On the Question of the Stability of a Class of Relay Systems

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Abstract

Relay control systems of the form

$$x^{(n)} + a_{nx}^{(n-1)} + \dots + a_2\dot{x} + a_1 x = u, u = -\operatorname{sign}(x^{(n-1)} + A_{n-1}x^{(n-2)} + \dots + A_2\dot{x} + A_1 x),$$

are considered, where x is the controlled coordinate; u is the control action; and $a_1, a_2 \dots, a_n$ and $A_1, A_2, \dots, A_{n-1}$ are real numbers. A characteristic property of these systems is the presence of "sliding" modes. The main content of the article consists of the stability analysis of the equilibrium position of the relay system using qualitative methods. In this work, sliding conditions are determined, the structure of the sliding region is studied, and necessary and sufficient conditions for the system' s representative point to reach the switching plane are provided. Bibliography: 11 items.

Full Text

Preamble

This paper, published in 1967 (Vol. 111, No. 8), examines the stability and dynamics of control systems described by differential equations. We consider a system of the form:

$$\dot{x}_i = x_{i+1} \quad (i = 1, 2, \dots, n-1)$$

$$\dot{x}_n = -\sum_{j=1}^{n} a_j x_j + u$$

where the control law is defined as $u = -\operatorname{sign}(A_1x_1 + \dots + A_{n-1}x_{n-1} + x_n)$. The state space is divided into two regions, (I) and (II), separated by the switching surface $A_1x_1 + \dots + A_{n-1}x_{n-1} + x_n = 0$. In region (I), the sum is positive, while in region (II), it is negative.

The analysis builds upon established methods in the theory of discontinuous systems [?, ?, ?, ?, ?, ?, ?, ?, ?]. Specifically, we investigate the behavior of trajectories near the switching surface and the conditions for the existence of sliding modes. The gradient of the switching surface is given by $\text{grad}(V) = \{A_1, \dots, A_{n-1}, 1\}$. By examining the derivative of the switching function along the trajectories of the system, we can determine the stability of the equilibrium point.

For the system to exhibit stable sliding motion on the surface $A_1 x_1 + \cdots + A_{n-1} x_{n-1} + x_n = 0$, the coefficients must satisfy certain algebraic relations. We define the characteristic equation associated with the sliding manifold as:

$$\sum_{j=1}^{n} \alpha_j \lambda^{j-1} = 0$$

where the coefficients α_j are derived from the system parameters a_j and the switching surface parameters A_j. As shown in previous works [?, ?], the stability of the $(n-1)$-th order dynamics within the sliding mode is a necessary condition for the overall convergence of the system.

We further analyze the asymptotic behavior of the system as $t \to \infty$. By employing Lyapunov' s direct method, we construct a quadratic form $V = \sum a_{ij} x_i x_j$ to prove the stability of the equilibrium $x_1 = x_2 = \cdots = x_n = 0$. The derivative of this Lyapunov function, $\dot{V}$, must be negative definite in the regions of continuity and satisfy the conditions for stability in the sense of Filippov [?] at the discontinuity surface.

The control law $u = -\text{sign}(\sigma)$ ensures that the state trajectory is forced toward the switching surface $\sigma = 0$. If the gain $K > 0$ is sufficiently large, the system enters a sliding mode, and the subsequent motion is determined solely by the parameters A_i. This makes the system robust against variations in the plant parameters a_i. The results presented here extend the findings of Barbashin [?], Alimov [?, ?, ?], and Drozdov [?] regarding the synthesis of variable structure systems.

In conclusion, we have established the conditions under which the multi-dimensional relay system remains stable. The relationship between the coefficients of the switching function and the characteristic roots of the system provides a practical framework for designing controllers that are insensitive to external disturbances and internal parameter fluctuations.

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Note: Figure translations are in progress. See original paper for figures.