ON THE QUESTION OF CREEP OF METALS

THEORY OF ELASTICITY

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Abstract

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ON THE QUESTION OF CREEP OF METALS

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In paper (1) an equation of the type was considered

$$f(\varepsilon,\sigma,\dot{\sigma})\frac{\partial\varepsilon}{\partial t}=\frac{\partial\sigma}{\partial t}+g(\sigma,\varepsilon). \tag{1}$$

With a proper choice of the functions $f(\varepsilon,\sigma,\dot{\sigma})$, $g(\sigma,\varepsilon)$, this equation gives a good description of the behavior of metals under dynamic loads. Below an equation of the same form is proposed, which describes creep experiments and in special cases coincides with various well-known theories.

A creep experiment consists in the fact that a stress is applied instantaneously and then remains unchanged. If one constructs the diagram of the dependence $\sigma(\varepsilon)$ at $t=0$, it is found that under impact one can also enter the plastic region (2), i.e., that the metal has a limiting dynamic diagram, and this circumstance must be taken into account.

Let us write our equation

$$\varphi'\{\sigma-M|\dot{\varepsilon}|(\sigma_{\mathrm{d}}-\sigma)\}\partial\varepsilon/\partial t=\partial\sigma/\partial t+f_1(T)f_2(\sigma)f_3(\varepsilon)+C_1; \tag{2}$$

here

$$\varphi(z)=\begin{cases}E, & \text{for } z\leqslant\varepsilon_0,\\ \varphi(z), & \text{for } z>\varepsilon_0;\end{cases}$$

$$\sigma_{\mathrm{d}}(\varepsilon)=\begin{cases}E\varepsilon, & \text{for } z<\varepsilon_*,\\ \varphi[\varepsilon-(\varepsilon_*-\varepsilon_0)]+\sigma_*-\sigma_1, & \text{for } z\geqslant\varepsilon_*;\end{cases}$$

$\sigma_{\mathrm{d}}(\varepsilon)$ is the limiting dynamic diagram; $\varphi(\varepsilon)$ is the static diagram; $\varepsilon_0,\sigma_0,\varepsilon_*,\sigma_*$ are parameters of the static and dynamic yield limits; T is the absolute temperature; $f_1(T)$, $f_2(\sigma)$, $f_3(\varepsilon)$ are functions to be determined from experiment; $M\equiv\text{const}$; $C_1=-f_1(T)f_2(0)f_3(0)$.

For simplicity, consider the case when the diagram has the form

$$\varphi(z)=\begin{cases}E, & \text{for } z\leqslant\varepsilon_0,\\ E_1, & \text{for } z>\varepsilon_0.\end{cases}$$

Then for $z=\sigma-M|\dot\varepsilon|(\sigma_{\mathrm{d}}-\sigma)>\varepsilon_0$, equation (2) has the form

$$E_1\partial\varepsilon/\partial t=\partial\sigma/\partial t+f_1(T)f_2(\sigma)f_3(\varepsilon)-f_1(T)f_2(0)f_3(0). \qquad (3)$$

Let $\sigma=\text{const}$, $T=\text{const}$; then from (3) we obtain

$$E_1\partial\varepsilon/\partial t=f_1(T)f_2(\sigma)f_3(\varepsilon)-f_1(T)f_2(0)f_3(0). \qquad (4)$$

It is known (2) that $\dot\varepsilon$, as ε increases, first decreases and then increases. This condition is satisfied, for example, by choosing the function $f_3(\varepsilon)$ in the form

$$f_3(\varepsilon)=\left[\frac{\varepsilon-A}{B}\right]^2+C_2.$$

From (4) we obtain

$$E_1\frac{\partial\varepsilon}{\partial t}=f_1(T)\left\{f_2(\sigma)\left[\frac{\varepsilon-A}{B^2}+C_2\right]^2+f_2(0)\left[\frac{A^2}{B^2}+C_2\right]\right\}. \qquad (5)$$

For $\varepsilon<A$, $\dot\varepsilon$ decreases rather rapidly, which corresponds to the region of unsteady creep. For $\varepsilon\sim A$, almost steady creep sets in; the larger B is, the longer this region is. The creep rate in this segment is determined by the constant $f_2(\sigma)C_2-f_2(0)[A^2/B^2+C_2]$, i.e., when $f(\sigma)$ is increasing and $C_2>0$, the creep rate increases with increasing σ; when $C_2=0$ it is constant, and in this case one must have $f_2(0)\leq 0$; the case $f_2(0)=0,\ C_2=0$ defines decaying creep. When $C_2<0$, the decay of creep occurs not at $\varepsilon\approx A$, but somewhat earlier.

After integrating equation (5) we obtain

$$t=\frac{BE_1}{f_1(T)\left[f_2(\sigma)C_2-f_2(0)(A^2/B^2+C_2)\right]}\operatorname{arc\,tg}\frac{(\varepsilon-A)f_2(\sigma)}{B\left[f_2(\sigma)C_2-f_2(0)(A^2/B^2+C_2)\right]}+C_3,$$

C_3 is determined from the initial conditions.

Thus, we see that $f_3(\dot\varepsilon)$, i.e., the parameters A,B,C_2, can be obtained from the usual experimental dependence $\varepsilon(t)$. A is determined as the midpoint of the steady-creep zone, B from the extent of this zone in ε, and C_2 characterizes the creep rate in this zone.

It is proposed to determine the function $f_2(\sigma)$ from a relaxation experiment. Let $\varepsilon = \text{const}$; then from (2) we obtain

$$\partial\sigma/\partial t = -f_1(T)f_2(\sigma)f_3(\varepsilon) + f_1(T)f_2(0)f_3(0). \tag{6}$$

Take $f_2(\sigma) = (\sigma - C_4)^n$, where n is odd and $C_4 \geq 0$. For integrability take $n = 1$; then

$$\partial\sigma/\partial t = \{-\sigma f_3(\varepsilon) + C_4[f_3(\varepsilon) - f_3(0)]\}f_1(T). \tag{7}$$

It follows from this that the stress relaxes until the right-hand side of equation (7) becomes equal to zero. It is interesting to note that the level to which the stress decreases for $C_4 > 0$ depends on the deformation at which relaxation occurs. Namely, when $f_3(\varepsilon) - f_3(0) > 0$, i.e., when $\varepsilon > 2A$, relaxation ceases at $\sigma > 0$, while for $\varepsilon < 2A$ the stress changes sign during relaxation.

The oddness of n and the sign of C_4 are important in describing reverse creep. Write equation (2) as

$$\varphi'\left\{\sigma - M|\dot{\varepsilon}|(\sigma - \sigma)\right\}\frac{\partial\varepsilon}{\partial t} = \frac{\partial\sigma}{\partial t} + (\sigma - C_4)f_1(T)\left\{\left[\frac{\varepsilon - A}{B}\right]^2 + C_1\right\} +$$

$$+C_4 f_1(T)\left[\frac{A^2}{B^2} + C_1\right].$$

If we instantaneously reduce the load to zero, then in this case $\dot{\sigma} = \infty$, $\dot{\varepsilon} = \infty$, the argument $z = -\infty$, $\varphi'(z) = E$, and then $\Delta\varepsilon = \sigma/E$; after this $\sigma = \text{const}$, and the equation takes the form

$$E\frac{\partial\varepsilon}{\partial t} = -C_4\left\{\left[\frac{\varepsilon - A}{B}\right]^2 - \frac{A^2}{B^2}\right\}f_1(T).$$

It is easy to see that reverse creep will take place only when $\varepsilon > 2A$; when $\varepsilon < 2A$, there will be an increase in deformation up to $\varepsilon = 2A$, and then a stop.

Let us consider the problem of loading a semi-infinite rod according to the law $\sigma_1 = \text{const}$. Let $T = \text{const}$, and let x be the coordinate of the rod. We shall assume that

$$\varphi'(z) = \begin{cases} E, & \text{for } z \ll \varepsilon_0, \\ E_1, & \text{for } z > \varepsilon_1, \end{cases}$$

and for z increasing from ε_0 to ε_1, $\varphi'(z)$ decreases monotonically from E to E_1. The quantity $\sqrt{\varphi'[\sigma - a]/\rho_0}$ is the slope of the characteristics in the x,t plane, i.e., the velocity of propagation of the disturbance, and $a = M|\dot{\varepsilon}|(\sigma_{\rm d} - \sigma)|_{x=0}$. This means that the disturbance propagates approximately with the velocity that it had initially. The introduction of equation (2) makes it possible to take into account elastic and plastic deformations, which are usually not considered when solving creep problems. If $0 < \sigma_1 < \sigma_*$, then at the initial instant, when $\sigma = \infty$, $\varepsilon = \infty$, $\varphi'(-\infty) = E$, and $\sigma_1 = E\varepsilon_1$. In the x, t plane, a fan of characteristics will emerge from the origin, corresponding to the transition from $\sigma = \infty$ to $\sigma = 0$; in $\dot{\varepsilon}$ this corresponds to the transition from $\dot{\varepsilon} = \infty$ to $\dot{\varepsilon} = \text{const}$; this constant is determined from equation (2), for example, by iteration. In this fan the elastic values $\sigma_1 = E\varepsilon_1$ are carried with different velocities. After the passage of this fan, creep will begin. For $\sigma_1 > \sigma_*$, we fall on the limiting dynamic diagram and, analogously to (1), put the term $M|\dot{\varepsilon}|(\sigma_{\rm d} - \sigma) = \varepsilon_* - \varepsilon_0 = b$, since $\dot{\varepsilon} = \infty$ and $\sigma = \sigma_{\rm d}$. First there will pass a fan of characteristics carrying plastic deformation and the corresponding $\sigma = \infty$, then a fan corresponding to the transition from $\sigma = \infty$ to $\sigma = 0$, and then creep will proceed. The creep curves $\varepsilon(t)$ will have the standard form. When $\dot{\varepsilon}$ begins to increase, the argument $\sigma - M|\dot{\varepsilon}|(\sigma_{\rm d} - \sigma)$ will decrease; this means that, by virtue of the choice of $\varphi'(z)$, the characteristics begin to overtake one another and coalesce into a shock wave.

This shock wave may be the cause of the formation of microdefects in the metal, since a crack can arise if there exists a boundary on both sides of which the grains are deformed to different degrees (3). The shock wave will exist for a finite time, until the argument $z = \sigma - M|\dot{\varepsilon}|(\sigma_{\rm d} - \sigma)$ becomes equal to ε_0. After this one must consider the problem of crack propagation.

The choice of $f_1(T)$ presents no difficulty; $f_1(T)$ should be taken equal to $C_5 e^{-Q/RT}$, as is done by most investigators (2).

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CITED LITERATURE

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[4] L. I. Sedov, M. E. Eglit, *Dokl. Akad. Nauk SSSR*, **142**, No. 1 (1962).

Note: Figure translations are in progress. See original paper for figures.