VACUUM QUANTUM FLUCTUATIONS IN CURVED SPACE AND THE THEORY OF GRAVITATION

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Abstract

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In Einstein' s theory of gravitation, a dependence of the action of space-time on curvature is postulated R$istheinvariantoftheRiccitensor$

$$S(R) = -\frac{1}{16\pi G}\int (dx)\sqrt{-g}\,R. \tag{1}$$

The presence of the action (1) leads to a "metric elasticity" of space, i.e., to the appearance of a generalized force that opposes the curvature of space.

Here we shall consider a hypothesis that identifies the action (1) with the change in the action of the quantum fluctuations of the vacuum when space is curved. Thus, we regard the metric elasticity of space as a kind of level-shift effect (compare also with (1a)*).

In modern quantum field theory it is assumed that the energy-momentum tensor of the quantum fluctuations of the vacuum $T_k^i(0)$ and the corresponding action $S(0)$, formally proportional to a divergent integral over the momenta of virtual particles of the fourth degree, of the form $\int k^3 dk$, are in fact equal to zero.

Recently Ya. B. Zel' dovich ([2]) suggested that gravitational interactions may lead to a certain "small" violation of this equality and thereby to a finite value of Einstein' s cosmological constant, in accordance with a recent interpretation of astrophysical data. We are interested here in the dependence of the action of quantum fluctuations on the curvature of space. Expanding the density of the Lagrange function in a series in powers of the curvature, we have (A and $B \sim 1$)

$$\mathcal{L}(R) = \mathcal{L}(0) + A\int k\,dk \cdot R + B\int \frac{dk}{k}\,R^2 + \cdots \tag{2}$$

The first term corresponds to Einstein' s cosmological constant. The second term corresponds, according to our hypothesis, to the action (1), i.e.,

$$G = -\frac{1}{16\pi A \int k\,dk}, \qquad A \sim 1. \tag{3}$$

The third term of the expansion, written here in a conventional form, leads to nonlinear corrections with respect to R in Einstein' s equations.**

The divergent integrals over the momenta of virtual particles in (2) and (3) are written on dimensional grounds. Knowing the numerical value of the gravitational constant G, we find that the effective cutoff of the integration in (3) is

$$k_i \sim 10^{28}\ \text{eV} \sim 10^{+33}\ \text{cm}^{-1}.$$

* Here the molecular attraction of condensed bodies is calculated as the result of a change in the spectrum of electromagnetic fluctuations. As the author points out, a special case of the attraction of metallic bodies was studied earlier by Casimir ([16]).

** A more exact form of this term:

$$\int \frac{dk}{k} \left\{ BR^2 + CR^{ik}R_{ik} + DR^{iklm}R_{iklm} + ER^{iklm}R_{ilkm} \right\} \quad (A,\ B,\ C,\ D,\ E \sim 1).$$

According to ([3,4]), $\int \frac{dk}{k} \sim 137$, therefore the third term is significant for $R \geq 1/137$ (in gravitational units), i.e., in the neighborhood of the singular point of the Friedmann model of the Universe.

In the gravitational system of units $G = \hbar = c = 1$. In this case $k_0 \sim 1$. At the suggestion of M. A. Markov, the quantity k_0 determines the mass of the heaviest particles existing in nature, which he called "maximons." It is also natural to suppose that the quantity k_0 determines the limit of applicability of modern ideas about space and causality.

Consideration of the density of the vacuum Lagrange function in a simplified "model" theory for noninteracting free fields with particle masses $M \sim k_0$ shows that, for certain mass ratios of real particles and "ghost" particles (i.e., hypothetical particles that make a contribution, opposite to that of real particles, to the action depending on R), a finite effect arises: a change in the action under curvature of space, proportional to M^2R, which we identify with R/G. Thus, the magnitude of the gravitational interaction is determined by the masses and laws of motion of free particles and also, probably, by the "cutoff momentum."

This approach to the theory of gravitation is analogous to the treatment of quantum electrodynamics in ([3]), where the possibility was noted of neglecting the Lagrangian of the free electromagnetic field in calculating the renormalization of the elementary electric charge. In the work of L. D. Landau and I. Ya.

Pomeranchuk the magnitude of the elementary charge is expressed in terms of the particle masses and the cutoff momentum; for a further development of these ideas see ([4]), where the possibility is substantiated of formulating the equations of quantum electrodynamics without the "bare" Lagrangian of the free electromagnetic field.

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Note: Figure translations are in progress. See original paper for figures.