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Abstract

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PHYSICS

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THE VELOCITY DISTRIBUTION FUNCTION OF GAS MOLECULES NEAR A SOLID WALL IN A NONUNIFORMLY HEATED GAS

In our work ⁽¹⁾ it was shown that, for the most rigorous determination of the thermal slip velocity of a gas along a solid wall in the presence of a temperature gradient tangential to the latter, it is necessary to compute the velocity distribution function of the gas molecules by solving the Boltzmann kinetic equation.

In ⁽¹⁾ an attempt was made to carry out this task by a rough approximation of the collision integral in the Boltzmann kinetic equation by the Bhatnagar-Gross-Krook method ⁽²⁾. In this case the distribution function, as is to be expected, depends on the distance to the solid wall, but the slip velocity turns out to be equal to

$$u = {}^{3/4} v d \ln T / dy, \quad (1)$$

which exactly coincides with the result obtained earlier by Maxwell ⁽³⁾.

In the present work the distribution function is determined considerably more correctly than in ⁽¹⁾. The distribution function will be determined as the sum of three terms, each of which satisfies the Boltzmann equation. This approach is more convenient (in the sense of overcoming mathematical difficulties) than the Gross-Ziering method ^(4,5).

Let us consider a gas situated in the field of a temperature gradient tangential to an infinite wall. We choose the origin of coordinates on the surface of the wall. The x -axis is directed perpendicular to the wall, and the y -axis along the surface. Along the y -axis there is a temperature gradient in the gas. Under these conditions the velocity distribution function of the gas molecules will, generally speaking, depend on the coordinates x and y , and can be found from the Boltzmann kinetic equation, which in the stationary case has the form

$$(\mathbf{c}\nabla)f = \partial f / \partial t_{\text{coll}}, \quad (2)$$

where $\mathbf{c} = (m/2kT)^{1/2}\mathbf{v}$ is the dimensionless velocity; $f = f(x, y, \mathbf{c})$ is the distribution function; $(\partial f/\partial t)_{\text{coll}}$ is the collision integral. The collision integral in Boltzmann form is given by the expression

$$\left(\frac{\partial f}{\partial t}\right)_{\text{coll}} = \int d\mathbf{v}_1 \int gb db d\varepsilon (f' f'_1 - f f_1), \quad (3)$$

where $g = |\mathbf{c}_1 - \mathbf{c}|$ is the relative velocity of two colliding molecules; b is the impact parameter of the collision; ε is the azimuthal scattering angle.

At large distances from the wall the gas is described by the Chapman-Enskog distribution ⁽⁶⁾, which may be written in the form

$$f = f^{(0)}[1 + 2c_y u + \tau c_y S_{3/2}^{(1)}(c^2) \partial \ln T / \partial y], \quad (4)$$

where

$$f^{(0)} = n(m/2\pi kT)^{3/2} \exp(-mv^2/2kT);$$

u is the dimensionless mass velocity; $\tau = 15/16 \lambda \pi^{1/2}$ (λ is the mean free path); $S_{3/2}^{(1)}(c^2) = 5/2 - c^2$.

It is assumed that at infinity from the wall there is a concentration gradient $\partial n/\partial y$, which in relation (4) is expressed through $\partial \ln T/\partial y$, as well as a mass transport velocity u .

We shall seek the distribution function in the form:

$$f^{\pm} = f^{(0)}[1 + \Psi(\infty, y, \mathbf{c}) + \Phi^{\pm}(x, y, \mathbf{c})], \quad (5)$$

where the signs \pm denote the distribution functions for molecules flying away from the wall and toward the wall, respectively; $f^{(0)}[1 + \Psi(\infty, y, \mathbf{c})]$ is the Chapman-Enskog distribution (4).

From the estimates given in work (1), it follows that

$$|\partial \Phi / \partial y| \ll |\partial \Phi / \partial x|,$$

which makes it possible to regard Φ as a function only of x and \mathbf{c} .

Taking into account the form of the distribution function at large distances from the wall (4), and the fact that at arbitrary distances the distribution function contains terms proportional to $c_{xc} y \partial u / \partial x$, we shall seek the correction to the distribution function in the form

$$\Phi^\pm(x, \mathbf{c}) = \Phi_0^\pm(x, \mathbf{c}) + \Phi_1^\pm(x, \mathbf{c}) + \Phi_2^\pm(x, \mathbf{c}) + \Phi_3^\pm(x, \mathbf{c}), \quad (6)$$

where

$$\Phi_0^\pm(x, \mathbf{c}) = a_0^\pm(x)c_y, \quad \Phi_1^\pm(x, \mathbf{c}) = a_1^\pm(x)c_{xc}y, \quad \Phi_2^\pm(x, \mathbf{c}) = a_2^\pm(x)c_{xc}y,$$

$$\Phi_3^\pm(x, \mathbf{c}) = a_3^\pm(x)c_{yS_{3/2}}^{(1)}(c^2).$$

Substituting the distribution function (5) into equation (2), we obtain

$$c_x \frac{\partial \Phi}{\partial x} + c_y \frac{1}{f^{(0)}} \frac{\partial f^{(0)}}{\partial y} = J(\Phi) + J(\Psi),$$

where $J(\Phi)$ and $J(\Psi)$ are the linearized collision integrals. The function $\Psi(\infty, y, \mathbf{c})$ satisfies the equation

$$c_y \frac{\partial f^{(0)}}{\partial y} = f^{(0)} J(\psi),$$

therefore $\Phi^\pm(x, \mathbf{c})$ will be found from the solution of the equation

$$c_x \partial \Phi / \partial x = J(\Phi). \quad (7)$$

We choose the functions $\Phi_i^\pm(x, \mathbf{c})$ so that they satisfy the equations:

$$c_x \frac{\partial}{\partial x} (\Phi_0 + \Phi_1) = J(\Phi_0 + \Phi_1), \quad (8)$$

$$c_x \frac{\partial}{\partial x} \Phi_2 = J(\Phi_2), \quad (9)$$

$$c_x \frac{\partial}{\partial x} \Phi_3 = J(\Phi_3). \quad (10)$$

The solution of equation (8) is contained in works (5, 7). Using the data of work (7), it is easy to write the expressions for the functions $a_0^\pm(x)$ and $a_1^\pm(x)$:

$$a_0^+ = \alpha_0 + c_1 \exp(-7.56x/\lambda), \quad a_0^- = c_1 \exp(-7.56x/\lambda), \quad (11)$$

$$a_1^+ = \alpha_1 + c_1 \exp(-7.56x/\lambda), \quad a_1^- = \alpha_1 - c_1 \exp(-7.56x/\lambda),$$

where $\alpha_0^+ = -4.487$; $\alpha_1^+ = 4.643$; $\alpha_1^- = 1.551$; c_1 is a constant determined from the boundary conditions at the wall.

Let us find the solution of equation (9). It is convenient to introduce the auxiliary function $\text{sign } c_x = +1$ for $c_x > 0$ and $\text{sign } c_x = -1$ for $c_x < 0$. With its aid Φ_2 is expressed through Φ_2^+ and Φ_2^- as follows:

$$\Phi_2 = \Phi_2^+ \frac{1 + \text{sign } c_x}{2} + \Phi_2^- \frac{1 - \text{sign } c_x}{2} = \frac{a_2^+ + a_2^-}{2} c_{xc} y + \frac{a_2^+ - a_2^-}{2} c_{xc} y \text{sign } c_x. \quad (12)$$

Since $J(\Phi_2)$ is a linear operator, we have

$$J(\Phi_2) = \frac{a_2^+ + a_2^-}{2} J(c_{xc} y) + \frac{a_2^+ - a_2^-}{2} J(c_{xc} y \text{sign } c_x). \quad (13)$$

Substituting (12) and (13) into equation (9), multiplying both sides of the resulting equation by $c_{xc} y (1 \pm \text{sign } c_x) \exp(-c^2) dc$, and integrating over the entire velocity space, we obtain

$$\frac{d}{dx} a_2^+ = a_2^+ \frac{I_1 + I_2}{\pi} + a_2^- \frac{I_1 - I_2}{\pi}, \quad \frac{d}{dx} a_2^- = -a_2^+ \frac{I_1 - I_2}{\pi} - a_2^- \frac{I_1 + I_2}{\pi}, \quad (14)$$

where

$$I_1 = [c_{xc} y, c_{xc} y] = \int c_{xc} y \exp(-c^2) J(c_{xc} y) dc = -0.400 \frac{\pi}{\lambda},$$

$$I_2 = [c_{xc} y \text{sign } c_x, c_{xc} y \text{sign } c_x] = -1.698 \frac{\pi}{\lambda}.$$

The solution of system (14) has the form

$$a_2^+ = 2.897 c_2 \exp(-1.65x/\lambda), \quad a_2^- = c_2 \exp(-1.65x/\lambda), \quad (15)$$

where c_2 is an arbitrary constant determined from the boundary conditions at the wall.

Equation (10) for the function Φ_3 is solved in the same way. The function Φ_3 is given by the expression

$$\Phi_3(x, c) = \frac{a_3^+ + a_3^-}{2} c_y^{(1)} S_{3/2}(c^2) + \frac{a_3^+ - a_3^-}{2} c_y^{(1)} S_{3/2}(c^2) \text{sign } c_x, \quad (16)$$

and the collision integral of Φ_3 has the form

$$J(\Phi_3) = \frac{a_3^+ + a_3^-}{2} J(c_{yS_{3/2}}^{(1)}(c^2)) + \frac{a_3^+ - a_3^-}{2} J[c_{yS_{3/2}}^{(1)}(c^2) \text{sign } c_x]. \quad (17)$$

Substituting (16) and (17) into equation (10), multiplying both sides of the resulting equation by

$$c_{yS_{3/2}}^{(1)}(c^2)(1 \pm \text{sign } c_x) \exp(-c^2) dc,$$

and integrating over the entire velocity space, we obtain

$$\begin{aligned} \frac{d}{dx} a_3^+ &= a_3^+ \frac{4(I_3 + I_4)}{13\pi} + a_3^- \frac{4(I_3 - I_4)}{13\pi}, \\ \frac{d}{dx} a_3^- &= -a_3^+ \frac{4(I_3 - I_4)}{13\pi} - a_3^- \frac{4(I_3 + I_4)}{13\pi}, \end{aligned} \quad (18)$$

where

$$I_3 = [c_{yS_{3/2}}^{(1)}(c^2), c_{yS_{3/2}}^{(1)}(c^2)] = -1.333\pi/\lambda,$$

$$I_4 = [c_{yS_{3/2}}^{(1)}(c^2) \text{sign } c_x, c_{yS_{3/2}}^{(1)}(c^2) \text{sign } c_x] = -1.262\pi/\lambda.$$

The solution of system (18) has the form:

$$a_3^+ = -73.09c_3 \exp(-0.798x/\lambda), \quad a_3^- = c_3 \exp(-0.798x/\lambda); \quad (19)$$

c_3 is an arbitrary constant of the system.

The arbitrary constants of the differential equations are determined from the boundary conditions at the wall, which for the complete distribution function have the form (1)

$$f^+(0, c) = qf^{(0)} + (1 - q)f^-(0, -c_x, c_y, c_z), \quad (20)$$

where q is the coefficient of diffuse reflection.

Substituting the distribution function (5) into (20), it is easy to obtain the boundary condition for the function $\Phi(x, c)$

$$\Phi^+(0, c) = -q\Psi(\infty, c) + (1 - q)\Phi^-(0, -c_x, c_y, c_z). \quad (21)$$

Equating the coefficients of c_y , $c_{xc}y$, $c_{yS_{3/2}}^{(1)}(c^2)$ in relation (21), we obtain the boundary conditions for the functions a_i^\pm

$$\begin{aligned} a_0^+(0) &= -q \cdot 2u + (1-q)a_0^-(0), & a_1^+(0) + a_2^+(0) &= \\ &= q - 1[a_1^-(0) + a_2^-(0)], & a_3^+(0) &= -q\tau \partial \ln T / \partial y + (1-q)a_3^-(0). \end{aligned} \quad (22)$$

Substituting expressions (11), (15), (19) into the boundary conditions (22), we obtain

$$\begin{aligned} c_1 &= -q \frac{2u}{q - 5.487}, & c_2 &= q \frac{2u(6.194 - 1.551q)}{-q^2 + 9.384q - 21.38}, \\ c_3 &= q \frac{\tau}{74.09 - q} \frac{\partial \ln T}{\partial y}. \end{aligned} \quad (23)$$

The correction to the distribution function has the form

$$\begin{aligned} \Phi(x, c) &= \frac{a_0^+ + a_0^-}{2} c_y + \frac{a_0^+ - a_0^-}{2} c_y \operatorname{sign} c_x + \frac{a_1^+ + a_1^-}{2} c_{xc}y + \frac{a_1^+ - a_1^-}{2} c_{xc}y \operatorname{sign} c_x + \\ &+ \frac{a_2^+ + a_2^-}{2} c_{xc}y + \frac{a_2^+ - a_2^-}{2} c_{xc}y \operatorname{sign} c_x + \frac{a_3^+ + a_3^-}{2} c_{yS_{3/2}}^{(1)}(c^2) + \\ &+ \frac{a_3^+ - a_3^-}{2} c_{yS_{3/2}}^{(1)}(c^2) \operatorname{sign} c_x. \end{aligned} \quad (24)$$

From (24), (11), (15), (19), after carrying out the calculations we shall have

$$\begin{aligned} \Phi(x, c) &= -3.487 \frac{c_1}{2} c_y \exp(-7.56x/\lambda) - \\ &- 5.487 \frac{c_1}{2} c_y \operatorname{sign} c_x \exp(-7.56x/\lambda) + 6.194 \frac{c_1}{2} c_{xc}y \exp(-7.56x/\lambda) + \\ &+ 3.092 \frac{c_1}{2} c_{xc}y \operatorname{sign} c_x \exp(-7.56x/\lambda) + 3.897 \frac{c_2}{2} c_{xc}y \exp(-1.65x/\lambda) + \\ &+ 1.897 \frac{c_2}{2} c_{xc}y \operatorname{sign} c_x \exp(-1.65x/\lambda) - 72.09 \frac{c_3}{2} c_{yS_{3/2}}^{(1)}(c^2) \exp(-0.798x/\lambda) - \\ &- 74.09 \frac{c_3}{2} c_{yS_{3/2}}^{(1)}(c^2) \operatorname{sign} c_x \exp(-0.798x/\lambda). \end{aligned}$$

The distribution function obtained differs substantially from the one derived by us earlier ⁽¹⁾ in that the distribution function for the incident molecules depends explicitly on the distance to the wall; consequently, one may expect that on the basis of the distribution function obtained there should result a slip velocity different from the expression obtained in ⁽¹⁾. This question will be the subject of our next paper.

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