for a model example of the field V_{zz} from two beds of infinite depth, without smoothing (a) and with smoothing (b) at each level. A random component of amplitude $0.5\,(V_{zz})_{\max}$ was introduced into the initial values of the field V_{zz}.

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Note: Figure translations are in progress. See original paper for figures.