DISPLACEMENT OF A RIGID SPHERE UNDER THE ACTION OF AN ACOUSTIC PRESSURE WAVE

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Abstract

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An exact solution of problems on the action of acoustic shock waves on elastic structures encounters considerable mathematical and computational difficulties, caused by the complex nature of the interaction. The available exact results concern the assessment of the motion of rigid bodies ([1–3]).

The problem of the displacement of a body of arbitrary shape immersed in a liquid under the action of an acoustic pressure wave was first investigated by V. V. Novozhilov ([1]), where an exact expression was obtained for the limiting displacement of a body possessing two planes of symmetry. Subsequently L. I. Slepyan ([2]) showed that the established expression for the limiting displacement is also valid for elastically deformable bodies.

The problem of the motion of a rigid sphere under the action of an acoustic shock wave was first solved by M. N. Lefonova ([3]) by the operational method; the exact solution, as well as an approximate variant of the calculation, were developed by B. V. Zamyshlyaev and Yu. S. Yakovlev ([3]). A solution of the analogous problem for an infinitely long circular cylinder under the action of a unit step wave was obtained numerically by Yu. V. Goryainov and Yu. A. Fedorovich ([3]). An exact solution of this same problem by Morphew is mentioned in ([4]).

In the present work a further refinement is given of the expressions obtained earlier by the authors ([5]) for hydrodynamic forces arising in the interaction of shock waves with elastic shells immersed in an ideal compressible liquid. These expressions can be successfully used for solving problems of the dynamic strength and stability of shells. As an illustration of the possibility of using the obtained formulas also for the study of the motion of solid bodies in a liquid, the motion of a rigid sphere is considered for simplicity. In the present case approximate methods do not possess great advantages in comparison with the exact solution (with good accuracy they are somewhat more complicated than exact ones), but in the study of bodies of another geometrical shape these advantages become substantial (even in the case of a cylinder).

§ 1. Formulation of the problem

Let a pressure wave propagate in the direction of the x-axis, whose origin we take at the point of the initially immobile body. Then the motion of an absolutely rigid sphere of radius R is described by the differential equation

$$M\frac{d^2V}{dt^2} = \rho \oint \frac{\partial\psi}{\partial t}\cos(n,x)\,dS + \rho \oint \frac{\partial\varphi_1}{\partial t}\cos(n,x)\,dS + \rho \oint \frac{\partial\varphi_2}{\partial t}\cos(n,x)\,dS. \tag{1,1}$$

Here $V(t)$ is the displacement of the body; φ_1 is the potential of the reflected waves; φ_2 is the potential of the radiated waves; ψ is the potential of the incident waves, propagating with the speed of sound c; ρ is the density of the liquid; M is the mass of the sphere; S is its surface; n is the direction of the outward normal to S; t is time.

In the authors' work ([5]), from consideration of the motion of the medium in a thin layer at the surface of the body, the following expression was obtained for φ_1:

$$\varphi_1 = c\int_0^t F(t-t_1)\left.\frac{\partial\psi}{\partial n}\right|_{n=0} dt_1, \tag{1,2}$$

$$F(t) = \exp(-kct) - kc\int_0^t \exp(-kc\sqrt{t^2-u^2})J_1(kcu)\,du, \qquad k = \frac{k_1+k_2}{2}, \tag{1.3}$$

where $J_1(kcu)$ is the Bessel function of the first kind, and k_i are the principal curvatures of the surface.

In the case of a cylinder $k = 1/2R$, where R is the radius of the cylinder; for a sphere $k = 1/R$; for a plate $k = 0$.

In the same form, through $F(t)$, the radiation potential φ_2 can also be represented [5]. In [5], when solving dynamic problems, it was proposed

Fig. 1
Fig. 1

Fig. 2
Fig. 2

in the first approximation to restrict oneself to the first term in (1.3), in view of the insignificant influence of the integral term. Numerical calculations for a sphere and a cylinder show that the function $F(t)$ can be approximated by the more accurate expression

$$F(t) = \exp(-kct)\cos(kct). \tag{1.4}$$

The variation of the function $F(\tau)$ with dimensionless time $\tau = ct/R$ for a sphere is presented in Fig. 1. Curve 1 corresponds to the exact value (1.3), curve 2 to (1.4), and curve 3 corresponds to the first term in (1.3). These curves will have an analogous form for a cylinder as well, differing only by a slower decay.

§ 2. **Displacement of a sphere in the direction of the wave.** If, in determining the potentials φ_1 and φ_2, one restricts oneself to the first term in (1.3), then by the operational method, from the solution of the equation of motion (1.1) for a step pressure wave p, one can obtain the displacement of the sphere for $0 \leq \tau \leq 2$

$$w(\tau) = a_1\tau^4 - a_2\tau^3 + a_3\alpha\tau\left(\tfrac{1}{2}\alpha\tau - 1\right) + a_3(1 - e^{-\alpha\tau}), \tag{2.1}$$

where

$$w = \frac{V\rho c^2}{Rp_0}, \qquad \alpha = \frac{1+\beta}{\beta}, \qquad a_1 = \frac{1}{16\alpha\beta},$$

$$a_2 = \frac{1+2\alpha}{4\alpha^2\beta}, \qquad a_3 = \frac{3(1+2\alpha+2\alpha^2)}{2\alpha^5\beta};$$

when $\tau \geq 2$

$$w(\tau) = \frac{1}{\alpha\beta}\left[2\tau - \frac{\alpha+2}{\alpha} + \frac{3}{2\alpha^4}\exp[-\alpha(\tau-2)] - \alpha\beta a_3\exp(-\alpha\tau)\right]. \tag{2.2}$$

Here β is the ratio of the mass of the sphere to the mass of the volume of liquid displaced by it; p_0 is the pressure at the wave front.

If one uses (1.4) for the approximation of $F(t)$, then we obtain the exact solution of the problem

$$w(\tau) = \frac{3}{\kappa\beta}\left[\tau - 1 + \exp\left(-\tfrac{1}{2}\kappa\tau\right)\left(\frac{1}{2\beta\omega}\sin\omega\tau + \cos\omega\tau\right)\right], \tag{2.3}$$

$$\kappa = \frac{2\beta+1}{\beta}, \qquad \omega = \sqrt{\kappa - \kappa^2/4}.$$

Such an unexpected result is explained by the fact that when (1.4) is used, the potential φ_2 coincides with the exact solution for translational motion, and the resulting force on a stationary sphere also coincides with the exact expression

Fig. 3

Figure 1: Fig. 3

Fig. 4

Figure 2: Fig. 4

(3) (the resulting force is an integral characteristic and is determined by the first form in the expansion of the potential φ_1 in Legendre polynomials). For a cylinder, such a simple result can no longer be obtained.

Fig. 3

Fig. 4

Let us note that if, for the potential of the radiated waves, one uses the exact expression, while for the potential of the reflected waves one takes one term in (1.3), then a formula for $w(\tau)$ is also obtained, but it has a considerably more cumbersome form.

§ 3. **Results of the calculation.** In Fig. 2 are shown the changes of the velocity $\dot{w}$ and acceleration $\ddot{w}$ ($\beta = 1$) with time under the action of a step impulse. Curves 1 are the exact solution, and curve 2 is the approximate solution (2.1), (2.2). For the velocity $\dot{w} = dw/dt$, the results according to (2.1) and (2.2) are not plotted because of the small error of the approximate solution. For other values of β this error becomes appreciable. Also shown here are the curves, taken from (3), of the variation of the velocity $\dot{w}$, computed on the basis of the approximate solution (curve 3) and according to the scheme of an incompressible fluid (curve 4).

When a wave of arbitrary profile acts, the solution can be obtained with the aid of Duhamel's integral. The results for an exponential pressure wave ($\beta = 1,\ \lambda = 1$) are given in Fig. 3 (λ characterizes the rate of decrease of pressure behind the wave front). In Fig. 4 the dependences of the limiting value of the displacement λw on the buoyancy β are given (1 —exact solution, 2 —approximate).

§ 4. **Conclusion.** The proposed method can be extended to a cylinder and other shells of revolution. In the case of a cylinder, the solution of the problem is obtained much more simply than the exact one. Let us note that the approximate formulas for the hydrodynamic pressure on the basis of (1.4), in solving dynamic interaction problems, make it possible to consider large time intervals, since they satisfy the limiting relations for the potentials φ_1 and φ_2. It should be noted that the radiation potential φ_2 when (1.4) is used (or only the first term in (1.3)) is strictly applicable to the lower modes of oscillation, but it is known that for large interaction times the lower modes of oscillation play the principal role.

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Note: Figure translations are in progress. See original paper for figures.