On a Condition for the Breakdown of a Region of Continuous Supersonic Flow in the Flow Past a Convex Profile with a Detached Shock Wave

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

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On a Condition for the Breakdown of a Region of Continuous Supersonic Flow in the Flow Past a Convex Profile with a Detached Shock Wave

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Let us consider the symmetric flow past a convex profile (smooth or with a corner point from which a sonic line issues to the shock wave) by a uniform supersonic flow of an ideal gas with a detached shock wave. As usual, we shall call the minimal domain of influence the domain containing a subdomain of subsonic flow and characterized by the fact that small disturbances of the contour of the profile bounding it propagate throughout this entire domain. Let the stream function be equal to zero on the critical streamline, consisting of a segment of the axis of symmetry and the contour of the profile. In view of symmetry we shall consider only the upper half-plane of the flow, in which the stream function is positive.

If the flow is continuous in the minimal domain of influence, then part of its boundary is always a characteristic AB of the first family, drawn from some point B of the profile to the sonic line. For a smooth profile this follows from the property that the sonic line on the profile in symmetric flow makes an acute angle with the direction of the streamline, along which the velocity increases (see, for example, ([1])). In the case when the sonic line issues from a corner point of the profile, the Vallée-Laurin study ([2]) has shown that the sonic line at this point is orthogonal to the limiting position of the tangent to the contour as one approaches the corner point from the region of subsonic velocities, and is turned, by the convexity at this point, toward the region of subsonic velocities (with respect to the lines $\varphi = \text{const}$—the orthogonal trajectories of the streamlines).

Fig. 1

The point A (see Fig. 1) may be located either in the flow field or on the shock wave. Let us draw from the point A to the contour of the profile the characteristic AC of the second family.

Fig. 2

Figure 2: Fig. 2

We shall assume the existence of such a profile that the flow behind the shock wave has the following properties:

1. The minimal domain of influence is unique.

2. The entropy in the minimal domain of influence is a monotonically decreasing function of the stream function.

3. The flow behind the shock wave is continuous.

Assumptions 1 and 2 are in agreement with all currently known results of solutions of the problem of flow past a convex profile by numerical methods. This also applies to the additional a priori assumptions that will be made below.

Let us first consider the flow past a smooth profile. Integrating the compatibility relations on the characteristics

$$\pm d\beta_{1,2} = \frac{\operatorname{ctg}\alpha}{\lambda}\,d\lambda + \frac{\sin 2\alpha}{2Rk}\,dS, \qquad \alpha = \arcsin\frac{1}{M},$$

along the corresponding characteristics in the direction of increasing entropy from point A to the contour of the profile, we obtain (see also (3))

$$\begin{aligned} \beta_B \geq \beta_A + I_1 = \beta_A + \frac{1}{2Rk}\int_{(AB)} \sin 2\alpha\,dS, \\ \beta_C \leq \beta_A - I_2 = \beta_A - \frac{1}{2Rk}\int_{(AC)} \sin 2\alpha\,dS. \end{aligned} \tag{1}$$

Fig. 2

Here λ is the speed coefficient, β is the angle of inclination of the velocity vector to the axis of symmetry, measured counterclockwise; S is entropy; R is the gas constant; k is the adiabatic exponent. Since both integrals are nonnegative, on the contour of the profile between points B and C there must exist a point D at which $\beta_D = \beta_A$ (Fig. 2).

Subject the profile to a continuous deformation, replacing the part of the contour located downstream from some point E tangent to the profile at this point and, in doing so, moving point E from the extreme rear point of the profile F upstream, up to point D. We shall show that point E cannot coincide with point D, because before that happens one of the assumptions made above will be violated.

Suppose the contrary. Consider the integral $\int \sin 2\alpha\, dS$ along the characteristic AC for an arbitrary position of point E, i.e. for $E \in [DE]$. This integral in a vortex flow is strictly positive, since it can vanish only in the case when the characteristic AC coincides with a sonic line; this is impossible, since then the sonic line would have to be orthogonal to the contour of the profile. Therefore, in the class of continuous solutions there exists

$$\varepsilon = \frac{1}{2Rk} \inf_{F\in[DC]} \int_{(AC)} \sin 2\alpha\, dS = \frac{1}{2Rk} \inf_{E\in[DC]} \int_{(AC)} \sin 2\alpha\, dS > 0.$$

Define the point G on the initial profile by the equality

$$\beta_G = \beta_D - \varepsilon.$$

In view of the convexity of the profile, point G lies between points D and C. If point E is placed between points D and G, i.e. if $\beta_D \geq \beta_E > \beta_G$, then we obtain

$$\beta_C = \beta_E > \beta_G = \beta_D - \varepsilon = \beta_A - \frac{1}{2Rk} \inf_{E\in[DC]} \int_{(AC)} \sin 2\alpha\, dS,$$

which contradicts the second inequality (1).

Denote by $\Delta(E)$ the maximal characteristic triangle with vertex at point E, whose base is a segment of the contour of the profile located downstream from point E.

Following (4), p. 56, we shall call problem 3 the problem of constructing, in a characteristic triangle, a continuous supersonic flow from a prescribed distribution of the velocity vector on the characteristic and the condition of no penetration on the adjacent solid wall. It follows from what has been said that on the initial profile there exists a point H, $\beta_H \leq \beta_G$,

that for $\beta_E < \beta_H$ problem 3 has a solution in the triangle $\Delta(E)$, whereas for $\beta_E \geqslant \beta_H$ it does not. (What is meant is the existence of a solution "as a whole." On the characteristic of the first family passing through the point E, an initial distribution of the velocity vector is prescribed.) This means that, when the point E is combined with the point H, a shock wave arises in $\Delta(E)$, or a local region of subsonic flow is formed.

Let us establish some estimates for the angle β_A. Denote by M_∞ the Mach number of the incident flow; let δ be the angle on the shock wave between the sonic line and the velocity vector, measured counterclockwise. It is known (see, for example, ([1])) that for $1 < M_\infty < M_0(k)$ the angle δ is obtuse, and for $M_\infty > M_0(k)$ it is acute. ($M_0(k)$ is a certain constant, $M_0 \approx 1.69$ for $k = 1.4$.) Suppose that for $M_\infty < M_0(k)$ on the sonic line there are no points K at which the sonic line is orthogonal to the velocity vector. Then, for $M_\infty < M_0(k)$,

Fig. 3

Figure 3: Fig. 3

the point A is a sonic point on the shock wave, and therefore the angle β_A is determined with the aid of the shock polar.

Fig. 3

In the case when $M_\infty > M_0(k)$, we shall assume that the segment of the sonic line between the shock wave and the point A contains no points K, nor points L, at which the curvature of the streamline, considered as a function of the arc length of the sonic line, changes sign. In this case, as shown in ([3]), when moving along the sonic line from the point A toward the shock wave, the velocity vector rotates counterclockwise, i.e., the angle β will increase. This means that for $M_\infty > M_0(k)$ the angle β_A will be smaller than the angle of inclination of the velocity vector at the sonic point on the shock wave.

Thus it has been shown that if, in the flow around an infinite blunt wedge by a uniform supersonic stream, the wedge opening angle γ is gradually increased or M_∞ is decreased (in such a way, however, that the flow behind the shock wave at infinity remains supersonic at all times), then for some value of the parameters (γ or M_∞) either a shock wave arises in the flow, or a local subsonic region of isentropically decelerated gas appears (the latter, apparently, is impossible in the general case). The shock wave that arises may, with further increase of the wedge opening angle, move upstream; when it enters the minimal domain of influence, that domain will be restructured.

The result obtained can be extended to the case of flow around a convex profile with an angular point; here, apparently, there is some connection with the work ([5]).

Without loss of generality one may assume that the characteristic of the first family AB, which bounds the minimal domain of influence, is contained inside the fan of characteristics of the first family issuing from the angular point (Fig. 3a, b), since the case in which the entire fan of characteristics belongs to the minimal domain of influence (Fig. 3c) is analogous to the case of flow around a smooth profile.

The deformation of the initial contour will be carried out in an analogous way, moving the point E upstream to the angular point, after which, if necessary, one may perform an additional deformation—a rotation of the rectilinear segment of the contour about the angular point counterclockwise; in this case the last characteristic of the expansion fan will move in the direction of the characteristic AB, which bounds the minimal domain of influence. It is easy to verify, analogously

it follows from what has been set forth that the segment of the profile contour downstream from the corner point will not have time to become parallel to the velocity vector at point A (and, in this case, the last characteristic of the expansion corner will not yet coincide with the characteristic AB) before one of assumptions 1, 2, 3 is violated.

Let us note that the requirement that the profile contour downstream from point E be rectilinear may be replaced by the weaker requirement that the change in the angle of inclination of the tangent to the profile downstream from point E be sufficiently small.

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REFERENCES

1. O. M. Belotserkovskii, PMM, **22**, issue 2 (1958).
2. R. Vaglio-Laurin, *J. Fluid Mech.*, No. 9 (1960).
3. E. G. Shifrin, *Izv. AN SSSR, Mekh. zhidk. i gazov*, No. 5 (1966).
4. N. E. Kochin, I. A. Kibel, N. V. Roze, *Theoretical Hydromechanics*, 2, Moscow, 1963.
5. H. Yoshihara, *J. Aeronaut. Sci.*, **24**, No. 9 (1957).

Note: Figure translations are in progress. See original paper for figures.