A. V. Bitsadze. Boundary value problems for second-order elliptic equations

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Abstract

Full Text

Preamble

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A. V. Bitsadze. "Boundary Value Problems for Second-Order Elliptic Equations." Nauka Publishing House, 1966. Circulation: 9,000 copies. 11 printed sheets. (Review)

This monograph is dedicated to a vital branch of modern mathematics: the theory of fundamental boundary value problems for uniformly elliptic linear second-order partial differential equations and systems. This theory is not only of independent mathematical interest but also finds direct and diverse applications in various problems of mechanics and physics. A significant portion of the results presented in the book belongs to the author himself. Bitsadze provides a sufficiently complete treatment of the solutions to several boundary value problems for these systems, describes the existing problem sets in the field, and poses urgent and fundamentally important problems for the further development of the theory, illustrating the emerging difficulties with a series of well-chosen examples.

The content of the monograph can be briefly summarized as follows, along with several observations made during the exposition. The first chapter presents the necessary background on the formulation of boundary value problems for second-order elliptic systems, the theory of linear equations in normed linear spaces, and Fredholm integral equations of the first and second kind. It also provides a classification of boundary value problems, including Hausdorff normally solvable problems, Noether problems, and Fredholm problems.

The second chapter covers certain qualitative and constructive properties of solutions to elliptic equations. This includes the extremum principle for a single elliptic equation, the Hopf and Zaremba-Giraud principles for local extrema of solutions, and the extremum principle for a specific class of elliptic systems. The author also studies fundamental solutions and, in connection with them, potential-type integrals. Naturally, the question of the existence of solutions for elliptic equations with a right-hand side (independent of boundary conditions) is also addressed. Furthermore, this chapter establishes formulas for the representation of the general solution for a class of elliptic systems on the plane—specifically, systems where the principal terms in each equation form the Laplace operator for the corresponding function, and the lower-order terms have analytic coefficients. It is perhaps worth noting that a phrase in the fourth paragraph of page 38 is somewhat unfortunate, as an inexperienced reader might incorrectly conclude that the concepts of the parametrix and the fundamental solution coincide for all elliptic equations.

The third chapter is devoted to the solution of the Dirichlet problem for a single elliptic equation, while the fourth chapter addresses the Dirichlet problem for elliptic systems. In Chapter III, the author also provides an exceptionally simple and elegant solution to the Hilbert problem for the Laplace equation. In Chapter IV, in addition to general results on the Dirichlet problem for a general elliptic system in the plane, several significantly more precise results are obtained for systems with constant coefficients. Chapter V investigates the oblique derivative problem (the Poincaré problem) for a single equation in cases where the direction of the derivative is not tangent to the boundary. The sixth chapter examines a similar problem for systems of equations.

It should be noted that while the Dirichlet and Poincaré problems are investigated relatively simply for a single equation, substantial fundamental difficulties arise in the case of systems. One of the primary questions—determining the conditions under which these boundary value problems for elliptic systems are Fredholm, Noether, or Hausdorff normally solvable—is still awaiting a complete solution. All investigations in the monograph are carried out using a unified method: the method of linear integral equations. Using this approach, the boundary value problems under consideration are reduced to finding the fixed point of a certain linear operator in an appropriate normed linear space.

The method of integral equations, as applied in A. V. Bitsadze' s monograph, allows for the study and construction of classical solutions to boundary value problems for elliptic systems, which are the most natural from the perspective of the theory of differential equations. The final seventh chapter is dedicated to the author' s research on the oblique derivative problem for harmonic functions with three independent variables. The solution to this problem is presented following a preliminary study of multidimensional singular integrals, which generalize classical Cauchy-type integrals to the multidimensional case. Their application to the solution of certain systems of singular equations is also provided. These results by the author are of undeniable independent scientific interest.

This brief summary covers only a portion of the topics addressed in the monograph. Despite its relatively small volume, the book contains extensive and scientifically significant material on linear elliptic equations and second-order systems. The publication of A. V. Bitsadze' s monograph is highly timely; it not only fills a substantial gap in the mathematical literature but will undoubtedly serve to attract the attention of researchers to the important and relevant range of issues presented within its pages.

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Note: Figure translations are in progress. See original paper for figures.