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Abstract

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ON ELECTROMAGNETIC WAVES

IN GENERAL RELATIVITY

(Presented by Academician L. I. Sedov, 25 III 1966)

We shall characterize the electromagnetic field in a hyperbolic Riemannian manifold by means of exterior differential forms

$$F = \frac{1}{2} F_{ik} dx^i \wedge dx^k, \qquad *F = \frac{1}{2} * F_{ik} dx^i \wedge dx^k;$$

F_{ik} is the tensor of the electromagnetic field. Then Maxwell' s equations in vacuum, in the absence of charged particles, may be represented in the form

$$dF = 0, \tag{1}$$

$$\delta F = 0, \tag{2}$$

where d is the operator of exterior differentiation, and δ is the operator of codifferentiation of forms ([1]).

From equations (1), (2) it follows that the 2-form F is closed and coclosed, and therefore also harmonic, i.e., it satisfies the generalized wave equation:

$$\Box F = 0, \tag{3}$$

where $\Box = d\delta + \delta d$ is the d' Alembert operator on a hyperbolic Riemannian manifold. The adjoint 2-form $*F$ is likewise harmonic, since the operators $\Box$ and $*$ commute.

In a curved space-time admitting the structure of a topological product $T \times V_3$, the system of equations of the electromagnetic field reduces to the following second-order equations:

$$\partial^2 e/\partial T^2 + \Delta^{(3)} e = 0,$$

$$\partial^2 h/\partial T^2 + \Delta^{(3)} h = 0.$$

The 1-forms e and h, whose coefficients are respectively the components of the vectors of the electric and magnetic field strengths, are defined on the Riemannian manifold V_3, while the operator $\Delta^{(3)} = d^{(3)}\delta^{(3)} + \delta^{(3)}d^{(3)}$ is the Laplace operator on this manifold ([1]).

Thus, the d' Alembert operator on a hyperbolic Riemannian manifold with structure $T \times V_3$ can be represented in the form

$$\Box = \frac{\partial^2}{\partial T^2} + \Delta^{(3)}.$$

The coefficients of the 2-form F, defined according to (3), satisfy the equations

$$-F^{ik;a}{}_{;a} - R^i{}_a F^{ak} - R^k{}_a F^{ia} + 2R^{aikb}F^{ab} = 0. \qquad (4)$$

It follows from this ([2]) that, in a curved space-time of a given metric structure, any flux of electromagnetic waves will not preserve its form and will develop a “tail,” so that in this case Huygens' principle is not fulfilled; but it will hold if space-time is curved in a definite way by the electromagnetic field.

Let us try to determine the structures of curved space-time in which the propagation of electromagnetic waves without the formation of a “tail” is possible. To this end we turn to consideration of the self-consistent system of equations (4) and the equations of the gravitational field

$$R_{ik} = -2F^{aF}_{iak}, \qquad (5)$$

assuming that

$$F_{ik}F^{ik} = 0, \qquad (6)$$

$$F_{ik} * F^{ik} = 0. \qquad (7)$$

Condition (7) is a necessary and sufficient condition for the simplicity of the bivector F_{ik}.

The structure of the Ricci tensor is determined on the basis of equations (5). In this case the second and third terms in equations (4) vanish.

It is also necessary to require the fulfillment of the following conditions:

$$R_{ikab}F^{ab} = 0. \tag{8}$$

Taking into account that in the case under consideration the tensor of conformal curvature has the form

$$P_{iklj} = R_{iklj} + \frac{1}{2}(g_{il}R_{kj} + g_{kj}R_{il} - g_{ij}R_{kl} - g_{kl}R_{ij}),$$

we find that conditions (8) are equivalent to the following conditions:

$$P_{ikab}F^{ab} = 0. \tag{9}$$

Introduce the metrized bivector space R_6

$$G_{AB} \to G_{iklj} \equiv (G_{il}G_{kj} - G_{ij}G_{kl}).$$

Then conditions (9), taking (6) into account, mean that the basis of a non-simple elementary divisor of the λ-matrix

$$(P_{AB} - \lambda G_{AB}),$$

to which there corresponds the isotropic eigenvector $F^B \to F^{ik}$ of the symmetric tensor P_{AB} in the space R_6, must be equal to zero.

We arrive at an analogous conclusion if we consider the generalized wave equation for the conjugate form $*F$. On this basis, taking into account

$$P_{kl} = P^i{}_{kli} = 0,$$

we conclude that the tensor of conformal curvature can belong only and exclusively to the second degenerate or third type according to the classification of A. Z. Petrov ([3]), or must be equal to zero. The latter case corresponds to a conformally Euclidean space.

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3. A. Z. Petrov, *Einstein Spaces*, Moscow, 1961.

Note: Figure translations are in progress. See original paper for figures.