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Abstract

Full Text

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CYBERNETICS AND CONTROL THEORY

S. D. ZEMLYAKOV, V. Yu. RUTKOVSKII

SYNTHESIS OF ALGORITHMS FOR CHANGING ADJUSTABLE COEFFICIENTS IN A SELF-TUNING CONTROL SYSTEM WITH A REFERENCE MODEL

(Presented by Academician B. N. Petrov, June 28, 1966)

Consider a search-free self-tuning system with a model ([1,2]), whose motion is described by the equations:

object

$$\sum_{\nu=0}^{m} q_\nu(t)\varphi^{(\nu)} = d(t)\mu, \tag{1}$$

actuator

$$\sum_{j=0}^{r} l_j \mu^{(j)} = -\sigma, \tag{2}$$

control law

$$\sigma = k^* \left[\sum_{s=0}^{n-1} k_s^* \varphi^{(s)} - k_g^* g(t) \right], \tag{3}$$

where φ is the controlled coordinate; μ is the regulator coordinate; $q_\nu(t)$, $d(t)$ are time-varying coefficients of the object; l_j are constants; k^*, k_s^*, k_g^* are adjustable coefficients of the regulator; $g(t)$ is the control action; $n = m + r$.

Without loss of generality one may take $q_m = 1$. Then (1)-(3) can be represented in the form

$$\varphi^{(n)} + \sum_{i=0}^{n-1} \left[b_i(t) + \frac{k^* d(t)}{l_r} k_i^* \right] \varphi^{(i)} = \frac{k^* d(t)}{l_r} k_g^* g(t), \tag{4}$$

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where $b_i(t)$ satisfy the identical equality

$$\frac{1}{l_r h}\sum_{j=0}^{r} l_j \sum_{\gamma=0}^{j}\left[C_j^{\gamma} h^{(\gamma)}\right]\left[\sum_{\nu=0}^{m}\sum_{y=0}^{j-\gamma} C_{j-\gamma}^{y} q_{\nu}^{(y)}(t)\,\varphi^{(\nu+j-\gamma-y)}\right] \equiv \sum_{i=0}^{n} b_i(t)\varphi^{(i)}; \quad h=\frac{1}{d(t)}. \tag{5}$$

Denote $k^* d(t)/l_r = a$; $k_i^* a = k_i$; $k_g^* a = k_g$. The variability of the efficiency of the regulating element $d(t)$ has a substantial effect on the control processes in system (1)-(3) and on the adaptation processes. However, there exist various methods of adjusting the overall gain coefficient k^* in such a way as to maintain, with sufficient accuracy, $k^* d(t) = \text{const}$ ([2]). Without considering such adjustment in detail, we shall assume $a = \text{const}$.

Represent the coefficients $b_i(t)$ and k_i in the form $b_i(t) = \bar{b}_i + \Delta b_i(t)$, $k_i = \bar{k}_i + \Delta k_i$, where $\bar{b}_i, \bar{k}_i$ are constant quantities; $\Delta b_i(t)$ are variable components; Δk_i are adjustable components. Denote $\bar{b}_i + \bar{k}_i = a_i$. Then equation (4) is written

$$\varphi^{(n)} + \sum_{i=0}^{n-1}\left[a_i + \Delta b_i(t) + \Delta k_i\right]\varphi^{(i)} = k_g g(t). \tag{6}$$

As the reference model, we shall choose the filter described by differential equation (6) for $\Delta b_i(t) = \Delta k_i \equiv 0$. We shall denote the output coordinate of the model by φ_{m}.

In the laws governing the variation of the adjustable coefficients, we shall consider only the integral components

$$\Delta k_i = z_i, \qquad z_i = \int_0^t \psi_i\left(t, \varepsilon, \dot{\varepsilon}, \dots, \varepsilon^{(n-1)}\right) dt. \tag{7}$$

It is required to synthesize the algorithms $\psi_i\left(t, \varepsilon, \dot{\varepsilon}, \dots, \varepsilon^{(n-1)}\right)$, where $\varepsilon = \varphi - \varphi_{\mathrm{m}}$, from the condition of stability of the motion of system (6) relative to the motion of the model, and stability of the motion of the coefficients of equation (6) relative to the corresponding coefficients of the model a_i, in the case of a quasi-stationary regime, i.e., $\Delta b_i(t) = \text{const}$ for $t \geq 0$, for any form of the control action $g(t)$.

The equation of the system with respect to the error ε will be

$$\varepsilon^{(n)} + \sum_{i=0}^{n-1} a_i \varepsilon^{(i)} = -\sum_{i=0}^{n-1}\left[\Delta b_i(t) + \Delta k_i\right]\varphi^{(i)}. \tag{8}$$

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Let us denote $\varepsilon^{(i)} = x_{i+1}$, $\Delta b_i(t) + \Delta k_i = y_{i+1}$ $(i = 0, 1, \dots, n-1)$, and write (8) in the form of a system of two equations in matrix form, taking into account $\Delta b_i(t) = \text{const}$:

$$\dot{\mathbf{x}} = A\mathbf{x} + \mathbf{u}, \qquad \dot{\mathbf{y}} = \psi,$$

where

$$\mathbf{x} = \begin{Vmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Vmatrix}, \qquad \mathbf{y} = \begin{Vmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{Vmatrix},$$

$$A = \begin{Vmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{Vmatrix}, \qquad \vec{\psi} = \begin{Vmatrix} \psi_0(t, \mathbf{x}) \\ \psi_1(t, \mathbf{x}) \\ \vdots \\ \psi_{n-1}(t, \mathbf{x}) \end{Vmatrix}, \tag{9}$$

$$\mathbf{u} = \begin{Vmatrix} 0 \\ \vdots \\ 0 \\ f(t, \mathbf{y}) \end{Vmatrix}, \qquad f(t, \mathbf{y}) = -\sum_{i=0}^{n-1} \varphi^{(i)} y_{i+1} \qquad (i = 0, 1, \dots, n-1).$$

When choosing $\psi_i(t, \mathbf{x})$, we shall impose the condition $\psi_i(t, \mathbf{x}) \equiv 0$ for $\mathbf{x} = 0$. From (9), evidently, $\mathbf{u} \equiv 0$ when $\mathbf{y} = 0$. Consequently,

$$\mathbf{x} = 0,$$

$$\mathbf{y} = 0 \tag{10}$$

are a solution of system (9). In accordance with the statement of the problem, we shall synthesize the vector ψ from the condition of stability of the zero solution (10). We note that the condition $\mathbf{y} = 0$ means compensation of the change in the variable coefficients of the plant by means of the adjustable coefficients of the regulator.

To solve the problem, choose a Lyapunov function in the form of a positive-definite quadratic form [3,4]

$$V(\mathbf{x}, \mathbf{y}) = \mathbf{x}' P \mathbf{x} + \mathbf{y}' E \mathbf{y}, \tag{11}$$

where

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$$P = \left\| \begin{matrix} p_{11} & \cdot & \cdot & p_{1n} \\ \vdots & & & \vdots \\ \vdots & & & \vdots \\ p_{n1} & \cdot & \cdot & p_{nn} \end{matrix} \right\|,$$

and E is the identity matrix of order n.

The derivative of $V(\mathbf{x}, \mathbf{y})$ by virtue of (9) has the form

$$\dot{V}(\mathbf{x}, \mathbf{y}) = \mathbf{x}' Q \mathbf{x} + 2[\mathbf{x}' P \mathbf{u} + \psi' E \mathbf{y}]. \tag{12}$$

Under the condition that A is a nonsingular matrix whose characteristic equation roots have negative real parts, $|A - \lambda E| = 0$, the negative-definite quadratic form $\mathbf{x}' Q \mathbf{x}$, where $Q = A' P + P A$, is always associated with the positive-definite quadratic form $\mathbf{x}' P \mathbf{x}$ [3]. In this case, the stability condition for the zero solution of (10) reduces to satisfying the nonpositivity of the expression

$$\mu(\mathbf{x}, \mathbf{y}, t) = \mathbf{x}' P \mathbf{u} + \psi' E \mathbf{y} \leqslant 0. \tag{13}$$

Carrying out the corresponding transformations, we find that condition (13) is satisfied when

$$\psi_i(t, \mathbf{x}) = \varphi^{(i)} \sum_{\beta=1}^{n} x_\beta p_{\beta n} \quad (i = 0, 1, \dots, n-1). \tag{14}$$

Thus, the following may be formulated.

Theorem. A sufficient condition for stability of the motion of system (6) with respect to the motion of the model, as well as of the coefficients of the differential equation of system (6) with respect to the corresponding coefficients of the model in the quasi-stationary regime ($\Delta b_i(t) = \text{const}$ for $t \geqslant 0$) with laws of variation of the adjustable coefficients (7), for any form of the control action $g(t)$, is that the algorithms $\psi_i(t, \mathbf{x})$ belong to class (14), where the parameters $p_{\beta n}$ are determined by the matrix in (11).

Institute of Automation and Telemechanics

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Note: Figure translations are in progress. See original paper for figures.

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