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Abstract

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DISSIPATION OF ENERGY IN A TURBULENT GAS CONTAINING SUSPENDED PARTICLES

A turbulent gas at sufficiently large Reynolds numbers, in which solid particles are suspended, is considered. With respect to the turbulent flow, an assumption is made concerning its local homogeneity, isotropy, and stationarity. The suspended particles considered in the present work have a size R sufficiently small in comparison with the internal scale of turbulence l that the Reynolds numbers of their motion relative to the gas would be less than unity. The motion of particles under the conditions indicated above was considered by us in the preceding paper ([1]), all notation from which we shall retain in the present work.

The quantity sought, ε—the total dissipation of energy per unit volume of the suspension per unit time—is composed of ε_1, the dissipation of energy due to the interaction of the gas and the solid particles, and ε_0, the dissipation of energy of the pure gas. If n denotes the mean number of particles per unit volume, and θ the mean dissipation of energy on one particle per unit time, then it is evident that $\varepsilon_1 = n\theta$. To calculate the quantity θ, taking into account that the dissipative part of the resistance force to the motion of the particles f_i is given by Stokes' formula, we obtain the relation

$$\theta = \overline{f_i V_i} = k_2 \rho_0 \nu R \overline{V_i(t) V_i(t)} = k_2 \rho_0 \nu R S(0). \tag{1}$$

It follows from (1) that, in order to find ε_1, it is necessary to know the mean square of the velocity of the relative motion of the particles, $S(0)$. In (1) a formula was obtained that makes it possible to determine $S(0)$, if $Q(\tau)$ is known —the Lagrangian correlation function of the gas containing suspended particles:

$$S(0) = (1-\alpha)^2 \left[Q(0) - \beta \int_0^\infty Q(\tau) e^{-\beta\tau}\, d\tau \right]. \tag{2}$$

If one makes the assumption that the influence of the particles on the motion of the gas may be neglected (the impurity may be regarded as "passive"), then in formula (2) one should substitute the correlation function of a pure gas not containing suspended particles.

In order to determine the conditions of applicability of this last assumption, let us examine in more detail the character of the interaction of particles with the turbulent medium in which they are suspended. Let us recall that the following consideration does not concern the influence of the gravitational field, and also assumes that $\gamma \ll 1$, i.e., that the mean distance between particles is much greater than the dimensions of the particles themselves.

The character of the interaction of a particle with turbulent pulsations of different scales is determined by the ratio of the frequency of the pulsations of the given scale to the characteristic frequency β, which depends on the properties of the particles and the characteristics of the medium, but does not depend on the structure of the turbulence. If $\beta \ll \omega_L$, i.e., the particle possesses sufficiently large inertia, then even the largest-scale pulsations practically do not have time, over the course of a period, to set it in motion. Such a heavy particle is almost motionless and therefore is flowed around by pulsations of all scales. In another

In the limiting case $\beta \gg \omega_l$, the particle is carried along by pulsations of all scales, and it tends to follow all of them. If, however, the quantity β is such that it is smaller than ω_l, but at the same time still large in comparison with ω_L, then it will be carried along by those pulsations whose frequencies are small in comparison with β, and will be flowed around by pulsations whose frequencies are large in comparison with β. Pulsations with frequencies close to β will partly carry along and partly flow around the suspended particle.

Obviously, in order that the influence of the presence of particles on the motion of the gas could be neglected, it is necessary that, in the equation of motion of the gas, the term characterizing the force acting on the gas from the side of the particles could be omitted. In other words, in order that the admixture may be regarded as "passive," it is necessary that the loss of momentum of the gas due to the particles be small in comparison with the total loss of momentum of this gas. Mathematically this means that the inequality

$$\frac{\rho_0 \dot{U}}{nf} \sim \frac{\rho_0 \dot{U}}{\rho\gamma\dot{W}} \sim \frac{\alpha}{\gamma}\frac{\dot{U}}{\dot{W}} \gg 1. \tag{3}$$

must be satisfied. The quantities $\dot{U}$ and $\dot{W}$ entering inequality (3) are, respectively, the accelerations of the gas and of the particles. If one passes to Fourier components in the equation relating the velocities of the particles and the flow (1), then inequality (3) takes the form

$$\frac{\alpha}{\gamma}\left[\frac{\omega^2+\beta^2}{\alpha^2\omega^2+\beta^2}\right]^{1/2} \gg 1. \tag{4}$$

It follows from condition (4) that if $\alpha/\gamma \gg 1$ (the mass of the suspended particles is small in comparison with the mass of the flow), then the admixture is always passive, since for all ω the quantity $\sqrt{(\omega^2+\beta^2)/(\alpha^2\omega^2+\beta^2)} \gtrsim 1$. If, however, $\alpha/\gamma \sim 1$, then inequality (4) is satisfied only for sufficiently high frequencies $\omega \gg \gamma/\alpha \sim \beta$. Thus, if we consider times much smaller than $1/\beta$, the admixture may be regarded as "passive." However, if one is interested in the behavior of the gas at times greater than $1/\beta$, then the influence of the particles on the motion of the gas cannot be neglected. This means that the presence in the gas of suspended particles whose mass is comparable with the mass of the gas leads to a change in the spectrum of large-scale pulsations and does not affect small-scale pulsations. Indeed, since large-scale pulsations carry the particles along completely, the particle velocities at large scales are practically equal to the gas velocity at these scales. In other words, the behavior of a suspension of gas on scales sufficiently large in comparison with the amplitudes of the relative motion of the particles and at times sufficiently large in comparison with $1/\beta$ can approximately be regarded as that of a homogeneous medium, but with macroscopic parameters different from the parameters of the pure gas. In the opposite limiting case, if we are interested in time intervals small in comparison with $1/\beta$, and in scales small in comparison with the amplitudes of the relative motion of the particles, the particles may be regarded as almost immobile, flowed around by small-scale pulsations whose accelerations are much greater than the accelerations of the particles. The loss of momentum of these pulsations on the particles is small in comparison with their total loss of momentum, and therefore the presence of particles has no effect on pulsations whose frequencies are large in comparison with β. Thus, if the gas is considered over time intervals small compared with $1/\beta$, then its characteristics (including the correlation function) will be the same as in a pure gas containing no suspended particles.

In considering the integral appearing in formula (2), it is not difficult to note that the main contribution to it is made by values of τ small in comparison with $1/\beta$; therefore, as $Q(\tau)$ one may use the correlation function of a pure gas. The form of this function for an arbitrary value of the correlation time is, generally speaking, unknown. However, since for op-

To determine $S'(0)$, it is necessary to know the behavior of the function $Q(\tau)$ only for $\tau < 1/\beta$; then, in two important limiting cases, this function can be determined. If the values of τ that make the main contribution to the integral in formula (2) are small in comparison with the periods of the large-scale pulsations $1/\omega_L$, but are still large in comparison with the smallest periods of the spectrum of pulsations $1/\omega_l$, then for the function Q one may use the known expression (2)

$$Q(\tau) = Q(0) - k_3 \frac{\varepsilon_0}{\rho_0}\tau. \tag{5}$$

Since $\omega_L \sim U_L/L,\ \omega_l \sim U_l/l \sim (\nu\rho_0/\varepsilon_0)^{-1/2}$, we have

$$\frac{\omega_L}{\beta} \sim \frac{U_{LR}^2}{La\nu} \sim \frac{R^2 U_{LL}}{aL^2\nu} \sim \frac{\mathrm{Re}_L R^2}{al^2 \mathrm{Re}_L^{3/2}} \sim \frac{1}{a\,\mathrm{Re}_L^{1/2}} \frac{R^2}{l^2},$$

$$\frac{\omega_l}{\beta} \sim \frac{\omega_l}{\omega_L} \frac{\omega_L}{\beta} \sim \mathrm{Re}_L^{1/2} \frac{1}{a\,\mathrm{Re}_L^{1/2}} \frac{R^2}{l^2} \sim \frac{1}{a} \frac{R^2}{l^2}.$$

From the estimates given, it is easy to see that, in order to satisfy the requirement $\omega_l^{-1} < \beta^{-1} < \omega_L^{-1}$, necessary for the use of formula (5), the particle sizes must satisfy the condition $a^{1/2} < R/l < a^{1/2}\,\mathrm{Re}_L^{1/4}$. If, however, the particle sizes are so small that the inequality $R < a^{1/2} l$ holds, then the main contribution to integral (2) is made by values of τ smaller than ω_l^{-1}. Therefore, instead of formula (5), for the correlation function one must use another formula (2)

$$Q(\tau) = Q(0) - k_4(\varepsilon_0^3/\rho_0^3\nu)^{1/2}\tau^2. \tag{6}$$

Substituting formulas (5) and (6) into (2) and carrying out the integration, we obtain the values of the mean-square velocities of the relative motion of the particles

$$(\overline{V^2})^{1/2} = k_5 \frac{\varepsilon_0^{1/2}|1-\alpha|}{\rho_0^{1/2} a^{1/2} \nu^{1/2}} R, \quad \text{if } a^{1/2} l < R < a^{1/2}\,\mathrm{Re}_L^{1/4}\, l; \tag{7}$$

$$(\overline{V^2})^{1/2} = k_6 \frac{\varepsilon_0^{3/4}|1-\alpha|}{\rho_0^{3/4} a \nu^{5/4}} R^2, \quad \text{if } R < a^{1/2} l. \tag{8}$$

Comparing formulas (7) and (8), one may easily see that for $R \approx a^{1/2} l$ they pass into one another. Moreover, taking into account that $\varepsilon_0/\rho_0 \sim U_L^3/L \sim U_L^2 \omega_L$, it is not difficult to verify that formula (7) is obtained in the limiting case $\omega_L \ll \beta$ from the relation derived by us earlier (1), where an empirical formula for the correlation function was used. The latter formula correctly describes the behavior of the true correlation function at sufficiently large correlation times, but becomes inapplicable near the point $\tau = 0$. From what has been said it is clear why formula (8) is not obtained in the limit from the corresponding formula of work (1).

Using (7), (8), and (1), we find the value of the total dissipation of energy per unit volume per unit time

$$\varepsilon = \varepsilon_0 \left[1 + k_7(1-\alpha)^2 \frac{\gamma}{\alpha}\right], \quad \text{if } a^{1/2} l < R < a^{1/2}\,\mathrm{Re}_L^{1/4}\, l; \tag{9}$$

$$\varepsilon = \varepsilon_0 \left[1 + k_8(1-\alpha)^2 \frac{\gamma}{\alpha} \frac{R^2}{al^2}\right], \quad \text{if } R < a^{1/2} l. \tag{10}$$

From the form of formulas (9) and (10) it follows that, if solid particles are suspended in a gas (in this case $a \sim 10^{-3}$), then already for $\gamma \sim 10^{-3}$ the additional energy dissipation in the mixture, due to the particles, becomes comparable with the dissipation in the pure gas. For the case of particles in a liquid, $a \sim 1$ and $\varepsilon = \varepsilon_0$, i.e., the presence of a small volume content of suspended particles practically does not change the magnitude of the dissipated energy.

The results obtained may be interpreted as follows. For a liquid suspension, the densities of the particles and of the liquid are comparable. As a result, the entrainment of the particles by the liquid will be almost complete, and the energy dissipation due to the relative motion of the liquid and the particles will be small. If, however, the particles are suspended in a gas, then, because of the large difference in the densities of the particles and the gas, the relative velocity of their motion becomes so large that the energy dissipated on the surface of the particles becomes comparable with the energy dissipated in the volume of the pure gas. The fact that formula (9) does not contain the viscosity of the gas is connected with the fact that viscosity plays a dual role in the mechanism of energy dissipation on the particle surface. On the one hand, as the viscosity increases, at a given velocity of relative motion, the energy dissipation on each particle increases. But, on the other hand, the entrainment of the particle by the gas is thereby increased, which leads to a decrease in the relative velocity; consequently, the two effects compensate each other for particles of size $a^{1/2}l < R < a^{1/2}\mathrm{Re}_L^{1/4}l$. Owing to this, there is no explicit dependence of the quantity ε on ν in formula (9). For smaller particles, the effect of viscosity on the change in relative velocity proves more substantial, and therefore, for them, as the viscosity increases, the quantity ε decreases.

In addition to the dissipation of the flow energy on the particle surface, associated with their inertia, there is also an energy dissipation ε_1' caused by the motion of the particles relative to the medium under the action of the gravitational field. Taking into account that the free-fall velocity of the particles is $U = g/\beta$, and also using formulas (7) and (8), let us compare the quantities ε_1 and ε_1'. If $a^{1/2}l < R < a^{1/2}\mathrm{Re}_L^{1/4}l$, then

$$\frac{\varepsilon_1}{\varepsilon_1'} = \frac{\overline{V^2}}{U^2} \sim \frac{\beta^2\varepsilon_0}{\rho g^2\beta} \sim \frac{a_l U_l^4}{g^2 R^2} \sim \left(\frac{a_l}{g}\right)^2 \frac{al^2}{R^2}. \tag{11}$$

If, however, $R < a^{1/2}l$, then

$$\varepsilon_1/\varepsilon_1' = \overline{V^2}/U^2 \sim \beta^2 a_l^2/g^2\beta^2 \sim (a_l/g)^2. \tag{12}$$

In formulas (11) and (12), a_l denotes the acceleration of pulsations of scale l. From the form of formulas (11) and (12) it follows that, if the condition $a_l < g$ is satisfied, then the inertial dissipation of flow energy on the particles ε_1 can always be neglected in comparison with the gravitational dissipation ε_1'. If, however, the reverse inequality $a_l > g$ is satisfied, then the inertial dissipation may prove greater than the gravitational dissipation.

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Note: Figure translations are in progress. See original paper for figures.