DAMPING OF RADIAL OSCILLATIONS OF THE EARTH

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Abstract

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DAMPING OF RADIAL OSCILLATIONS OF THE EARTH

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In papers (1) a way was indicated for considering a set of real Earth models on the basis of some initial real Earth model by means of a small parameter. For the spheroidal oscillations of the Earth (both the fundamental tone and the overtones), tables of derivatives of the eigenfrequencies with respect to the model parameters (2) were calculated; these make it possible, without further calculations, to pass from one real Earth model to another, and also to consider the question of damping. Application of the data obtained to the physics of the Earth's mantle made it possible to establish, in broad outline, the distribution of the dissipative function Q in the mantle. At present, tables of derivatives have been calculated for the fundamental radial tone and its four overtones. Below we give the formulas by which the computation was carried out, as well as the results of calculations of the damping of radial oscillations for the distributions Q obtained in (3).

As the initial Earth model, the Gutenberg-Bullen model A (4) was chosen. In this model the crust and mantle are divided into 34 layers with piecewise-constant parameters. The Earth' s core is liquid and is divided into 7 layers: the inner core, a transition layer, and the outer core, consisting of 5 layers. All calculations were carried out in dimensionless variables

$$u = az_1, \quad \sigma_r = \overline{K}z_2, \quad r = ax, \quad N_1 = \overline{\mu}/\overline{K}, \quad N = {}^4/_3 N_1, \quad N_0 = {}^2/_3 N_1,$$
$$\mu^0 = \overline{\mu}\mu, \quad K^0 = K\overline{K}, \quad \rho^0 = \overline{\rho}\rho, \quad \nu = 4\overline{g}\overline{\rho}a/\overline{K}, \quad \chi^2 = \omega^2 a^2 \overline{\rho}/\overline{K}, \quad (1)$$

where u and σ_r are the dimensional displacement and stress in radial oscillations; r is the current radius; a is the Earth' s radius; quantities with superscript zero are the dimensional model parameters (μ^0, K^0, ρ^0); quantities with a bar are certain normalizing constants $(\overline{\mu}, \overline{K}, \overline{\rho}, \overline{g})$ such that the dimensionless modulus of rigidity μ, modulus of compression K, density ρ, and acceleration of gravity $g = g^0/\overline{g}$ are everywhere less than or equal to unity; ω and χ are the dimensional and dimensionless circular frequencies. Integration and numbering of the layers were carried out from the center. The boundary conditions are

$$z_1(0) = 0, \quad z_2(1) = 0, \tag{2}$$

with $z_1(x)$ and $z_2(x)$ continuous throughout the entire region of integration $0 \le x \le 1$. At the center of the Earth, z_2 was conventionally taken to be unity ($z_2(0) = 1$). The system of differential equations for the central region, whose density ρ_1 is constant,

$$\dot{z}_1 = -\frac{2}{x} z_1 + \frac{1}{K_1} z_2, \quad \dot{z}_2 = -\rho_1(\chi^2 + \nu\rho_1 D), \quad D = \frac{4\pi G\bar{\rho}a}{3\bar{g}} \tag{3}$$

has an exact solution in terms of Bessel functions

$$z_1(\zeta) = (K_1 m_1)^{-1}\sqrt{\frac{\pi}{2}}\zeta^{-1/2} j_{3/2}(\zeta), \quad z_2(\zeta) = \sqrt{\frac{\pi}{2}}\zeta^{-1/2} j_{1/2}(\zeta),$$

$$m_1^2 = \frac{\rho_1}{K_1}(\chi^2 + \nu D\rho_1), \quad \zeta = m_1 x. \tag{4}$$

In the remaining segments of the liquid core

$$\dot{z}_1 = -\frac{2}{x} z_1 + \frac{1}{K} z_2, \qquad \dot{z}_2 = -\left(\varkappa^2\rho + \frac{\nu}{x}\rho g\right) z_1 \tag{5}$$

and in the solid regions of the mantle and crust

$$\begin{aligned} \dot{z}_1 &= -\frac{2}{x}\frac{K - N\mu}{K + N\mu} z_1 + \frac{1}{K + N\mu} z_2, \\ \dot{z}_2 &= -\frac{4}{x} N_1\mu \frac{1}{K + N\mu} z_2 + \left[\frac{12}{x^2}\frac{N_1\mu K}{K + N\mu} - \varkappa^2\rho - \frac{\nu}{x}\rho g\right] z_1. \end{aligned} \tag{6}$$

The integration was carried out numerically by the Runge–Kutta method. Suppose that, in passing from the initial model $\{\rho_0(x), \mu_0(x), K_0(x)\}$, which we shall distinguish by a subscript zero, to a nearby model,

$$\rho = \rho_0 + \Delta\rho, \qquad \mu = \mu_0 + \Delta\mu, \qquad K = K_0 + \Delta K, \qquad \Delta\rho \ll \rho_0, \quad \Delta\mu \ll \mu,$$

$$\Delta K \ll K_0 \tag{7}$$

the dimensionless frequency $\varkappa_0$ acquires the increment $\Delta\varkappa$

$$\varkappa = \varkappa_0 + \Delta\varkappa, \tag{8}$$

where

$$\Delta\varkappa = \sum_{i=1}^{41} \left(\varkappa_{\rho i}\Delta\rho_i + \varkappa_{Ki}\Delta K_i + \varkappa_{\mu i}\Delta\mu_i\right), \qquad (9)$$

where i is the layer number, and the additions ($\Delta\rho_i, \Delta K_i$ and $\Delta\mu_i$), as well as the initial functions (ρ_0, K_0, μ_0), are assumed piecewise constant. The calculation formulas for the "derivatives" $\varkappa_{\rho i}, \varkappa_{Ki}$ and $\varkappa_{\mu i}$ have the following form:

in the first segment,

$$\varkappa_{\rho 1} = \varkappa'_{\rho 1} + \varkappa''_{\rho 1},$$

$$\varkappa'_{\rho 1} = -\frac{(\varkappa_0^2 + \nu\rho_{10}D)}{2\varkappa_0 J}\int_0^{x_1} x^2 z_{10}^2\,dx, \qquad \varkappa''_{\rho 1} = -\frac{\nu\rho_{10}D\int_0^{x_1} x^2 z_{10}^2\,dx + \nu D x_1^3\int_{x_1}^1 \frac{\rho_0 z_{10}^2\,dx}{x}}{2\varkappa_0 J},$$

$$\varkappa_{K1} = (2K_{10}^2\varkappa_0 J)^{-1}\int_0^{x_1} x^2 z_{20}^2\,dx, \qquad J = \int_0^1 \rho_0 x^2 z_{10}^2\,dx, \qquad \varkappa_{\mu 1} = 0; \quad (10)$$

in the remaining segments of the liquid core

$$\varkappa_{\rho i} = \varkappa'_{\rho i} + \varkappa''_{\rho i}, \qquad i = 2, \dots, 7, \qquad \varkappa_{\mu i} = 0,$$

$$\varkappa'_{\rho i} = -\frac{1}{2\varkappa_0 J}\int_{x_{i-1}}^{x_i}\left(\varkappa_0^2 x^2 + \nu x g_{i0}\right) z_{10}^2\,dx,$$

$$\varkappa''_{\rho i} = -\frac{\nu D}{2\varkappa_0 J}\left\{\rho_{i0}\int_{x_{i-1}}^{x_i}\frac{x^3 - x_{i-1}^3}{x} z_{10}^2\,dx + \left(x_i^3 - x_{i-1}^3\right)\int_{x_i}^1 \rho_0\,\frac{1}{x} z_{10}^2\,dx\right\}, \quad (11)$$

$$\varkappa_{Ki} = \frac{1}{2K_{i0}^2\varkappa_0 J}\int_{x_{i-1}}^{x_i} x^2 z_{20}^2\,dx$$

and, finally, in the solid regions of the mantle and crust

$$\varkappa_{\rho i} = \varkappa'_{\rho i} + \varkappa''_{\rho i},$$

$$\varkappa'_{\rho i} = -(2\varkappa_0 J)^{-1} \int_{x_{i-1}}^{x_i} (\varkappa_0^2 x^2 + \nu g_{0i} x)\, z_{10}^2\, dx,$$

$$\varkappa''_{\rho i} = -\frac{\nu D}{2\varkappa_0 J} \left\{ \rho_{0i} \int_{x_{i-1}}^{x_i} \frac{x^3 - x_{i-1}^3}{x}\, z_{10}^2\, dx + (x_i^3 - x_{i-1}^3) \int_{x_i}^{1} \rho_0 \, \frac{1}{x} z_{10}^2\, dx \right\};$$

$$\chi_{Ki} = (2\chi_0 J M_{0i}^2)^{-1} \int_{x_{i-1}}^{x_i} dx\, x^2 \left\{ \frac{16 N_1^2 \mu_0}{x^2} z_{10}^2 + z_{20}^2 + \frac{8 N_1 \mu_0}{x} z_{10} z_{20} \right\}, \tag{12}$$

$$\chi_{\mu i} = (2\chi_0 J M_{0i}^2)^{-1} N \int_{x_{i-1}}^{x_i} x^2 dx \left\{ \frac{9 K_0^2}{x^2} z_{10}^2 + z_{20}^2 - \frac{6 K_0 z_{10} z_{20}}{x} \right\},$$

where

$$M_{0i} = K_{0i} + N\mu_{0i}, \qquad i = 8 \div 41.$$

In (10)–(12) the derivative χ_ρ is divided into two parts; $\chi'_{\rho i}$ determines the direct change in the frequency χ_0 when the density of the i-th layer changes by $\Delta\rho_i$; $\chi''_{\rho i}$ gives the change in χ_0 due to the change in the gravitational field g_0.

In paper ([1]), where the damping of radial oscillations for a model of an average homogeneous Earth was considered, it was shown why the damping of normal oscillations is determined by dissipative processes in the shell, while dissipation in the liquid core may be neglected. It was also indicated there that, in considering damping, the relaxation of the bulk modulus K, in comparison with the relaxation of the shear modulus μ, may be neglected. As in ([1,3]), taking the frequency and the shear modulus to be complex,

$$\omega = \omega_0(1+i\Phi), \qquad \Delta\omega = i\omega_0\Phi, \qquad \Phi = {}^1/_2 Q^{-1}, \qquad \Delta\chi = i(\bar\rho/\bar K)^{1/2}/2a\omega_0 Q^{-1},$$

$$\mu_i = \mu_{0i}(1 + iQ_i^{-1}), \qquad \Delta\mu_i = i\mu_{0i} Q_i^{-1},$$

and substituting these $\Delta\chi$ and $\Delta\mu_i$ into (9) ($\Delta\rho_i = \Delta K_i = 0$), we obtain the formula

$$Q^{-1} = \frac{2}{\chi_0} \sum_{i=8}^{41} R_i Q_i^{-1}, \qquad R_i = \chi_{\mu i} \mu_{0i}, \tag{13}$$

which relates the dissipative function for radial oscillations Q^{-1} to the dissipative parameters of the Earth's interior Q_i^{-1}. Formula (13) is too detailed. In concrete calculations ([3]) it is "coarsened" :

$$R^1 = \sum_{i=40}^{41} R_i, \qquad R^2 = \sum_{i=27}^{39} R_i, \qquad R^3 = \sum_{i=18}^{26} R_i, \qquad R^4 = \sum_{i=8}^{17} R_i. \tag{14}$$

R^1 characterizes the damping in the Earth' s crust ($0 \le l \le 38$ km), R^2($38 \le l \le 300$ km)—in the upper mantle, R^3($300 \le l \le 1000$ km)—in the transition Golitsyn layer, and R^4($1000 \le l \le 2898$ km)—in the lower mantle (l is depth). Thus, the computational relation takes the form

$$Q^{-1} = \frac{2}{\chi_0} \sum_{j=1}^{4} R^j Q_j^{-1}. \tag{15}$$

The results of calculations of Q for the fundamental radial mode ${}_0S_0$ and the first four overtones ${}_{jS}0$ ($j = 1, 2, 3, 4$) are presented in Table 1.

In the table, ${}_0Q_{s0}$ denotes the mechanical quality factor (dissipative function) for the fundamental radial oscillation, and ${}_1Q_{s0}, {}_2Q_{s0}, {}_3Q_{s0}, {}_4Q_{s0}$—for the 1st, 2nd, 3rd, and 4th overtones. In ([3]) we stopped at distribution 6 of Table 1. The damping of radial oscillations is determined to a considerable extent by the mechanical quality factor of the lower mantle (Q_4). The damping of torsional oscillations depends not very strongly on Q_4; therefore, in ([3]) it is determined with insufficient confidence. This makes it possible to assume that the values ${}_{jQ_{s0}}$ of variant 32 of Table 1 represent an upper limit for the dissipative function of radial oscillations.

The values ${}_{jQ_{s0}}$ in the last row of the table may be regarded as a lower limit. We see that ${}_0Q_{s0}$ is approximately twice as large as ${}_1Q_{s0}$. Dat-

given in the present communication may be regarded as a theoretical explanation of the experimental results of Ness, Harrison, and Slichter[4], according to which ${}_0Q_{s0}$ is an order of magnitude greater than ${}_0Q_{s9}$ and ${}_0Q_{s12}$ for the fundamental ninth ${}_0S_9$ and twelfth ${}_0S_{12}$ spheroidal tones.

In conclusion, let us note that a careful experimental determination of the quantities ${}_0Q_{s0}$ and

Table 1

Variant	Q_1	Q_2	Q_3	Q_4	$10^{-3}{}_0Q_{s0}$	$10^{-3}{}_1Q_{s0}$	$10^{-3}{}_2Q_{s0}$	$10^{-3}{}_3Q_{s0}$	$10^{-3}{}_4Q_{s0}$
21	450	50	∞	∞	9.9	200	21	8.4	4.4
22	450	100	∞	∞	19.5	390	43	17	8.9
23	450	150	∞	∞	29	570	64	25	13

Variant	Q_1	Q_2	Q_3	Q_4	$10^{-3}\,{}_0Q_{s0}$	$10^{-3}\,{}_1Q_{s0}$	$10^{-3}\,{}_2Q_{s0}$	$10^{-3}\,{}_3Q_{s0}$	$10^{-3}\,{}_4Q_{s0}$
24	450	200	∞	∞	33	740	85	34	19
31	450	50	500	∞	8.1	40	6.3	3.7	2.6
32	450	100	500	∞	14	44	7.4	4.7	3.7
34	450	200	500	∞	21	46.5	8.1	5.4	4.7
41	450	50	500	1000	7.4	5.4	3.6	2.8	2.2
42	450	100	500	1000	12	5.5	4	3.3	2.8
44	450	200	500	1000	17	5.5	4.2	3.7	3.4
5	450	100	500	1500	12	7.8	4.7	3.7	3.1
6	450	100	300	1000	10	5.1	3.1	2.5	2.2

${}_1Q_{s0}$ in experiment would make it possible to test the assumption, usually made and adopted by us in discussing the results, that the quantities Q_j $(j = 1, 2, 3, 4)$ are independent of period, and would also make it possible to determine reliably the value of Q_4 for the Earth' s lower mantle. In radial oscillations a significant fraction of the total energy is contained in oscillations of the gravitational field, while the dissipation is entirely due to inelastic processes in the shell. The "pumping" of gravitational energy into elastic energy is what gives rise to the anomalously large values of ${}_jQ_{s0}$ for the Earth' s radial oscillations.

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Note: Figure translations are in progress. See original paper for figures.