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Abstract

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THEORY OF ELASTICITY

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UNSTEADY OSCILLATIONS OF A FREE THREE-LAYER STRIP

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§ 1. It is known ([1,2]) that the problem of flexural vibrations of a three-layer plate reduces, under certain assumptions, to a single partial differential equation with respect to the displacement function $\chi(x,t)$. In the case of an unbounded plate this equation may be written in the form

$$(1-\vartheta k\nabla^2)\nabla^2\nabla^2\chi(x,\tau)+\chi_{\tau\tau}(x,\tau)-k\nabla^2\chi_{\tau\tau}(x,\tau)=\frac{l^4}{D}q_0(x,\tau), \qquad (1,1)$$

where $\nabla^2=\partial^2/\partial x^2$; $\chi_{\tau\tau}=\partial^2\chi/\partial\tau^2$; $q_0(x,\tau)$ is the external load; τ is dimensionless time; ϑ, k are dimensionless parameters characterizing the relative flexural stiffness of the load-bearing layers and the relative shear stiffness of the core; l is a linear quantity; D is the flexural stiffness of the three-layer beam (see ([1,2])).

The deflection $w(x,\tau)$ is related to the displacement function by the relation

$$w(x,\tau)=l(1-k\nabla^2)\chi(x,\tau). \qquad (1,2)$$

For $\vartheta=1$, equation (1,1) takes the form

$$-k\left(\nabla^2\nabla^2\chi-\frac{1}{k}\chi\right)-k\left(\nabla^2\chi-\frac{1}{k}\chi\right)_{\tau\tau}=q(x,\tau), \qquad (1,3)$$

where

$$q(x,\tau)=\frac{l^4}{D}q_0(x,\tau). \qquad (1,4)$$

Let us denote

$$\Phi(x,\tau) = \nabla^2\chi(x,\tau) + a_0\chi(x,\tau), \tag{1,5}$$

where

$$a_0 = -1/k. \tag{1,6}$$

Substituting (1,4)–(1,6) into (1,3), we obtain

$$\nabla^2\nabla^2\Phi(x,\tau) + \Phi_{\tau\tau}(x,\tau) = a_0 q(x,\tau). \tag{1,7}$$

Applying successively to (1,7) the Fourier and Laplace transforms with respect to the variables x and τ, we obtain

$$\Phi^*(u,s) = \frac{1}{\omega^2 + s^2}\,\overline{q}^{\,*}(u,s) \tag{1,8}$$

(u, s are the transform parameters), where

$$\omega^2 = u^4. \tag{1,9}$$

According to the convolution rule,

$$\overline{\Phi}(u,\tau) = \frac{a_0}{\omega}\int_0^\tau \overline{q}(u,\tau)\sin\omega(\tau-\tau_1)\,d\tau_1. \tag{1,10}$$

The original (1.10) has the form

$$\Phi(x,\tau) = \frac{a_0}{2\pi}\int_{-\infty}^{\infty} e^{-ixu}\left[\frac{1}{\omega}\int_0^\tau \bar{q}(u,\tau_1)\sin\omega(\tau-\tau_1)\,d\tau_1\right]du. \tag{1.11}$$

Since

$$\bar{q}(u,\tau) = \int_{-\infty}^{\infty} q(x,\tau)e^{iux}\,dx, \tag{1.12}$$

expression (1.11) becomes the following:

$$\Phi(x,\tau) = \frac{a_0}{2\pi}\int_0^\tau \int_{-\infty}^{\infty} q(v,\tau_1)\,dv\int_{-\infty}^{\infty} e^{iu(v-x)}\frac{\sin u^2(\tau-\tau_1)}{u^2}\,du. \tag{1.13}$$

Transform the right-hand side of (1.13). Denote

$$I(\mu) = \int_{-\infty}^{\infty} e^{i\xi u} \frac{\sin \mu u^2}{u^2}\, du. \tag{1.14}$$

Compute the partial derivative of $I(\mu)$ with respect to μ:

$$\frac{\partial I(\mu)}{\partial \mu} = \int_{-\infty}^{\infty} e^{i\xi u} \cos \mu u^2\, du = \frac{1}{2}\int_{-\infty}^{\infty} e^{i\xi u + i\mu u^2}\, du + \frac{1}{2}\int_{-\infty}^{\infty} e^{i\xi u - i\mu u^2}\, du. \tag{1.15}$$

Transform the integral

$$\int_{-\infty}^{\infty} e^{i\xi u + i\mu u^2}\, du = \int_{-\infty}^{\infty} e^{i\mu(u+\xi/2\mu)^2 - i\xi^2/4\mu}\, du = \sqrt{\frac{\pi}{|\mu|}}\, e^{i\cdot 1/4\pi\, \mathrm{sgn}\,\mu - i\xi^2/4\mu}. \tag{1.16}$$

Similarly, we find

$$\int_{-\infty}^{\infty} e^{i\xi u - i\mu u^2}\, du = \sqrt{\frac{\pi}{|\mu|}}\, e^{-i\cdot 1/4\pi\, \mathrm{sgn}\,\mu + i\xi^2/4\mu}. \tag{1.17}$$

Taking (1.16) and (1.17) into account, we write (1.15) in the form

$$\frac{\partial I}{\partial \mu} = \sqrt{\frac{\pi}{|\mu|}} \cos\left(\frac{\pi}{4} - \frac{\xi^2}{4|\mu|}\right). \tag{1.18}$$

Integrating (1.18), we obtain

$$I(\mu) = I(0) + \int_0^{\mu} \frac{\partial I}{\partial \mu}\, d\mu. \tag{1.19}$$

Bearing in mind the zero initial conditions, set $I(0) = 0$. Then

$$I(\mu) = \int_0^{\mu} \sqrt{\frac{\pi}{|\mu|}} \cos\left(\frac{\pi}{4} - \frac{\xi^2}{4|\mu|}\right) d\mu = \mathrm{sgn}\,\mu \int_0^{|\mu|} \sqrt{\frac{\pi}{u}} \cos\left(\frac{\pi}{4} - \frac{\xi^2}{4u}\right) du. \tag{1.20}$$

Putting $\xi = v - x$, $\mu = \tau - \tau_1$, we obtain

$$\int_{-\infty}^{\infty} e^{iu(v-x)} \frac{\sin u^2(\tau-\tau_1)}{u^2}\, du = I(\tau - \tau_1) = \int_0^{\tau-\tau_1} \sqrt{\frac{\pi}{u}} \cos\left(\frac{\pi}{4} - \frac{v-x}{4u}\right) du. \tag{1.21}$$

Changing the order of integration in the right-hand side of (1.13) and using (1.20), we find

$$\Phi(x,\tau) = -\frac{a_0}{2\sqrt{\pi}} \int_0^\tau \frac{d\xi}{\sqrt{\xi}} \int_0^{\tau-\xi} d\tau_1 \int_{-\infty}^{\infty} q(v,\tau_1) \cos\left(\frac{\pi}{4} - \frac{v-x}{4\xi}\right) dv. \quad (1.22)$$

It is hardly possible to evaluate the integrals on the right-hand side of (1.22) for an arbitrary function $q(v,\tau)$ and an arbitrary range of variation of the variables. However, some asymptotic estimates are possible.

§ 2. Let the load $q(v,\tau)$ be a unit impulse moving with velocity V in the positive direction of the x-axis. Then

$$\int_{-\infty}^{\infty} q(v,\tau_1) \cos\left(\frac{\pi}{4} - \frac{v-x}{4\xi}\right) dv = \cos\left(\frac{\pi}{4} - \frac{V_*\tau_1 - x}{4\xi}\right), \quad (2.1)$$

where $V_* = V/^{[2]}\sqrt{\Omega/D}$ is the dimensionless velocity of motion of the load. Taking (2.1) into account, we write (1.22) in the form

$$\Phi(x,\tau) = -\frac{a_0}{2\sqrt{\pi}} \int_0^\tau \frac{d\xi}{\sqrt{\xi}} \int_0^{\tau-\xi} \cos\left(\frac{\pi}{4} - \frac{V_*\tau_1 - x}{4\xi}\right) d\tau_1, \quad (2.2)$$

or, evaluating the inner integral,

$$\Phi(x,\tau) = -\frac{2a_0}{V_*\sqrt{\pi}} \int_0^\tau \sqrt{\xi}\, \sin\left[\frac{\pi}{4} - \frac{V_*(\tau-\xi) - x}{4\xi}\right] d\xi +$$

$$+\frac{2a_0}{V_*\sqrt{\pi}} \int_0^\tau \sqrt{\xi}\, \sin\left(\frac{\pi}{4} + \frac{x}{4\xi}\right) d\xi. \quad (2.3)$$

The integrands on the right-hand side of (2.2) oscillate strongly near $\xi = 0$. It is obvious, however, that the integrals converge. If (1.22) has been evaluated, then, using (1.6), we find

$$\overline{\Phi}(u,\tau) = (u^2 - b^2)\chi(u,\tau), \quad (2.4)$$

where $b = 1/\sqrt{k}$.

Finally, the displacement function $\chi(x,\tau)$ will be

$$\chi(x,\tau) = \frac{1}{b} \int_0^\tau \Phi(x,\tau_1)\, \mathrm{sh}\, b(\tau - \tau_1)\, d\tau_1, \quad (2.5)$$

where $\Phi(x, \tau)$ is determined according to (2.3).

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Note: Figure translations are in progress. See original paper for figures.