A method for the approximate realization of a motion along a given trajectory

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Abstract

The problem of approximate motion along a given trajectory is considered for the system

$$\frac{dy}{dt} = Y(y, t, v) + g(t),$$

where y is an n-dimensional vector of phase coordinates, and v is an r-dimensional vector of control forces; $Y(y, t, v)$ is a given vector function characterizing the dynamic properties of the system; $g(t)$ is an n-dimensional vector of perturbing forces. To solve the problem, a method based on the use of a computing device is proposed. Using the computing device, an analysis of the perturbing forces acting on the system is performed, and a control vector is selected to ensure sufficient proximity between the actual and desired motions. Bibliography: 5 items. Illustrations: 2.

Full Text

Preamble

This work addresses the stabilization of dynamical systems under the influence of external disturbances. We consider a system of the form:

$$\frac{dy}{dt} = Y(y, t, v) + g(t) \tag{1.1}$$

where y is an n-dimensional state vector, Y is a vector function, $g(t)$ is an n-dimensional vector representing external disturbances, and v is an r-dimensional control vector.

Following the methodology established by E. A. Barbashin [**?**, **?**, **?**], we first consider the nominal system in the absence of disturbances:

$$\frac{dy}{dt} = Y(y, t, v) \tag{1.2}$$

with initial and boundary conditions $y(0) = y^0$ and $y(t_1) = y^1$. Let the optimal control for this nominal system be denoted by $v = v^*(t, y^0, y^1)$, which generates the optimal trajectory $y^*(t, y^0, y^1)$.

When the disturbance $g(t)$ is present, we define the deviation from the optimal trajectory as $x = y - y^*$ and the control deviation as $u = v - v^*$. The dynamics of the error system can then be expressed as:

$$\frac{dx}{dt} = f(x, t, u) + g(t) \tag{1.6}$$

where the function f is defined as:

$$f(x, t, u) = Y(y^* + x, t, v^* + u) - Y(y^*, t, v^*) \tag{1.7}$$

Our objective is to maintain the state $x = 0$ over the interval $[0, t_1]$. To achieve this, we discretize the time domain into intervals of length T, such that $t_k = kT$ for $k = 1, 2, \dots, N$. Within each interval $[(k-1)T, kT]$, we seek a control u_k that minimizes the deviation $x(t)$.

Section 2. Estimation of the Disturbance

To compensate for the disturbance $g(t)$, we must first estimate its integral effect over the current interval. Integrating the system equation (2.1) over the k-th interval $[(k-1)T, kT]$, we obtain:

$$x_k - x_{k-1} = \int_{(k-1)T}^{kT} f(x(t), t, u_k)dt + \int_{(k-1)T}^{kT} g(t)dt \tag{2.2}$$

where $x_k = x(kT)$. From this, the integral of the disturbance can be isolated:

$$\int_{(k-1)T}^{kT} g(t)dt = x_k - x_{k-1} - \int_{(k-1)T}^{kT} f(x(t), t, u_k)dt \tag{2.3}$$

Using numerical integration techniques, specifically the trapezoidal rule as discussed in [**?**], the integral of f can be approximated as:

$$\int_{(k-1)T}^{kT} f(x(t), t, u_k)dt \approx \frac{T}{2}[f(x((k-1)T), (k-1)T, u_k) + f(x(kT), kT, u_k)] + R(f) \tag{2.5}$$

where $R(f)$ is the approximation error. Let $F(t, T)$ represent the average value of the disturbance over an interval T:

$$F(t, T) = \frac{1}{T}\int_{t-T}^{t} g(\tau)d\tau \tag{2.8}$$

By applying a Taylor series expansion to $F(t, T)$ and neglecting terms of $O(T^2)$, we can derive a predictive formula for the disturbance in the subsequent interval. Specifically, the predicted disturbance for the $(k+1)$-th step, denoted as $F^0((k+1)T, T)$, is calculated using values from preceding steps:

$$F^0((k+1)T, T) = F(kT, T) + [F(kT, T) - F((k-1)T, T)] \tag{2.10}$$

Section 3. Control Synthesis

For the $(k+1)$-th interval, we aim to determine the control u_{k+1} that minimizes the predicted deviation. The state at the end of the interval is approximated using a Runge-Kutta type scheme:

$$x_{k+1} = x_k + \frac{1}{6}(z_1 + 4z_2 + z_3) + O(T^4) \tag{3.3}$$

where the coefficients z_i depend on the system dynamics f and the predicted disturbance $\xi_{k+1} = F^0((k+1)T, T)$. This leads to a discrete-time transition mapping:

$$x_{k+1} = A(x_k, T, \xi_{k+1}, u_{k+1}) \tag{3.5}$$

The optimal control u_{k+1} is found by minimizing the quadratic norm of the terminal error:

$$\|x_{k+1}\|^2 = \min_u \sum (x_i)^2 \tag{3.6}$$

This minimization yields a system of equations for the control components u_j:

$$\frac{\partial}{\partial u_j} \sum (x_i)^2 = 0 \tag{3.9}$$

For a linear system of the form $\dot{x} = Lx + Mu + g(t)$, the control law can be expressed in a feedback form:

$$u_{k+1} = Px_k + Q\xi_{k+1} \tag{3.13}$$

where P and Q are gain matrices determined by the system parameters and the sampling period T.

Section 4. Numerical Example

Consider a second-order system:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_1 + u + g(t) \end{aligned} \tag{4.1}$$

Applying the discretization and control synthesis described above, we obtain the iterative relations for x_1 and x_2. The disturbance is assumed to be a harmonic function $g(t) = \sin t$.

[FIGURE: 1] [FIGURE: 2]

Figures 1 and 2 illustrate the trajectories of the system starting from different initial conditions. In the first case, $x_1(0) = 1, x_2(0) = 0$, and in the second case, $x_1(0) = 0, x_2(0) = 0$. The results demonstrate that the proposed predictive control effectively compensates for the external disturbance, maintaining the system state near the origin.

References

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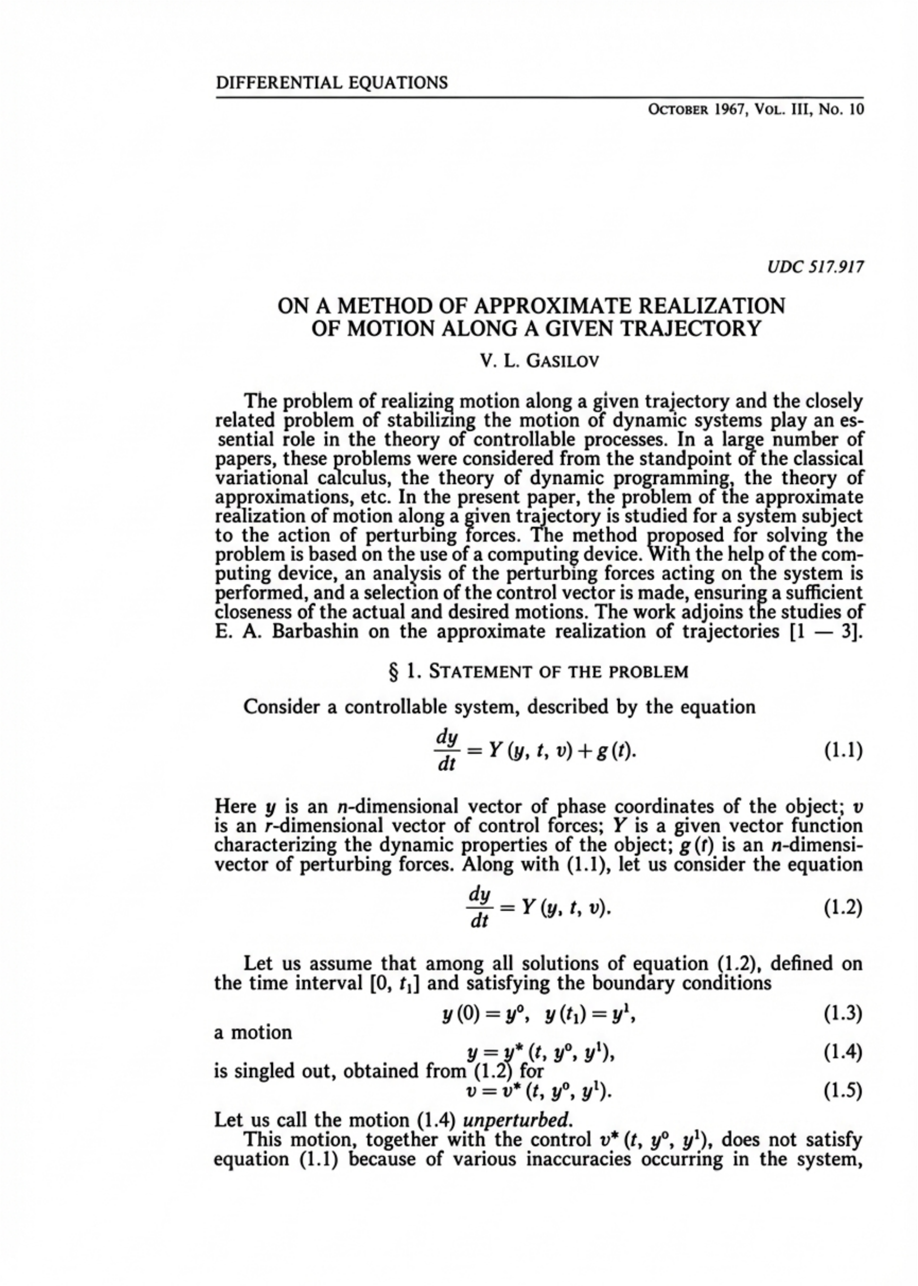
4. Demidovich, B. P., Maron, I. A., *Fundamentals of Computational Mathematics*, Fizmatgiz, 1962.

Figures



DIFFERENTIAL EQUATIONS

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ON A METHOD OF APPROXIMATE REALIZATION OF MOTION ALONG A GIVEN TRAJECTORY

V. L. GASILOV

The problem of realizing motion along a given trajectory and the closely related problem of stabilizing the motion of dynamic systems play an essential role in the theory of controllable processes. In a large number of papers, these problems were considered from the standpoint of the classical variational calculus, the theory of dynamic programming, the theory of approximations, etc. In the present paper, the problem of the approximate realization of motion along a given trajectory is studied for a system subject to the action of perturbing forces. The method proposed for solving the problem is based on the use of a computing device. With the help of the computing device, an analysis of the perturbing forces acting on the system is performed, and a selection of the control vector is made, ensuring a sufficient closeness of the actual and desired motions. The work adjoins the studies of E. A. Barbashin on the approximate realization of trajectories [1 — 3].

§ 1. STATEMENT OF THE PROBLEM

Consider a controllable system, described by the equation

$$\frac{dy}{dt} = Y(y, t, v) + g(t). \tag{1.1}$$

Here y is an n-dimensional vector of phase coordinates of the object; v is an r-dimensional vector of control forces; Y is a given vector function characterizing the dynamic properties of the object; $g(t)$ is an n-dimensivector of perturbing forces. Along with (1.1), let us consider the equation

$$\frac{dy}{dt} = Y(y, t, v). \tag{1.2}$$

Let us assume that among all solutions of equation (1.2), defined on the time interval $[0, t_1]$ and satisfying the boundary conditions

$$y(0) = y^0, \quad y(t_1) = y^1, \tag{1.3}$$

a motion

$$y = y^*(t, y^0, y^1), \tag{1.4}$$

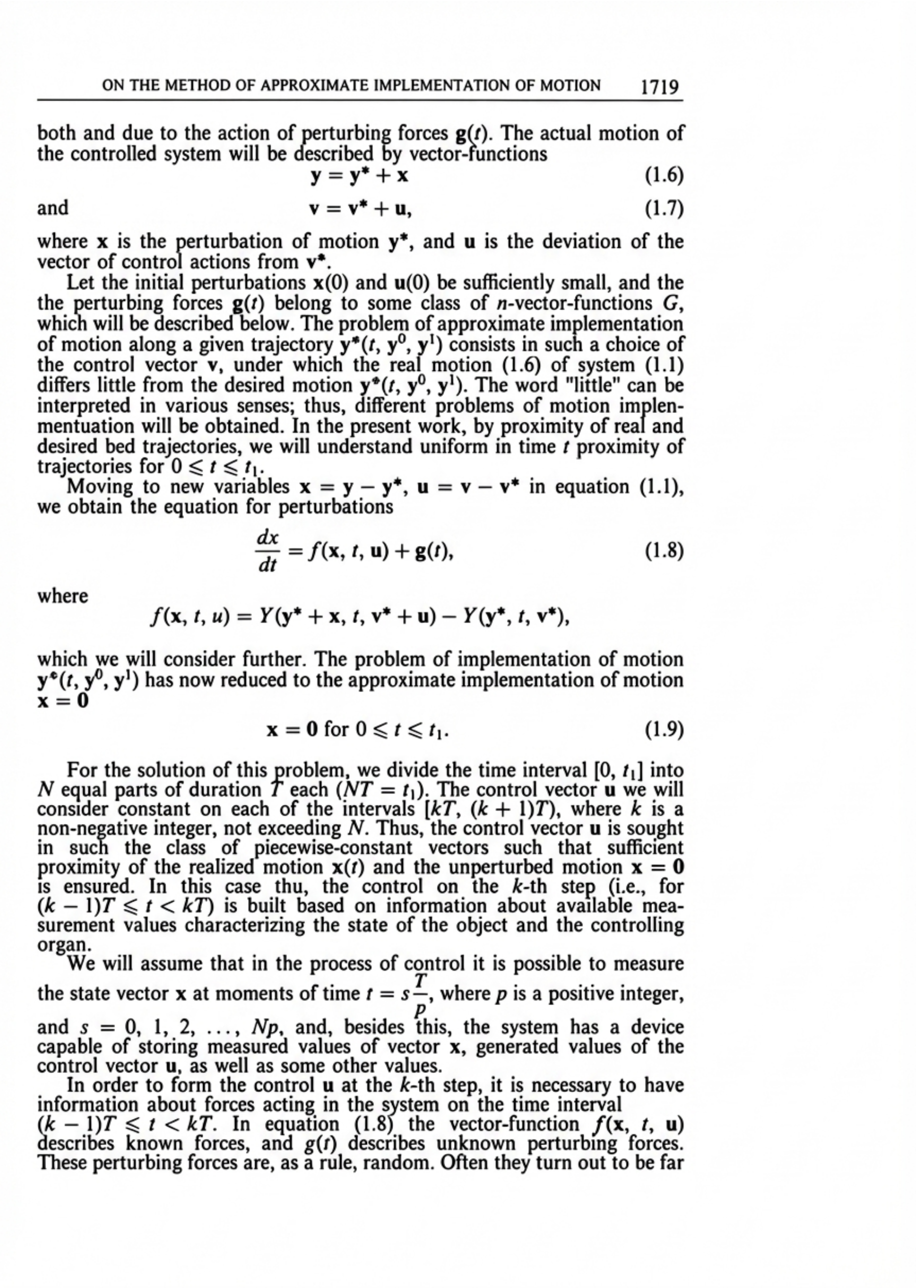
is singled out, obtained from (1.2) for

$$v = v^*(t, y^0, y^1). \tag{1.5}$$

Let us call the motion (1.4) *unperturbed*.

This motion, together with the control $v^*(t, y^0, y^1)$, does not satisfy equation (1.1) because of various inaccuracies occurring in the system,

Figure 1: Figure 1



both and due to the action of perturbing forces $\mathbf{g}(t)$. The actual motion of the controlled system will be described by vector-functions

$$\mathbf{y} = \mathbf{y}^* + \mathbf{x} \tag{1.6}$$

and

$$\mathbf{v} = \mathbf{v}^* + \mathbf{u}, \tag{1.7}$$

where $\mathbf{x}$ is the perturbation of motion $\mathbf{y}^*$, and $\mathbf{u}$ is the deviation of the vector of control actions from $\mathbf{v}^*$.

Let the initial perturbations $\mathbf{x}(0)$ and $\mathbf{u}(0)$ be sufficiently small, and the the perturbing forces $\mathbf{g}(t)$ belong to some class of n-vector-functions G, which will be described below. The problem of approximate implementation of motion along a given trajectory $\mathbf{y}^*(t, \mathbf{y}^0, \mathbf{y}^1)$ consists in such a choice of the control vector $\mathbf{v}$, under which the real motion (1.6) of system (1.1) differs little from the desired motion $\mathbf{y}^*(t, \mathbf{y}^0, \mathbf{y}^1)$. The word "little" can be interpreted in various senses; thus, different problems of motion implenmentuation will be obtained. In the present work, by proximity of real and desired bed trajectories, we will understand uniform in time t proximity of trajectories for $0 \leqslant t \leqslant t_1$.

Moving to new variables $\mathbf{x} = \mathbf{y} - \mathbf{y}^*$, $\mathbf{u} = \mathbf{v} - \mathbf{v}^*$ in equation (1.1), we obtain the equation for perturbations

$$\frac{dx}{dt} = f(\mathbf{x}, t, \mathbf{u}) + \mathbf{g}(t), \tag{1.8}$$

where

$$f(\mathbf{x}, t, u) = Y(\mathbf{y}^* + \mathbf{x}, t, \mathbf{v}^* + \mathbf{u}) - Y(\mathbf{y}^*, t, \mathbf{v}^*),$$

which we will consider further. The problem of implementation of motion $\mathbf{y}^*(t, \mathbf{y}^0, \mathbf{y}^1)$ has now reduced to the approximate implementation of motion $\mathbf{x} = \mathbf{0}$

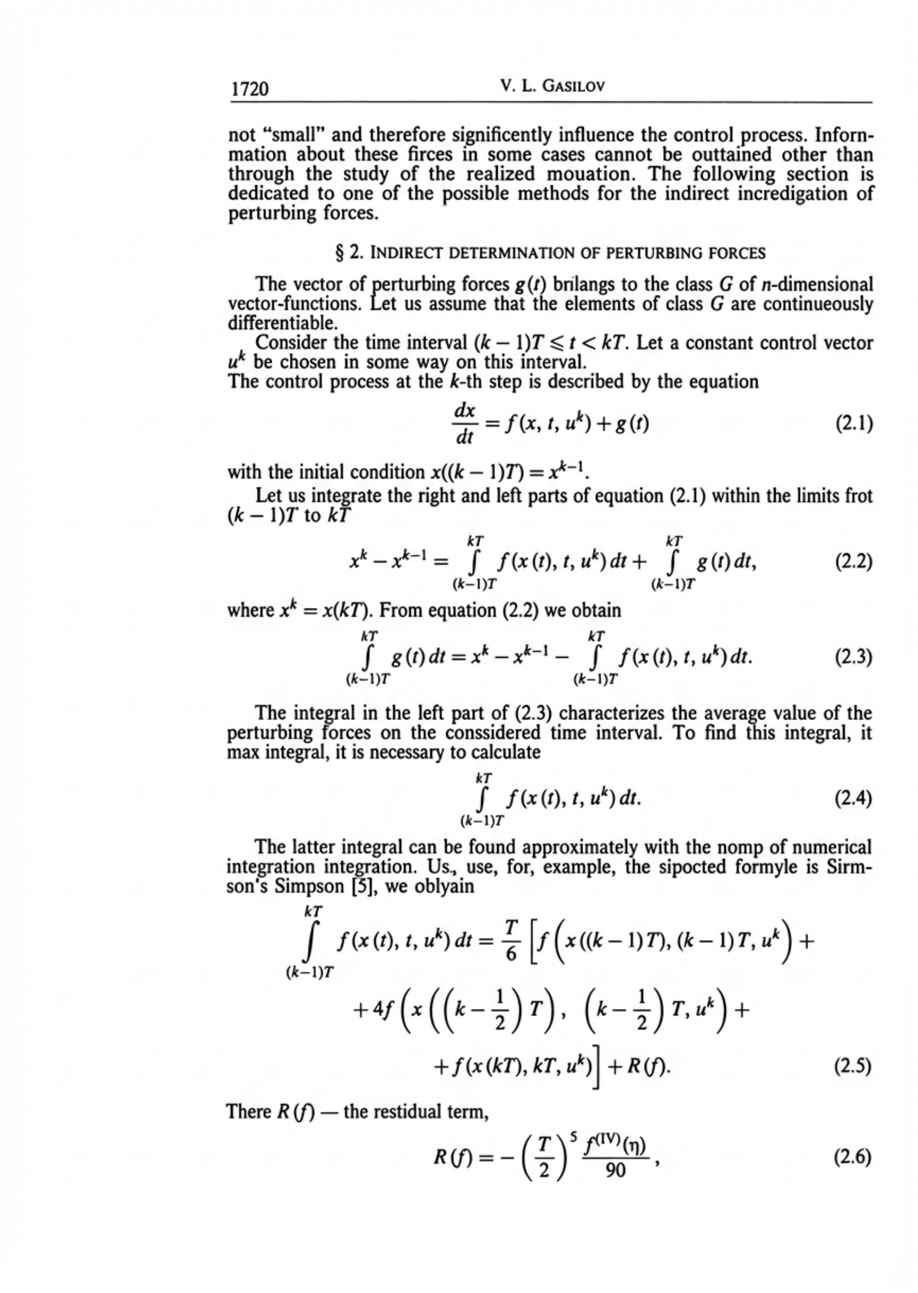
$$\mathbf{x} = \mathbf{0} \text{ for } 0 \leqslant t \leqslant t_1. \tag{1.9}$$

For the solution of this problem, we divide the time interval $[0, t_1]$ into N equal parts of duration T each ($NT = t_1$). The control vector $\mathbf{u}$ we will consider constant on each of the intervals $[kT, (k+1)T)$, where k is a non-negative integer, not exceeding N. Thus, the control vector $\mathbf{u}$ is sought in such the class of piecewise-constant vectors such that sufficient proximity of the realized motion $\mathbf{x}(t)$ and the unperturbed motion $\mathbf{x} = \mathbf{0}$ is ensured. In this case thu, the control on the k-th step (i.e., for $(k-1)T \leqslant t < kT$) is built based on information about available measurement values characterizing the state of the object and the controlling organ.

We will assume that in the process of control it is possible to measure the state vector $\mathbf{x}$ at moments of time $t = s\frac{T}{p}$, where p is a positive integer, and $s = 0, 1, 2, \ldots, Np$, and, besides this, the system has a device capable of storing measured values of vector $\mathbf{x}$, generated values of the control vector $\mathbf{u}$, as well as some other values.

In order to form the control $\mathbf{u}$ at the k-th step, it is necessary to have information about forces acting in the system on the time interval $(k-1)T \leqslant t < kT$. In equation (1.8) the vector-function $f(\mathbf{x}, t, \mathbf{u})$ describes known forces, and $g(t)$ describes unknown perturbing forces. These perturbing forces are, as a rule, random. Often they turn out to be far

Figure 2: Figure 2



not "small" and therefore significently influence the control process. Infornmation about these firces in some cases cannot be outtained other than through the study of the realized mouation. The following section is dedicated to one of the possible methods for the indirect incredigation of perturbing forces.

§ 2. INDIRECT DETERMINATION OF PERTURBING FORCES

The vector of perturbing forces $g(t)$ brilangs to the class G of n-dimensional vector-functions. Let us assume that the elements of class G are continueously differentiable.

Consider the time interval $(k-1)T \leqslant t < kT$. Let a constant control vector u^k be chosen in some way on this interval.

The control process at the k-th step is described by the equation

$$\frac{dx}{dt} = f(x, t, u^k) + g(t) \tag{2.1}$$

with the initial condition $x((k-1)T) = x^{k-1}$.

Let us integrate the right and left parts of equation (2.1) within the limits frot $(k-1)T$ to kT

$$x^k - x^{k-1} = \int_{(k-1)T}^{kT} f(x(t), t, u^k)\,dt + \int_{(k-1)T}^{kT} g(t)\,dt, \tag{2.2}$$

where $x^k = x(kT)$. From equation (2.2) we obtain

$$\int_{(k-1)T}^{kT} g(t)\,dt = x^k - x^{k-1} - \int_{(k-1)T}^{kT} f(x(t), t, u^k)\,dt. \tag{2.3}$$

The integral in the left part of (2.3) characterizes the average value of the perturbing forces on the conssidered time interval. To find this integral, it max integral, it is necessary to calculate

$$\int_{(k-1)T}^{kT} f(x(t), t, u^k)\,dt. \tag{2.4}$$

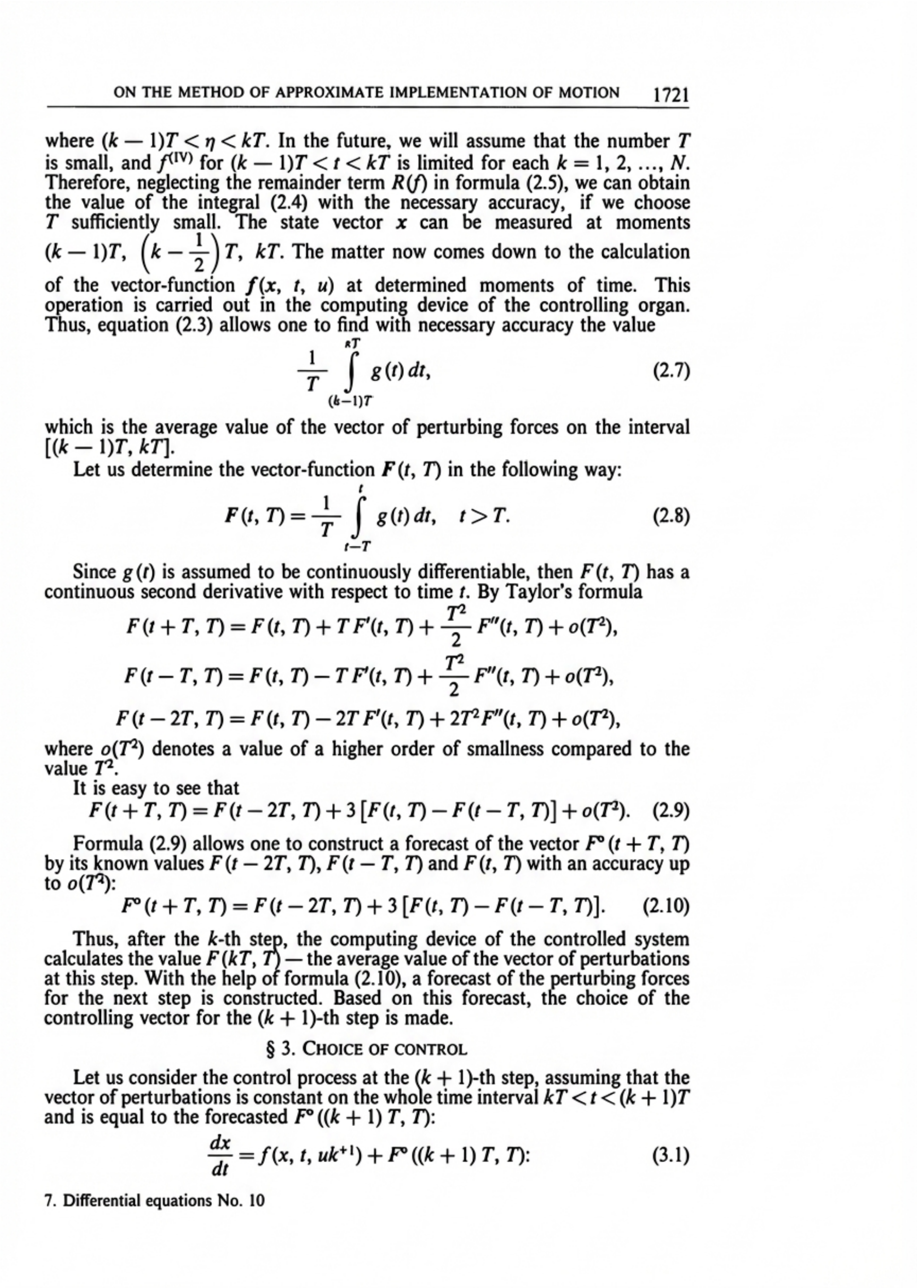
The latter integral can be found approximately with the nomp of numerical integration integration. Us., use, for, example, the sipocted formyle is Sirmson's Simpson [5], we oblyain

$$\int_{(k-1)T}^{kT} f(x(t), t, u^k)\,dt = \frac{T}{6}\left[f\Big(x((k-1)T), (k-1)T, u^k\Big) + 4f\left(x\left(\left(k-\frac{1}{2}\right)T\right),\ \left(k-\frac{1}{2}\right)T, u^k\right) + f(x(kT), kT, u^k)\right] + R(f). \tag{2.5}$$

There $R(f)$ — the residual term,

$$R(f) = -\left(\frac{T}{2}\right)^5 \frac{f^{(\mathrm{IV})}(\eta)}{90}, \tag{2.6}$$

Figure 3: Figure 3



where $(k-1)T < \eta < kT$. In the future, we will assume that the number T is small, and $f^{(IV)}$ for $(k-1)T < t < kT$ is limited for each $k = 1, 2, \dots, N$. Therefore, neglecting the remainder term $R(f)$ in formula (2.5), we can obtain the value of the integral (2.4) with the necessary accuracy, if we choose T sufficiently small. The state vector $\boldsymbol{x}$ can be measured at moments $(k-1)T,\ \left(k-\frac{1}{2}\right)T,\ kT$. The matter now comes down to the calculation of the vector-function $\boldsymbol{f}(\boldsymbol{x}, t, u)$ at determined moments of time. This operation is carried out in the computing device of the controlling organ. Thus, equation (2.3) allows one to find with necessary accuracy the value

$$\frac{1}{T}\int\limits_{(k-1)T}^{kT} g(t)\,dt, \tag{2.7}$$

which is the average value of the vector of perturbing forces on the interval $[(k-1)T, kT]$.

Let us determine the vector-function $\boldsymbol{F}(t, T)$ in the following way:

$$\boldsymbol{F}(t, T) = \frac{1}{T}\int\limits_{t-T}^{t} g(t)\,dt, \quad t > T. \tag{2.8}$$

Since $g(t)$ is assumed to be continuously differentiable, then $F(t, T)$ has a continuous second derivative with respect to time t. By Taylor's formula

$$F(t+T, T) = F(t, T) + T\,F'(t, T) + \frac{T^2}{2}F''(t, T) + o(T^2),$$

$$F(t-T, T) = F(t, T) - T\,F'(t, T) + \frac{T^2}{2}F''(t, T) + o(T^2),$$

$$F(t-2T, T) = F(t, T) - 2T\,F'(t, T) + 2T^2F''(t, T) + o(T^2),$$

where $o(T^2)$ denotes a value of a higher order of smallness compared to the value T^2.

It is easy to see that

$$F(t+T, T) = F(t-2T, T) + 3\left[F(t, T) - F(t-T, T)\right] + o(T^2). \tag{2.9}$$

Formula (2.9) allows one to construct a forecast of the vector $F^{\circ}(t+T, T)$ by its known values $F(t-2T, T)$, $F(t-T, T)$ and $F(t, T)$ with an accuracy up to $o(T^2)$:

$$F^{\circ}(t+T, T) = F(t-2T, T) + 3\left[F(t, T) - F(t-T, T)\right]. \tag{2.10}$$

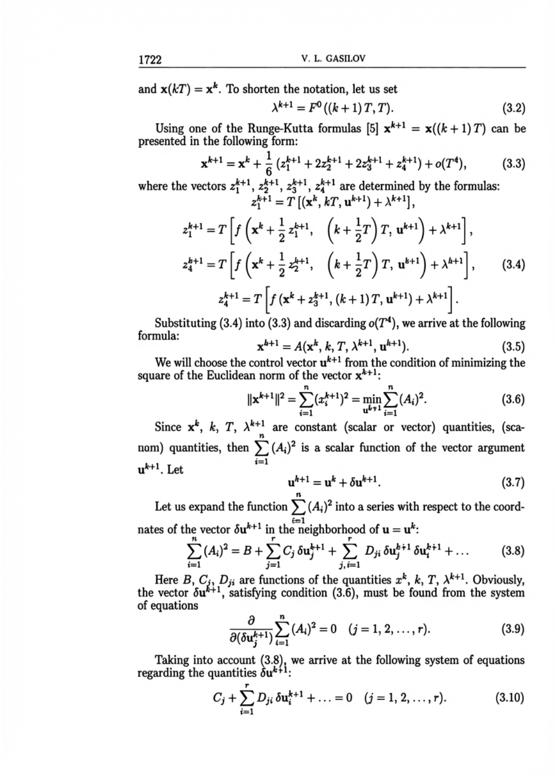
Thus, after the k-th step, the computing device of the controlled system calculates the value $F(kT, T)$ — the average value of the vector of perturbations at this step. With the help of formula (2.10), a forecast of the perturbing forces for the next step is constructed. Based on this forecast, the choice of the controlling vector for the $(k+1)$-th step is made.

§ 3. CHOICE OF CONTROL

Let us consider the control process at the $(k+1)$-th step, assuming that the vector of perturbations is constant on the whole time interval $kT < t < (k+1)T$ and is equal to the forecasted $F^{\circ}((k+1)\,T, T)$:

$$\frac{dx}{dt} = f(x, t, uk^{+1}) + F^{\circ}((k+1)\,T, T): \tag{3.1}$$

Figure 4: Figure 4



and $\mathbf{x}(kT) = \mathbf{x}^k$. To shorten the notation, let us set

$$\lambda^{k+1} = F^0((k+1)T, T). \tag{3.2}$$

Using one of the Runge-Kutta formulas [5] $\mathbf{x}^{k+1} = \mathbf{x}((k+1)T)$ can be presented in the following form:

$$\mathbf{x}^{k+1} = \mathbf{x}^k + \frac{1}{6}(z_1^{k+1} + 2z_2^{k+1} + 2z_3^{k+1} + z_4^{k+1}) + o(T^4), \tag{3.3}$$

where the vectors z_1^{k+1}, z_2^{k+1}, z_3^{k+1}, z_4^{k+1} are determined by the formulas:

$$z_1^{k+1} = T\left[(\mathbf{x}^k, kT, \mathbf{u}^{k+1}) + \lambda^{k+1}\right],$$

$$z_1^{k+1} = T\left[f\left(\mathbf{x}^k + \frac{1}{2}z_1^{k+1}, \quad \left(k + \frac{1}{2}T\right)T, \mathbf{u}^{k+1}\right) + \lambda^{k+1}\right],$$

$$z_4^{k+1} = T\left[f\left(\mathbf{x}^k + \frac{1}{2}z_2^{k+1}, \quad \left(k + \frac{1}{2}T\right)T, \mathbf{u}^{k+1}\right) + \lambda^{k+1}\right], \tag{3.4}$$

$$z_4^{k+1} = T\left[f(\mathbf{x}^k + z_3^{k+1}, (k+1)T, \mathbf{u}^{k+1}) + \lambda^{k+1}\right].$$

Substituting (3.4) into (3.3) and discarding $o(T^4)$, we arrive at the following formula:

$$\mathbf{x}^{k+1} = A(\mathbf{x}^k, k, T, \lambda^{k+1}, \mathbf{u}^{k+1}). \tag{3.5}$$

We will choose the control vector $\mathbf{u}^{k+1}$ from the condition of minimizing the square of the Euclidean norm of the vector $\mathbf{x}^{k+1}$:

$$\|\mathbf{x}^{k+1}\|^2 = \sum_{i=1}^{n}(x_i^{k+1})^2 = \min_{\mathbf{u}^{k+1}}\sum_{i=1}^{n}(A_i)^2. \tag{3.6}$$

Since $\mathbf{x}^k$, k, T, λ^{k+1} are constant (scalar or vector) quantities, (scanom) quantities, then $\sum_{i=1}^{n}(A_i)^2$ is a scalar function of the vector argument $\mathbf{u}^{k+1}$. Let

$$\mathbf{u}^{k+1} = \mathbf{u}^k + \delta\mathbf{u}^{k+1}. \tag{3.7}$$

Let us expand the function $\sum_{i=1}^{n}(A_i)^2$ into a series with respect to the coordnates of the vector $\delta\mathbf{u}^{k+1}$ in the neighborhood of $\mathbf{u} = \mathbf{u}^k$:

$$\sum_{i=1}^{n}(A_i)^2 = B + \sum_{j=1}^{r} C_j\,\delta\mathbf{u}_j^{k+1} + \sum_{j,i=1}^{r} D_{ji}\,\delta\mathbf{u}_j^{k+1}\,\delta\mathbf{u}_i^{k+1} + \ldots \tag{3.8}$$

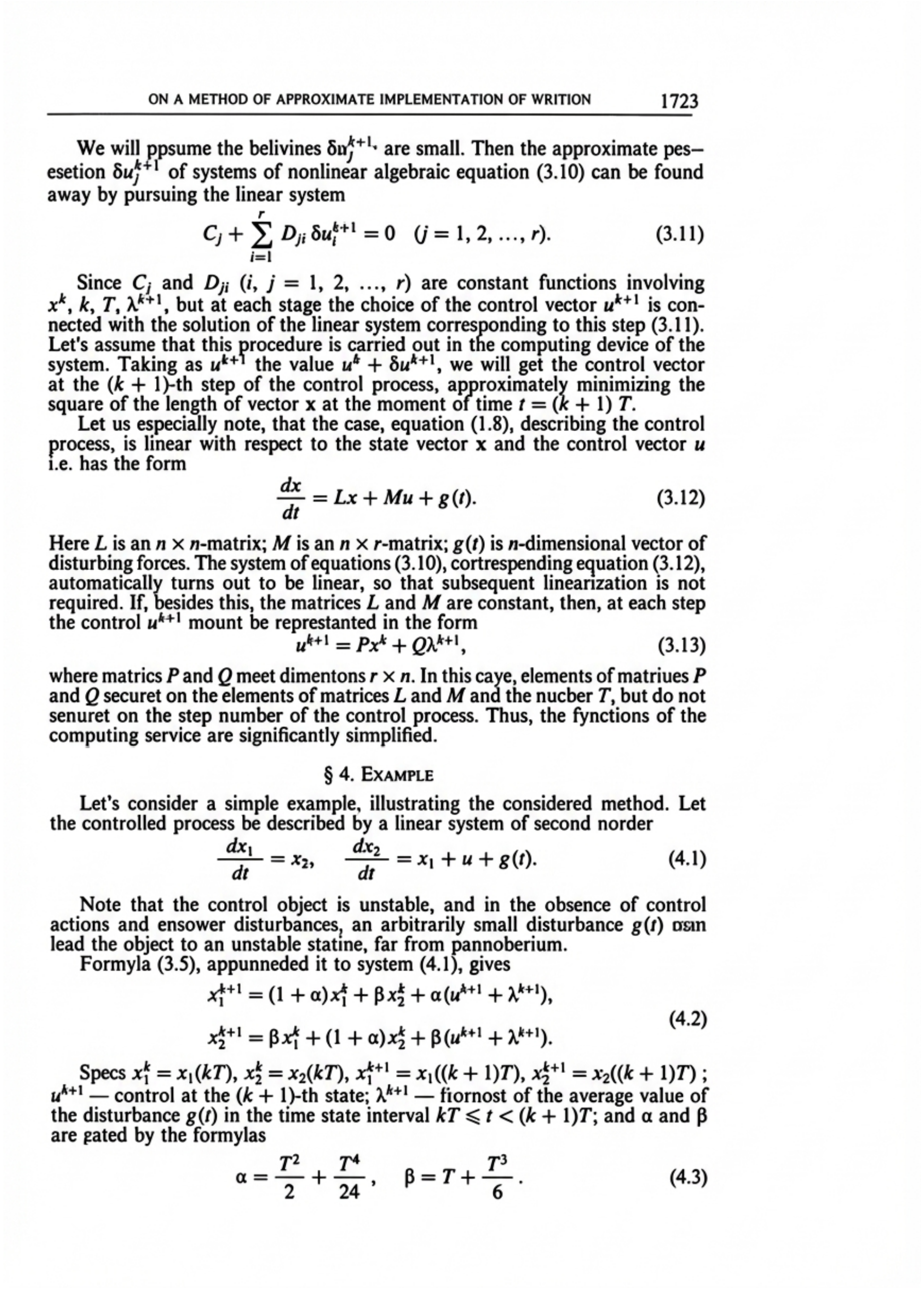
Here B, C_j, D_{ji} are functions of the quantities x^k, k, T, λ^{k+1}. Obviously, the vector $\delta\mathbf{u}^{k+1}$, satisfying condition (3.6), must be found from the system of equations

$$\frac{\partial}{\partial(\delta\mathbf{u}_j^{k+1})}\sum_{i=1}^{n}(A_i)^2 = 0 \quad (j = 1, 2, \ldots, r). \tag{3.9}$$

Taking into account (3.8), we arrive at the following system of equations regarding the quantities $\delta\mathbf{u}^{k+1}$:

$$C_j + \sum_{i=1}^{r} D_{ji}\,\delta\mathbf{u}_i^{k+1} + \ldots = 0 \quad (j = 1, 2, \ldots, r). \tag{3.10}$$

Figure 5: Figure 5



We will ppsume the belivines δu_j^{k+1} are small. Then the approximate pes–esetion δu_j^{k+1} of systems of nonlinear algebraic equation (3.10) can be found away by pursuing the linear system

$$C_j + \sum_{i=1}^{r} D_{ji}\,\delta u_i^{k+1} = 0 \quad (j = 1, 2, \ldots, r). \tag{3.11}$$

Since C_j and D_{ji} $(i, j = 1, 2, \ldots, r)$ are constant functions involving x^k, k, T, λ^{k+1}, but at each stage the choice of the control vector u^{k+1} is connected with the solution of the linear system corresponding to this step (3.11). Let's assume that this procedure is carried out in the computing device of the system. Taking as u^{k+1} the value $u^k + \delta u^{k+1}$, we will get the control vector at the $(k+1)$-th step of the control process, approximately minimizing the square of the length of vector $\mathbf{x}$ at the moment of time $t = (k+1)\,T$.

Let us especially note, that the case, equation (1.8), describing the control process, is linear with respect to the state vector $\mathbf{x}$ and the control vector $\boldsymbol{u}$ i.e. has the form

$$\frac{dx}{dt} = Lx + Mu + g(t). \tag{3.12}$$

Here L is an $n \times n$-matrix; M is an $n \times r$-matrix; $g(t)$ is n-dimensional vector of disturbing forces. The system of equations (3.10), cortrespending equation (3.12), automatically turns out to be linear, so that subsequent linearization is not required. If, besides this, the matrices L and M are constant, then, at each step the control u^{k+1} mount be represtanted in the form

$$u^{k+1} = Px^k + Q\lambda^{k+1}, \tag{3.13}$$

where matrics P and Q meet dimentons $r \times n$. In this caye, elements of matriues P and Q securet on the elements of matrices L and M and the nucber T, but do not senuret on the step number of the control process. Thus, the fynctions of the computing service are significantly simplified.

§ 4. EXAMPLE

Let's consider a simple example, illustrating the considered method. Let the controlled process be described by a linear system of second norder

$$\frac{dx_1}{dt} = x_2, \qquad \frac{dx_2}{dt} = x_1 + u + g(t). \tag{4.1}$$

Note that the control object is unstable, and in the obsence of control actions and ensower disturbances, an arbitrarily small disturbance $g(t)$ can lead the object to an unstable statine, far from pannoberium.

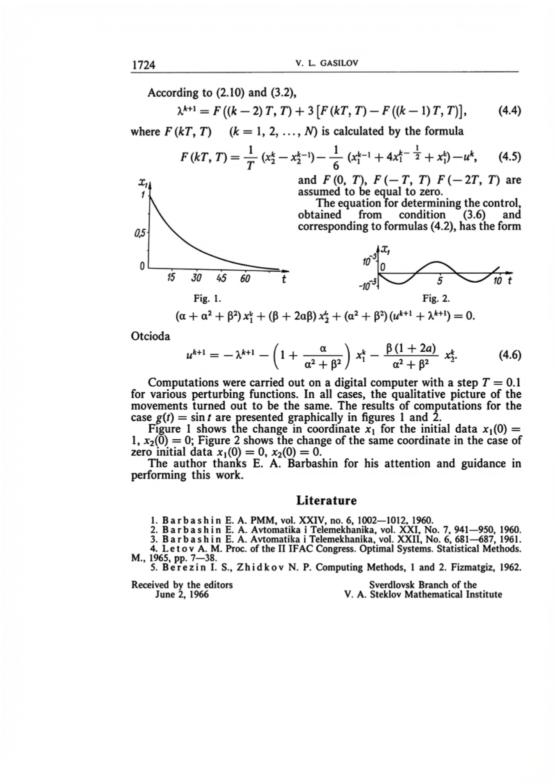
Formyla (3.5), appunneded it to system (4.1), gives

$$\begin{aligned} x_1^{k+1} &= (1+\alpha)x_1^k + \beta x_2^k + \alpha(u^{k+1} + \lambda^{k+1}), \\ x_2^{k+1} &= \beta x_1^k + (1+\alpha)x_2^k + \beta(u^{k+1} + \lambda^{k+1}). \end{aligned} \tag{4.2}$$

Specs $x_1^k = x_1(kT)$, $x_2^k = x_2(kT)$, $x_1^{k+1} = x_1((k+1)T)$, $x_2^{k+1} = x_2((k+1)T)$; u^{k+1} — control at the $(k+1)$-th state; λ^{k+1} — fiornost of the average value of the disturbance $g(t)$ in the time state interval $kT \leqslant t < (k+1)T$; and α and β are gated by the formylas

$$\alpha = \frac{T^2}{2} + \frac{T^4}{24}, \quad \beta = T + \frac{T^3}{6}. \tag{4.3}$$

Figure 6: Figure 6



According to (2.10) and (3.2),

$$\lambda^{k+1} = F((k-2)T, T) + 3\left[F(kT, T) - F((k-1)T, T)\right], \qquad (4.4)$$

where $F(kT, T) \quad (k = 1, 2, \ldots, N)$ is calculated by the formula

$$F(kT, T) = \frac{1}{T}\left(x_2^k - x_2^{k-1}\right) - \frac{1}{6}\left(x_1^{k-1} + 4x_1^{k-\frac{1}{2}} + x_1^k\right) - u^k, \qquad (4.5)$$

and $F(0, T)$, $F(-T, T)$ $F(-2T, T)$ are assumed to be equal to zero.

The equation for determining the control, obtained from condition (3.6) and corresponding to formulas (4.2), has the form

Fig. 1.

Fig. 2.

$$(\alpha + \alpha^2 + \beta^2)\, x_1^k + (\beta + 2\alpha\beta)\, x_2^k + (\alpha^2 + \beta^2)\left(u^{k+1} + \lambda^{k+1}\right) = 0.$$

Otcioda

$$u^{k+1} = -\lambda^{k+1} - \left(1 + \frac{\alpha}{\alpha^2 + \beta^2}\right) x_1^k - \frac{\beta(1 + 2a)}{\alpha^2 + \beta^2}\, x_2^k. \qquad (4.6)$$

Computations were carried out on a digital computer with a step $T = 0.1$ for various perturbing functions. In all cases, the qualitative picture of the movements turned out to be the same. The results of computations for the case $g(t) = \sin t$ are presented graphically in figures 1 and 2.

Figure 1 shows the change in coordinate x_1 for the initial data $x_1(0) = 1$, $x_2(0) = 0$; Figure 2 shows the change of the same coordinate in the case of zero initial data $x_1(0) = 0$, $x_2(0) = 0$.

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Figure 7: Figure 7