ON THE PROBABILITY DISTRIBUTION OF THE SQUARE OF THE TEMPERATURE DIFFERENCE AT TWO POINTS OF A TURBULENT FLOW

HYDROMECHANICS

1967

SovietRxiv

Abstract

Full Text

UDC 532.517.4

HYDROMECHANICS

A. S. GURVICH

ON THE PROBABILITY DISTRIBUTION OF THE SQUARE OF THE TEMPERATURE DIFFERENCE AT TWO POINTS OF A TURBULENT FLOW

(Presented by Academician A. N. Kolmogorov on 26 III 1966)

In connection with the works of A. N. Kolmogorov ([1]) and A. M. Obukhov ([2]), interest has recently increased in the study of the fine structure of turbulent flow at large Reynolds numbers. Experimental investigations ([3]) were aimed at giving at least a very rough estimate of the corrections to the two-thirds law in connection with the variability of the rate of dissipation of kinetic energy in a turbulent flow. As experiment has shown, this correction is small, and it is not easy to detect it by direct measurements of spectra or structural functions of the velocity field. The results of experimental investigations of correlation moments of the fourth ([3,4]) and sixth ([5]) orders for the velocity gradient proved to diverge sharply from the conclusions obtained on the assumption of a Gaussian probability distribution for the velocity field. The ideas developed by E. A. Novikov and R. W. Stewart ([6,7]) concerning the alternation of regions of the flow where fluctuations of the velocity gradient are large with quiet regions made it possible to explain the results of measurements ([3–5]). We note that in ([6,7]) the existence was assumed of finite regions where the velocity gradient vanishes. A. M. Yaglom ([8]) developed the ideas of A. M. Obukhov and A. N. Kolmogorov on the logarithmically normal distribution for fluctuations of the energy-dissipation rate ε_r, averaged over some volume with characteristic size r, and showed that the explanation of the results ([3,4]) can be given within the framework of these ideas. At the same time A. M. Yaglom showed that the physical ideas about the cascade process of fragmentation of turbulent disturbances, which underlie the entire theory of local structure, naturally lead to a logarithmically normal probability distribution of the square of the velocity gradient.

The aim of the present work is an experimental study of the probability distribution of the square of the temperature difference at two points of a turbulent flow. Measurement of the temperature difference was carried out with the aid of a fluctuation thermometer, whose measuring bridge included two temperature sensors made of tungsten wire 5 μ thick. The size of the sensitive element of the

Fig. 1. Segment of a record of instantaneous values of the temperature difference. a–$\Delta T = 0$, noise record; b–segment of the working record

Figure 1: Fig. 1. Segment of a record of instantaneous values of the temperature difference. a–$\Delta T = 0$, noise record; b–segment of the working record

sensor was about 3 mm, and the time constant about 0.8 msec ([9]). The sensors were placed at a height $Z = 4$ m above the earth' s surface at a distance of 2 cm from one another and were oriented so that the straight line passing through the sensitive elements was parallel to the direction of the mean wind $\langle u \rangle$. A change in the temperature of the sensors led to a change in their electrical resistance, and they were connected into the measuring bridge in such a way that the output signal from the bridge was proportional to the temperature difference. In fabricating and selecting the sensors, special attention was paid to their identity, in order to exclude the influence of large-scale temperature fluctuations on the readings of the instrument. The achieved degree of identity made it possible to reduce the distance between the sensors to 2 cm. Instantaneous values of the output signal of the microthermometer were recorded on a loop oscillograph. The time constant τ of the entire electronic part of the circuit, including the loop, was approximately $1 \div 1.5$ msec. This constant

of time corresponds to the spatial scale $\langle u \rangle \tau$, which, at the wind velocities of $5 \div 7$ m/sec that occurred during our measurements, did not exceed 1 cm. Comparison of all the characteristic scales permits the conclusion that, practically without substantial distortion, the temperature difference ΔT was measured at two points separated by 2 cm in the direction of the mean wind velocity. In what follows the obvious condition $\langle \Delta T \rangle = 0$ was used.

Fig. 1. Segment of a record of instantaneous values of the temperature difference. a–$\Delta T = 0$, noise record; b–segment of the working record.

Figure 1 gives an example of a short segment of the obtained record, best illustrating the intermittency phenomenon. From a large number of tapes with records, the two most suitable for processing were selected; on them the amplitude of the record was sufficiently large, but at the same time throughout its entire length the record did not go beyond the edge of the tape. The records were about 1 min long (length about 10 m) each and were processed with the aid of a simple device that made it possible to determine the probability $P_1(\xi < \xi_0)$ that the deviation ξ on the tape is less than a specified level value ξ_0. The spacing of the levels ξ_0 was chosen empirically, so that they were located most often where $dP_1/d\xi_0$ is large. Processing with the device was equivalent to taking approximately $8000 \div 10000$ independent values from the record.

The distributions $P_1(\xi_0)$ are shown in Fig. 2. Along the ordinate axis is plotted the probability scale z, which is related to P by the following formula

Figure 2

Figure 2: Figure 2

$$P = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-t^2/2}\, dt.$$

A Gaussian probability distribution on this scale is represented by a straight line whose slope is inversely proportional to the variance, and whose mean value corresponds to $P_1 = 0.5$ or $z = 0$. From the graph in Fig. 2 it is seen that the probability distributions obtained as a result of processing the records differ strongly from the Gaussian distribution in that the probability of large excursions and of values close to the mean ($\langle T \rangle = 0$) is greater than for a Gaussian with the same variance. The studied range of probability values P_1, approximately from 0.002 to 0.998, proved insufficient for this reason for a reliable calculation of the third and higher moments of the distribution. Nevertheless, it proved possible to use the obtained data to determine the type of probability distribution of the square of the temperature difference $(\Delta T)^2$.

To find the probability distribution P_2 of the square of the temperature difference from the obtained data, the values were approximately calculated by the trapezoidal method:

$$\langle \xi \rangle = \int_{-\infty}^{\infty} \xi\, dP_1(\xi).$$

The calculations of $\langle \xi \rangle$ were necessary because the thermometric bridge could not be balanced exactly, precisely because of pulsations of the temperature difference. The calculated values $\langle \xi \rangle$ make it possible to determine the level $\xi_0 = \langle \xi \rangle$, to which the value $\langle \Delta T \rangle = 0$ corresponds. The values $P_2(\zeta < \zeta_0)$, where $\zeta = (\xi - \langle \xi \rangle)^2$, were found from the formula

$$P_2(\xi < \xi_0) = \int_{\langle \xi \rangle - \sqrt{\xi_0}}^{\langle \xi \rangle + \sqrt{\xi_0}} dP_1(\xi) = P_1(\langle \xi \rangle + \sqrt{\xi_0}) - P_1(\langle \xi \rangle - \sqrt{\xi_0}).$$

However, it is more convenient to use the dimensionless quantity $y = (\Delta T)^2 / \langle (\Delta T)^2 \rangle$, rather than $\xi \sim (\Delta T)^2$. The final processing results are presented in Fig. 3, where the values of P_2 are plotted along the ordinate in a probability scale, and the abscissa axis is logarithmic. In these coordinates a lognormal distribution law is represented by a straight line. The values $\langle (\Delta T)^2 \rangle \sim \langle \xi \rangle$ were calculated from the distributions.

Fig. 2. Probability distribution $P_1(\xi < \xi_0)$ of the temperature difference. *a*—recording of 12 VIII; *b*—recording of 17 VIII.

Of particular interest is clarification of the behavior of $P_2(y)$ for values of y small in comparison with unity. An obstacle to studying this region is the noise of the measuring device and interference arising during recording. To estimate the probability that the signal on the recording was below the noise level, the time was measured during which the signal was not noticeable on the recording because of the interference. Relating this interval to the total duration of the recording, we obtained an estimate of the probability that the signal was below the interference level. These estimates are given on the graph in Fig. 3 (points in circles), with the interference level taken as the abscissa. It is evident from the graph that these measurements fit satisfactorily on the straight lines passing through the remaining points.

The results shown in Fig. 3 make it possible to conclude that, in the investigated range of probability values ($0.15 < P_2 < 0.995$), the experimental data on the probability distribution of the square of the temperature difference agree with the assumption of a lognormal probability distribution law*

$$P_2(y) = \frac{1}{\sqrt{2\pi}\sigma} \int_0^y \exp\left[-\frac{1}{2}\left(\frac{\ln t/y_0}{\sigma}\right)^2\right] \frac{dt}{t},$$

where $\langle y \rangle = 1$, $\sigma^2 = \langle(\ln y - \langle \ln y \rangle)^2\rangle$, $y_0 = e^{-\sigma^2/2} = \langle \ln y \rangle$. The values of σ were determined from the slopes of the straight lines drawn in Fig. 3 through the experimental points. We give the values of σ, together with the characteristics of the measurement conditions—the Reynolds numbers $\mathrm{Re} = Z\langle u \rangle/\nu$ and Richardson numbers Ri, which were calculated from the vertical profiles of temperature and wind speed:

	Ri	Re	σ
12VIII 1965, 16 h 20 min	−0.078	$1.8 \cdot 10^6$	1.7
17VIII 1965, 13 h 15 min	−0.097	$1.4 \cdot 10^6$	2.7

As noted above, the experimental probability distributions obtained do not make it possible to calculate the fourth moment $\langle(\Delta T)^4\rangle$,

* A comparison of the experimental data with the probability distribution proposed in (6), $P(y) = \Gamma(a, ay)/\Gamma(a)$, where $\Gamma(a, ay)$ is the incomplete gamma function, $\Gamma(a) = \Gamma(a, \infty)$, and a is a parameter characterizing the intermittency, $0 < a < 1$, showed that this law diverges strongly from the data obtained by us.

needed to determine the excess

$$\gamma = \langle(\Delta T)^4\rangle/[\langle(\Delta T)^2\rangle]^2 - 3.$$

Fig. 3. Probability distributions $P(\zeta < \zeta_0)$ of the square of the temperature difference. *a*—record of 12 VIII; *b*—record of 17 VIII

Figure 3: Fig. 3. Probability distributions $P(\zeta < \zeta_0)$ of the square of the temperature difference. *a*—record of 12 VIII; *b*—record of 17 VIII

However, if it is assumed that a lognormal law is valid for all values of $(\Delta T)^2$, then a rough estimate of the magnitude of the excess can be given. For a lognormal process $\langle y^2 \rangle = e^{\sigma^2}$ and $\langle (\Delta T)^4 \rangle / [\langle (\Delta T)^2 \rangle]^2 = \langle y^2 \rangle$, hence $\gamma = e^{\sigma^2} - 3$. On the basis of the obtained values $\sigma = 1.7$, we have $\gamma = 15$, and for $\sigma = 2.7$ we obtain $\gamma = 1400$. The large range of the estimated values evidently indicates that the excess can in fact be very large and, consequently, in experimental determinations of γ very stringent requirements must be imposed on the apparatus, so as not to introduce distortions into the measurement results. Such a large excess is a consequence of strong intermittency. We note that in (5) the value $\gamma = 17$ is given for the velocity gradient; at the same time it is noted that this is apparently an estimate of the lower bound of possible values.

Fig. 3. Probability distributions $P(\zeta < \zeta_0)$ of the square of the temperature difference. *a*—record of 12 VIII; *b*—record of 17 VIII

Strictly speaking, the experimental results obtained cannot be compared directly with the conclusions of papers ([1],[2],[7],[8]), in which only probability distributions averaged over some volume of squares of velocity gradients are considered specifically. It seems natural, however, to suppose that if the cascade mechanism of successive fragmentation of turbulent structures can be described mathematically as was done in ([8]), then it should also lead to a lognormal probability distribution of the random variable $(\Delta T)^2$. As A. M. Obukhov noted, the latter conclusion can be further explained as follows. It may be assumed that the values of the square of the temperature difference $(\Delta T)^2$ at two points separated from each other by a distance r, exceeding the inner scale of turbulence, satisfy the relation $(\Delta T)^2 \sim N_r \varepsilon_r^{-1/3} r^{2/3}$, where N_r and ε_r are the rates of smoothing of temperature and of energy dissipation, averaged over a volume containing the observation points, with characteristic size r. If ε_r and N_r are distributed lognormally, then the product $N_r \varepsilon_r^{-1/3}$ (and consequently also $(\Delta T)^2$) will be distributed in the same way. This consideration makes it possible to interpret the results we have obtained as experimental confirmation of the assumption, made in ([1],[2],[8]), of a lognormal probability distribution of ε_r.

In conclusion the authors express their deep gratitude to A. M. Obukhov and A. M. Yaglom for a number of useful comments, and also to E. A. Novikov for his constant attention to the work and discussion of the results.

Institute of Atmospheric Physics
Academy of Sciences of the USSR

Received

16 II 1966

CITED LITERATURE

1. A. N. Kolmogorov, *J. Fluid Mech.*, **13**, No. 1, 82 (1962).
2. A. M. Obukhov, *J. Fluid Mech.*, **13**, No. 1, 77 (1962).
3. A. S. Gurvich, S. L. Zubkovskii, *Izv. AN SSSR, ser. geofiz.*, No. 12, 1856 (1963).
4. A. S. Gurvich, S. L. Zubkovskii, *Izv. AN SSSR, ser. Physics of the Atmosphere and Ocean*, **1**, No. 8, 797 (1965).
5. S. Pond, R. U. Stewart, *Izv. AN SSSR, ser. Physics of the Atmosphere and Ocean*, **1**, No. 9, 914 (1965).
6. E. A. Novikov, *Izv. AN SSSR, ser. Physics of the Atmosphere and Ocean*, **1**, No. 8, 788 (1965).
7. E. A. Novikov, R. U. Stewart, *Izv. AN SSSR, ser. geofiz.*, No. 3, 408 (1964).
8. A. M. Yaglom, *DAN*, **166**, No. 1, 50 (1966).
9. A. S. Gurvich, T. K. Kravchenko, *Tr. Inst. fiziki atmosfery AN SSSR*, No. 4, 144 (1962).

Note: Figure translations are in progress. See original paper for figures.