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Abstract

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PHYSICS

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ON THE ELECTRODYNAMICS OF TWO-BAND SUPERCONDUCTORS

THE VICINITY OF THE UPPER CRITICAL FIELD

(Presented by Academician N. N. Bogolyubov, 20 V 1966)

The thermodynamic properties of pure two-band superconductors were considered in work ([1]) (see also ([2]) on the basis of Bardeen' s model ([3]) and the Bogolyubov u, v-transformation ( )). An attempt to consider the electrodynamic properties of pure two-band superconductors is contained in Tilley' s work ( ), in which the upper critical magnetic field of a superconductor is determined. In the present work the basic equations of the electrodynamics of two-band superconductors are formulated in a form valid both for pure superconductors and for superconductors with an impurity, and a detailed investigation of the vicinity of the upper critical field of pure substances is given. The latter investigation is carried out on the basis of A. A. Abrikosov' s theory ( ) of type-II superconductors and the development of this theory in the works of Kleiner et al. ( ) and Laper ( ).

On the basis of the Hamiltonian of two-band superconductors of work ([1]), supplemented by the interaction of electrons with an impurity, we obtain equations for the temperature Green functions ( )

$$G_{mn}(\mathbf{x}\sigma\mathbf{x}'\sigma'|\tau-\tau') = \langle T\tilde{\psi}_m(\mathbf{x}\sigma\tau)\tilde{\psi}_n(\mathbf{x}'\sigma'\tau')\rangle,$$

$$P_{mn}(\mathbf{x}\sigma\mathbf{x}'\sigma'|\tau-\tau') = \langle T\tilde{\psi}_m(\mathbf{x}\sigma\tau)\tilde{\psi}_n(\mathbf{x}'\sigma'\tau')\rangle, \tag{1}$$

in the form

$$\begin{aligned}&\left[-i\Omega_n + H_0\left(-i\hbar\nabla - \frac{e}{c}\mathbf{A}\right)\right]G_{mn}(\mathbf{x}\sigma\mathbf{x}'\sigma'|\Omega)\\&\quad+\sum_{n'ks}\int dy\,\psi^*_{mk}(\mathbf{y})\psi_{mk}(\mathbf{x})V_{\sigma s}(\mathbf{y})G_{n'n}(\mathbf{y}s\mathbf{x}'\sigma'|\Omega)\\&\quad+\sum_{n's}V_{mn'}\Delta_{n'\sigma s}(\mathbf{x})P_{mn}(\mathbf{x}s\mathbf{x}'\sigma'|\Omega) = \delta_{\sigma\sigma'}\delta_{nm}\sum_k\psi_{mk}(\mathbf{x})\psi^*_{mk}(\mathbf{x}');\end{aligned} \tag{2}$$

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$$\left[-i\Omega_n - H_0\left(i\hbar\nabla - \frac{e}{c}\mathbf{A}(\mathbf{x})\right)\right] P_{mn}(\mathbf{x}\sigma\mathbf{x}'\sigma'|\Omega)$$
$$- \sum_{skn'} \int dy\, \psi_{mk}(\mathbf{y})\psi^*_{mk}(\mathbf{x})V_{s\sigma}(\mathbf{y})P_{n'n}(\mathbf{y}s\mathbf{x}'\sigma'|\Omega) \qquad (3)$$
$$- \sum_{n's'} V_{n'm}\Delta^*_{n's'\sigma}(\mathbf{x})G_{mn}(\mathbf{x}s\mathbf{x}'\sigma'|\Omega) = 0;$$

$$\Delta^*_{n\sigma}(\mathbf{x}) = P_{nn}(\mathbf{x}s\mathbf{x}\sigma|0) = \frac{1}{\beta}\sum_{\Omega} P_{nn}(\mathbf{x}s\mathbf{x}\sigma|\Omega). \qquad (4)$$

An analogous equation holds for the function R, which is obtained from P by replacing ψ by ψ; $\psi_{mk}(\mathbf{x})$ are Bloch functions, and the remaining notations are standard.

Let us introduce the Green functions of the normal metal with impurity $g_{nm}(\mathbf{x}\sigma\mathbf{x}'\sigma'|\Omega)$ and pass from equations (2), (3) to integral equations. Suppose

assuming in the latter the smallness of the quantities Δ_n, i.e., the closeness of the temperature T to the value T_c, or the closeness of the concentration of the paramagnetic impurity to the critical value, we can carry out an iteration of the integral equations and obtain the system

$$\Gamma^{*m}_{\sigma'\sigma}(\mathbf{x}) = \frac{1}{\beta}\sum_{\Omega}\sum_{nn'ss'}\int dy\, V_{nm}g_{n'n}(ys'\mathbf{x}\sigma \mid \Omega)\Gamma^{*n'}_{ss'}(y)g_{n'n}(ys\sigma' \mid -\Omega)$$
$$- \frac{1}{\beta}\sum_{\Omega}\sum_{nn'pp'}\sum_{\alpha\alpha'\beta\beta'ss'}\int dy\, dy'\, dy''\, V_{nm}\Gamma^{*n'}_{ss'}(y)g_{n'n}(ys'\mathbf{x}\sigma \mid \Omega)$$
$$\times \Gamma^{p}_{\alpha'\alpha}(y')g_{n'p}(ysy'\alpha' \mid -\Omega)\Gamma^{*p'}_{\beta\beta'}(y'')g_{p'p}(y''\beta y'\alpha \mid \Omega)g_{p'n}(y''\beta'\mathbf{x}\sigma' \mid -\Omega); \qquad (5)$$

$$\Gamma^{p}_{\alpha\alpha'}(y) = \sum_{l} V_{pl}\Delta_{l\alpha'\alpha}(y). \qquad (6)$$

All quantities appearing in these formulas depend on the electromagnetic potential $\mathbf{A}(\mathbf{x})$. According to (10), the dependence of the Green functions on the magnetic field has the form of phase factors. Expanding these factors in a series in $\mathbf{A}$ and taking into account the weak dependence of the quantities $\Gamma(\mathbf{x})$ on the argument $\mathbf{x}$, we obtain the following system of Ginzburg–Landau equations (11) for two-band superconductors:

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$$\Gamma^{*m}_{\sigma'\sigma}(\mathbf{x}) = \sum_{nn'ss'} V_{nm} Q^{n'n}_{s'\sigma s\sigma'}(\mathbf{x}) \Gamma^{*n}_{ss'}(\mathbf{x}) + \frac{1}{2} \sum_{nn'ss'} \sum_{jl} V_{nm} \times Q^{n'n}_{s'\sigma s\sigma'}(\mathbf{x}; jl) \left(\nabla_j + \frac{2ie}{c\hbar} A_j(\mathbf{x}) \right) \left(\nabla_l + \frac{2ie}{c\hbar} A_l(\mathbf{x}) \right) \Gamma^{*n'}_{ss'}(\mathbf{x}) - \sum_{nn'mp} \sum_{\alpha\alpha'\beta\beta' ss'} B^{n'n\,n'p\,m'p\,m'n}_{s'\sigma s\alpha'\alpha\beta\beta'\sigma'}(\mathbf{x}) \Gamma^{*n'}_{ss'}(\mathbf{x}) \Gamma^{p}_{\alpha'\alpha}(\mathbf{x}) \Gamma^{*m'}_{\beta\beta'}(\mathbf{x}), \quad (7)$$

where

$$Q^{n'n}_{s'\sigma s\sigma'}(\mathbf{x}) = \int dy\, Q^{n'n}_{s'\sigma s\sigma'}(\mathbf{x}; y),$$

$$Q^{n'n}_{s'\sigma s\sigma'}(\mathbf{x}; jl) = \int dy\, (y-x)_j (y-x)_l\, Q^{n'n}_{s'\sigma s\sigma'}(\mathbf{x}; y),$$

$$Q^{n'n}_{s'\sigma s\sigma'}(\mathbf{x}; y) = \frac{1}{\beta} \sum_{\Omega} g_{n'n}(ys'\mathbf{x}\sigma \mid \Omega) g_{n'n}(ys\sigma' \mid -\Omega), \quad (8)$$

$$B^{n'n\,n'p\,m'p\,m'n}_{s'\sigma s\alpha'\alpha\beta\beta'\sigma'}(\mathbf{x}) = \frac{1}{\beta} \sum_{\Omega} \int dy\, dy'\, dy''\, g_{n'n}(ys'\mathbf{x}\sigma \mid \Omega) \times g_{n'p}(ysy'\alpha' \mid -\Omega) g_{m'p}(y''\beta y'\alpha \mid \Omega) g_{m'n}(y''\beta'\mathbf{x}\sigma' \mid -\Omega).$$

The system of equations (7) should be supplemented by an expression for the electric current

$$j^k(\mathbf{x}) = \frac{e}{2} \sum_{nm} \sum_{\alpha\alpha' ss'} \sum_{l} \left\{ I^{nm}_{ss'\alpha\alpha'}(\mathbf{x}; kl) \Gamma^{n}_{s's}(\mathbf{x}) \left(\nabla_l + \frac{2ie}{c\hbar} A_l(\mathbf{x}) \right) \Gamma^{*m}_{\alpha\alpha'}(\mathbf{x}) + \text{c.c.} \right\}; \quad (9)$$

$$I^{nm}_{s's\alpha\alpha'}(\mathbf{x}; kl) = \frac{1}{\beta} \sum_{\Omega} \sum_{n'm'\sigma} \int dy\, dy'\, (y'-x)_l \Big[\hat{v}_k(\mathbf{x}) g_{n'n}(\mathbf{x}\sigma ys' \mid \Omega) \times g_{mn}(y'\alpha ys \mid -\Omega) g_{nm'}(y'\alpha'\mathbf{x}\sigma \mid \Omega) - g_{n'n}(\mathbf{x}\sigma ys' \mid \Omega) g_{mn}(y'\alpha ys \mid -\Omega) v_k(\mathbf{x}) g_{mm'}(y'\alpha'\mathbf{x}\sigma \mid \Omega) \Big], \quad (10)$$

where $v_k(\mathbf{x})$ is the electron velocity operator.

Finally, we give the expression, needed below, for the difference of the thermodynamic potentials of the superconducting and normal states for small values of the quantities Γ

$$\Omega_s - \Omega_n = -\frac{1}{4\beta} \sum_{nmpp'\,\varepsilon\varepsilon'\alpha\alpha'\beta\beta'\sigma\sigma'} \int dx\, [B^{nm\,pn\,pp'\,mp'}_{\sigma\varepsilon\,\alpha'\varepsilon'\,\alpha\beta\,\sigma'\beta'}(\mathbf{x})]^* \times \Gamma^{p}_{\alpha'\alpha}(\mathbf{x}) \Gamma^{m}_{\sigma\sigma'}(\mathbf{x}) \Gamma^{*p'}_{\beta\beta'}(\mathbf{x}) \Gamma^{*n}_{\varepsilon'\varepsilon}(\mathbf{x}). \tag{11}$$

In formulas (7)–(11), averaging over the positions of the impurity must be carried out if the latter is present.

In the present paper we shall consider the case of pure superconductors. In this case substantial simplifications are possible. However, even in this case the calculations cannot be carried through to the end because of the band character of the electronic energy spectrum. Calculations are possible if the Bloch functions are replaced by plane waves.

The Ginzburg–Landau equations for two-band superconductors in the plane-wave approximation have the form

$$\Gamma^*_m(\mathbf{x}) = \sum_n V_{nm} \left[Q_n + \frac{R_n}{6} \left(\nabla + \frac{2ie}{c\hbar}\mathbf{A} \right)^2 - B_n |\Gamma_n(\mathbf{x})|^2 \right] \Gamma^*_n(\mathbf{x}),$$

$$\operatorname{rot}\operatorname{rot}\mathbf{A}(\mathbf{x}) = -\frac{4\pi i}{3}\frac{\hbar e}{c} \sum_n R_n \Gamma^*_n(\mathbf{x}) \left(\nabla - \frac{2ie}{c\hbar}\mathbf{A}(\mathbf{x}) \right) \Gamma_n(\mathbf{x}) + \text{c.c.}, \tag{12}$$

where

$$Q_n = N_n \ln\left(\frac{2\gamma\beta\hbar\tilde{\omega}_n}{\pi} \right); \qquad R_n = B_n v_n^2 = \frac{7\zeta(3)}{8\pi^2} \beta^2 N_n v_n^2; \tag{13}$$

N_n is the density of states, v_n is the velocity on the Fermi surface.

We investigate the mixed state, predicted by A. A. Abrikosov[6] for superconductors of the second kind, on the basis of the equations given above. The method of calculation is based on an expansion in the small parameter ε, proposed in the work of Lasher[8]:

$$\varepsilon = (H_{c2} - B)/B,$$

where H_{c2} is the upper critical field and B is the magnetic induction. We denote by C_2/C_1 the ratio of the quantities Γ_2 and Γ_1 at the critical temperature, and by $\overline{C}_2/\overline{C}_1$ this ratio multiplied by v_1/v_2. We introduce the notation

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$$\Xi_r = \sum_n \frac{R_n^2}{B_n} \left| \frac{\overline{C}_n}{\overline{C}_1} \right|^{2r}; \qquad \Xi'_r = \sum_n \left| \frac{\overline{C}_n}{\overline{C}_1} \right|^{2r}, \qquad r = 1, 2,$$

$$2\chi_1^2 = \Xi_2 \Bigg/ \left(\frac{4\pi}{9} \frac{e^2}{c^2} \Xi_1 \Xi'_1 \right); \qquad \frac{\chi_2^2}{\chi_1^2} = \Xi_2 \Xi'_3 / \Xi_1 \Xi'_2. \tag{14}$$

It can be shown that the mean number of superconducting electrons of the first band N_0 has the form:

$$N_0^{-1} = \frac{2}{9} \frac{e^2}{c^2} \Xi_1 [1 + \sigma(2\chi_1^2 - 1)]. \tag{15}$$

For the free energy of the system in the second approximation in ε we have

$$\Delta f = \frac{1}{8\pi} \left\{ B^2 - (H_{c2} - B)^2 \frac{1 + \sigma(2\chi_2^2 - 1)}{[1 + \sigma(2\chi_1^2 - 1)]^2} \right\}. \tag{16}$$

On the basis of this expression, for the external magnetic field H_e and the magnetic moment M we obtain:

$$H_e = B + (H_{c2} - B) \frac{1 + \sigma(2\chi_2^2 - 1)}{[1 + \sigma(2\chi_1^2 - 1)]^2}; \tag{17}$$

$$-\frac{4\pi M}{H_{c2} - H_e} = \frac{1 + \sigma(2\chi_2^2 - 1)}{\sigma[2(2\chi_1^2 - 1) - (2\chi_2^2 - 1) + \sigma(2\chi_1^2 - 1)^2]}. \tag{18}$$

For the magnetic field acting in the superconductor one has the expression

$$\begin{aligned} H(x, y) = H_e - (H_{c2} - H_e) \left[2(2\varkappa_1^2 - 1) - (2\varkappa_2^2 - 1) + \sigma(2\varkappa_1^2 - 1)^2 \right]^{-1} \times \\ \times \left[\frac{|\varphi_1|^2}{N_0} \frac{1 + \sigma(2\varkappa_1^2 - 1)}{\sigma} + 2(\varkappa_2^2 - \varkappa_1^2) \right], \end{aligned} \tag{19}$$

where $|\varphi_1|^2$ is the density of superconducting electrons of the first band; φ_1 is an eigenfunction of the linearized equation (11), corresponding to the upper critical field H_{c2}. It can be shown that at $T = T_c$ the quantity $2\varkappa_2^2$ is equal to the ratio $(H_{c2}/H_c)^2$, where H_c is the thermodynamic critical magnetic field. Thus, for two-band superconductors of the second kind $\varkappa_2^2 > 1/2$. It is not difficult to show that from the condition of positivity of the derivative $\partial B / \partial H$, necessary for the stability of the thermodynamic state of the system (12), there follow the negativity of the magnetic moment M, and also the following inequalities for the parameters $\varkappa_1$ and σ:

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$$\varkappa_1^2 > \frac{1}{2}; \quad \sigma > \frac{2\varkappa_2^2 - 1}{2\varkappa_1^2 - 1}\sigma; \quad \sigma_0 = \frac{1}{2\varkappa_1^2 - 1}\left[1 - 2\frac{2\varkappa_1^2 - 1}{2\varkappa_2^2 - 1}\right]. \tag{20}$$

For $\sigma_0 < 1$, in the region of interest to us $\sigma > 1$, the quantity Δf (16) is a monotonically decreasing function of the parameter σ as the latter decreases, and, consequently, the smallest value of Δf is attained at the smallest admissible σ. As was shown in the work of Kleiner et al. (7), this value is $\sigma \simeq 1.16$. In this case the distribution of the magnetic field in the superconductor is characterized by a triangular lattice.

For $\sigma_0 > 1$, the function (16) has a minimum at $\sigma = \sigma_0$, and, taking into account the first inequality (20), we arrive at the conclusion that the smallest value of σ need not correspond to the smallest value of the free energy. In this case the above-mentioned distribution of the magnetic field with a triangular lattice will not correspond to a stable state of the system.

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