Representation of electromagnetic fields generated by dipoles in the presence of a perfectly conducting half-plane via Macdonald integrals

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Abstract

A representation of Hertz vectors (electric or magnetic) and their corresponding electromagnetic fields **E** and **H**, generated by arbitrarily oriented dipoles (electric or magnetic, respectively) in the presence of a perfectly conducting half-plane, has been obtained in terms of Macdonald integrals. The theory of Macdonald integrals, as special functions of two variables, was recently developed by the author to an extent that allows for the creation of programs to compute these functions. The use of Macdonald integrals in the diffraction problems under consideration enables numerical field calculations over a wide range of parameters. Bibliography: 15 items.

Full Text

Preamble

This section presents a mathematical analysis of electromagnetic fields in the presence of specific boundary conditions, building upon the foundational work of T. Senior [**?**] and Yu. V. Vandakurov [**?**]. The methods developed by these authors have been further refined in subsequent studies [**?**, **?**, **?**, **?**], as well as in the works of G. D. Malyuzhinets [**?**, **?**, **?**].

We consider a source located at the point $r_0 = (\rho_0, \phi_0, z_0)$ in a cylindrical coordinate system. The primary field is generated by a dipole with moment P, which can be expressed as:

$$P = P(\cos\theta_0 e_1 + \sin\theta_0 e_2)$$

where e_1 and e_2 are unit vectors. Using the representation (1.12) for the function $G_\nu(k, R)$, we have:

$$G_{1/2}(k, R) = \sqrt{\frac{\pi}{2kR}} H^{(1)}_{1/2}(kR) = \frac{e^{ikR}}{R}$$

(1.13) The potential $\Pi(\rho, \phi, z)$ can be expressed as:

$$\Pi(\rho, \phi, z) = G_{1/2}(k, R(\alpha))e_\alpha + ...$$

(1.14) where $R(\alpha)$ is the distance function. The components of the electric and magnetic fields E and H are derived in Section 2 and Section 3, following the methodology established in [**?**].

2. Potential Representation

The potential $\Pi(\rho, \phi, z)$ is defined as:

$$\Pi(\rho, \phi, z) = \frac{iP}{8\pi} \cos\frac{\pi}{n} \int_C \left[e(\phi/2 - \phi_0), e_j\right] H_0^{(1)}(kR)\, d\alpha$$

$$\Pi(\rho, \phi, z) = \frac{iP}{8\pi} \sin\frac{\pi}{n} \int_C \left[e(\phi/2 - \phi_0) H_0^{(1)}(kR) + ...\right]$$

(2.1) For the case of a wedge, the potential $\Pi(\rho, \phi, z)$ is given by:

$$\Pi(\rho, \phi, z) = \frac{iP}{8\pi} \sin\frac{\pi}{n} \left[e(\phi/2 - \phi_0) H_0^{(1)}(kR(\phi - \phi_0)) + ...\right]$$

(2.2) Using the results from [**?**, **?**], we can express the integral in terms of the function:

$$H_0^{(1)}(z) = \frac{2}{i\pi} \int_0^\infty e^{iz\cosh t}\, dt$$

(2.3) The function $\Pi(\rho, \delta)$ is then represented as:

$$\Pi(\rho, \delta) = \sum_k \frac{1}{2k+1} G_{1/2}(k, R(\phi + \delta)) \frac{\cos\frac{\pi}{n}}{\cosh t + \cos(\phi + \delta)}$$

$$\int_0^\infty \frac{G_{1/2}(k, R(\pi + it))(\cosh t - 1)\cosh\frac{t}{n}\, dt}{\cosh t + \cos(\phi + \delta)}$$

(2.4) According to (1.14), the potential $\Pi(\rho, \delta)$ can be decomposed as:

$$\Pi(\rho, \delta) = \Gamma(\rho, \delta) + \Delta(\rho, \delta)$$

(2.5) where the geometric part $\Gamma(\rho, \delta)$ is:

$$\Gamma(\rho, \delta) = \sum G_{1/2}(k, R(\alpha)) e_{\alpha+\phi} \cot\frac{\alpha + \phi + \delta}{2n}$$

(2.6) and the diffraction part $\Pi_d(\rho, \delta)$ is:

$$\Pi_d(\rho, \delta) = -\frac{1}{4\pi n} \int_{-\pi+i\infty}^{\pi+i\infty} G_{1/2}(k, R(\alpha)) e_{\alpha+\phi+\delta} \cot\frac{\alpha + \phi + \delta}{2n}\, d\alpha$$

(2.7) As shown by A. A. Tuzhilin in (2.6), the poles of the cotangent occur at $\alpha = -\phi - \delta + 4\pi n$. For the range $-\pi < \text{Re}\ \alpha < \pi$, this condition is satisfied only for specific values of n.

4. Analysis of $G_{1/2}(k, R)$

The function $G_{1/2}(k, R(\phi + \delta))$ is periodic. Since $R(-\phi - \delta + 4\pi n) = R(\phi + \delta)$, we have:

$$\Gamma(\rho, \delta) = \sum G_{1/2}(k, R(\phi + \delta))e_\alpha$$

(2.8) The diffraction term $\Delta(\rho, \delta)$ is evaluated along the contours $\alpha = \pm\pi + it$. Using the symmetry $R(\pm\pi \pm it) = R(\pi + it)$, we obtain:

$$\Delta(\rho, \delta) = -\frac{1}{4\pi n} \int_0^\infty G_{1/2}(k, R(\pi + it)) \left[... \right] dt$$

(2.9) where the bracketed term involves the cotangent functions evaluated at the shifted arguments. This leads to the expression:

$$\frac{\sin \frac{\pi}{n} \cosh \frac{t}{n}}{\cosh t + \cos(\phi + \delta)}$$

which matches the form in (2.4). For $m = 0$ and $\nu > 0$, the integral (2.10) can be evaluated. As $a \to \infty$, the limit of the function $f(a)$ is zero. For $m > 0$ and $\cos a = -1$, we use the identity:

$$\int_0^\infty (\cosh t - 1)^m \cosh \frac{t}{n} \, dt = (1 + \cos a) \frac{2^{3/2m}}{\Gamma(m + 1/2)} ...$$

The final evaluation of the integral $J(m)$ yields:

$$J(m) = \frac{(-1)^{m-1} b^{m-1} \Gamma(l + 1/2) \Gamma(m + 1/2)}{\Gamma(1/2) J(0)}$$

(2.11) For $m = 0$, the expression simplifies. The function $f(a)$ is then:

$$f(a) = \frac{(-1)^{m-1} 2^{m-1}}{\sqrt{\pi}} \int \cdots + ...$$

(2.10) This allows for the determination of the field behavior near the edge.

For $\nu > 0$, the function $G_\nu(k, R(\pi + it))$ can be expanded. For $m = 0$:

$$\int_0^\infty G_\nu(k, R(\pi + it))(\cosh t - 1)^m \cosh \frac{t}{n} \, dt$$

(2.12) where G_ν is defined by (1.13) and $R_0 = R(\pi)$. As shown by A. A. Tuzhilin, (2.12) holds for Im $k \geq 0$. For $k \neq 0$, the integral converges. If Im $k > 0$ and $m = 0$, we use the substitution $k = i\kappa$, leading to:

$$G_\nu(i\kappa, R(\pi + it)) = \exp\left[-i\frac{\pi}{2}(\nu + 1)\right] K_\nu(\kappa R)$$

Using the properties of the modified Bessel function $K_\nu(z)$ from [**?**], valid for $|\arg z| < \pi/4$:

$$K_\nu(z) = \sqrt{\frac{\pi}{2z}} e^{-z} ...$$

The integral representation for G_ν becomes:

$$\int_0^\infty \exp\left[-\kappa\sqrt{R^2(\pi)+t^2}\right](\cosh t-1)^m \cosh\frac{t}{n}\,dt$$

Substituting the asymptotic forms, we find the behavior for $\cos\frac{a}{2} > 0$ and $\cos\frac{a}{2} < 0$. The potential near the edge is dominated by terms of the form:

$$\frac{P\rho_0\cos(a/2)}{\kappa R(a)^{v+3/2}}K_{v+1/2}(\kappa R(a))$$

By setting $\kappa = -ik$, we recover the Hankel function representation $H_\nu^{(1)}(kR)$ as seen in (2.3) and (2.12).

For the specific case $v = 1/2$ and $\mu = 0, 1$, the potential Π simplifies to:

$$\Pi = \frac{e^{ikR(\phi+\delta)}}{R(\phi+\delta)} + ...$$

(2.13) This result is consistent with the primary field (1.12) and the boundary conditions (2.1), (2.2). Section 3 details the electric and magnetic field components E and H. We define the position vectors:

$$r = \rho(\cos\phi i + \sin\phi j) + zk$$

(3.1)

$$r_0(\alpha) = \rho_0(\cos\alpha i + \sin\alpha j) + z_0 k$$

(3.2) where i, j, k are the Cartesian unit vectors. The distance function is $R(\alpha-\phi) = |r - r_0(\alpha)|$ as per (1.8) and (3.3).

Figures

Note: 2 additional figures available online.

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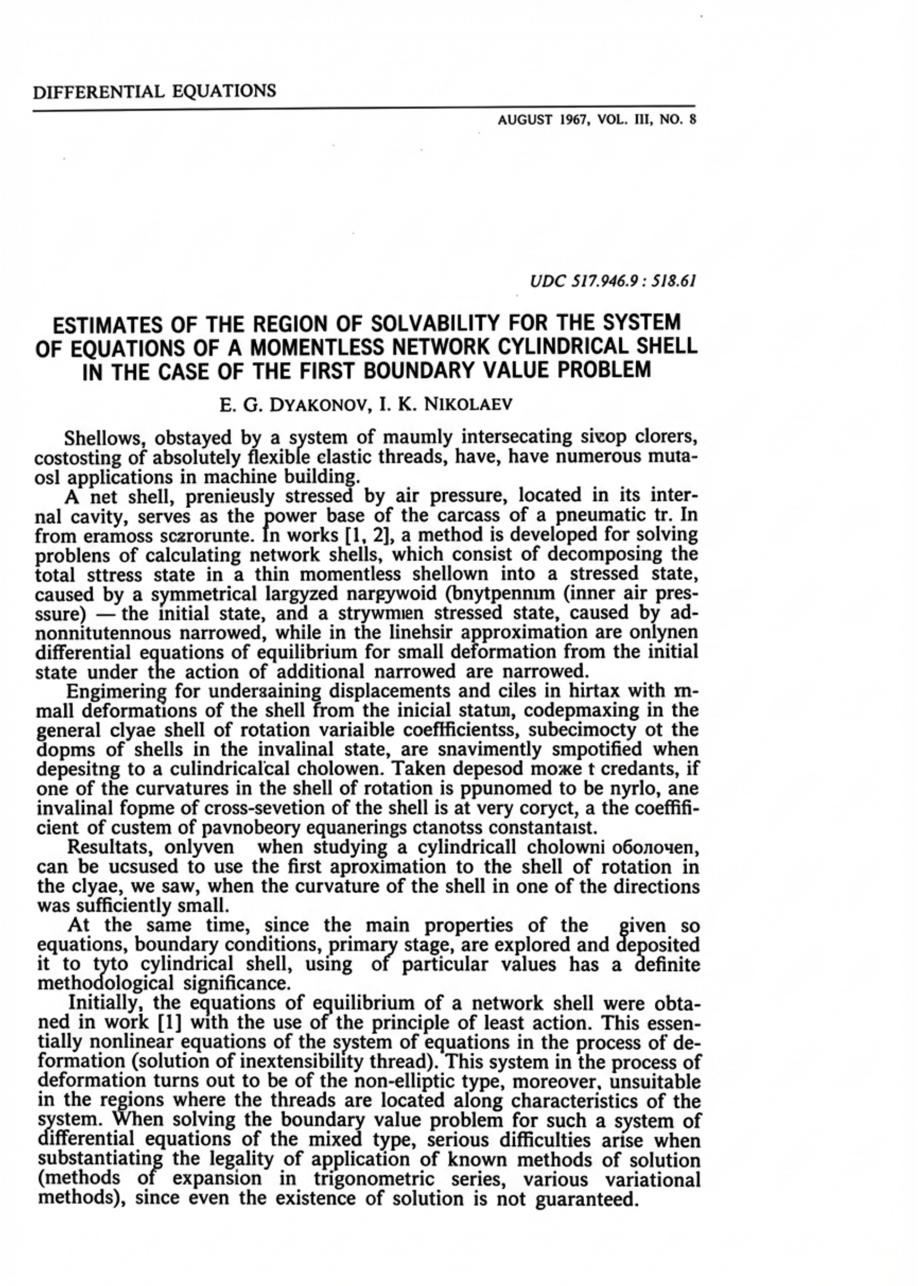

DIFFERENTIAL EQUATIONS

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ESTIMATES OF THE REGION OF SOLVABILITY FOR THE SYSTEM OF EQUATIONS OF A MOMENTLESS NETWORK CYLINDRICAL SHELL IN THE CASE OF THE FIRST BOUNDARY VALUE PROBLEM

E. G. DYAKONOV, I. K. NIKOLAEV

Shellows, obstayed by a system of maumly intersecating sivop clorers, costosting of absolutely flexible elastic threads, have, have numerous muta-osl applications in machine building.

A net shell, prenieusly stressed by air pressure, located in its internal cavity, serves as the power base of the carcass of a pneumatic tr. In from eramoss sczrorunte. In works [1, 2], a method is developed for solving problens of calculating network shells, which consist of decomposing the total sttress state in a thin momentless shellown into a stressed state, caused by a symmetrical largyzed nargywoid (bnytpennım (inner air pressure) — the initial state, and a strywmıen stressed state, caused by ad-nonnitutennous narrowed, while in the linehsir approximation are onlynen differential equations of equilibrium for small deformation from the initial state under the action of additional narrowed are narrowed.

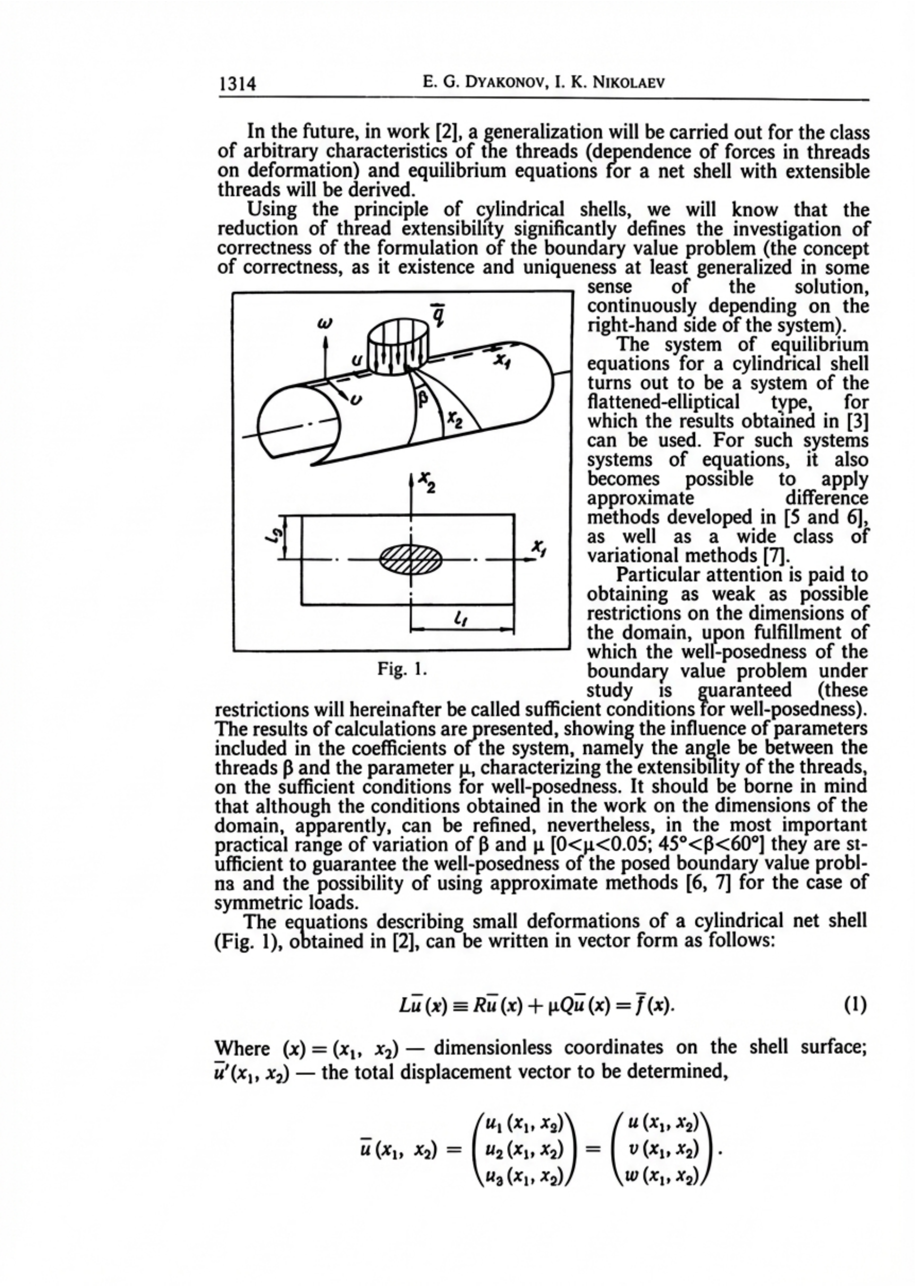
Engimering for underaaining displacements and ciles in hirtax with m-mall deformations of the shell from the inicial statun, codepmaxing in the general clyae shell of rotation variaible coeffficientss, subecimocty ot the dopms of shells in the invalinal state, are snavimently smpotified when depesitng to a culindricalcal cholowen. Taken depesod moжe t credants, if one of the curvatures in the shell of rotation is ppunomed to be nyrlo, ane invalinal fopme of cross-sevetion of the shell is at very coryct, a the coeffficient of custem of pavnobeory equanerings ctanotss constantaist.

Resultats, onlyven when studying a cylindricall cholowni оболочеп, can be ucsused to use the first aproximation to the shell of rotation in the clyae, we saw, when the curvature of the shell in one of the directions was sufficiently small.

At the same time, since the main properties of the given so equations, boundary conditions, primary stage, are explored and deposited it to tyto cylindrical shell, using of particular values has a definite methodological significance.

Initially, the equations of equilibrium of a network shell were obtaned in work [1] with the use of the principle of least action. This essentially nonlinear equations of the system of equations in the process of deformation (solution of inextensibility thread). This system in the process of deformation turns out to be of the non-elliptic type, moreover, unsuitable in the regions where the threads are located along characteristics of the system. When solving the boundary value problem for such a system of differential equations of the mixed type, serious difficulties arise when substantiating the legality of application of known methods of solution (methods of expansion in trigonometric series, various variational methods), since even the existence of solution is not guaranteed.

Figure 1: Figure 1



In the future, in work [2], a generalization will be carried out for the class of arbitrary characteristics of the threads (dependence of forces in threads on deformation) and equilibrium equations for a net shell with extensible threads will be derived.

Using the principle of cylindrical shells, we will know that the reduction of thread extensibility significantly defines the investigation of correctness of the formulation of the boundary value problem (the concept of correctness, as it existence and uniqueness at least generalized in some sense of the solution, continuously depending on the right-hand side of the system).

The system of equilibrium equations for a cylindrical shell turns out to be a system of the flattened-elliptical type, for which the results obtained in [3] can be used. For such systems systems of equations, it also becomes possible to apply approximate difference methods developed in [5 and 6], as well as a wide class of variational methods [7].

Fig. 1.

Particular attention is paid to obtaining as weak as possible restrictions on the dimensions of the domain, upon fulfillment of which the well-posedness of the boundary value problem under study is guaranteed (these restrictions will hereinafter be called sufficient conditions for well-posedness). The results of calculations are presented, showing the influence of parameters included in the coefficients of the system, namely the angle be between the threads β and the parameter μ, characterizing the extensibility of the threads, on the sufficient conditions for well-posedness. It should be borne in mind that although the conditions obtained in the work on the dimensions of the domain, apparently, can be refined, nevertheless, in the most important practical range of variation of β and μ [$0<\mu<0.05$; $45°<\beta<60°$] they are sufficient to guarantee the well-posedness of the posed boundary value problems and the possibility of using approximate methods [6, 7] for the case of symmetric loads.

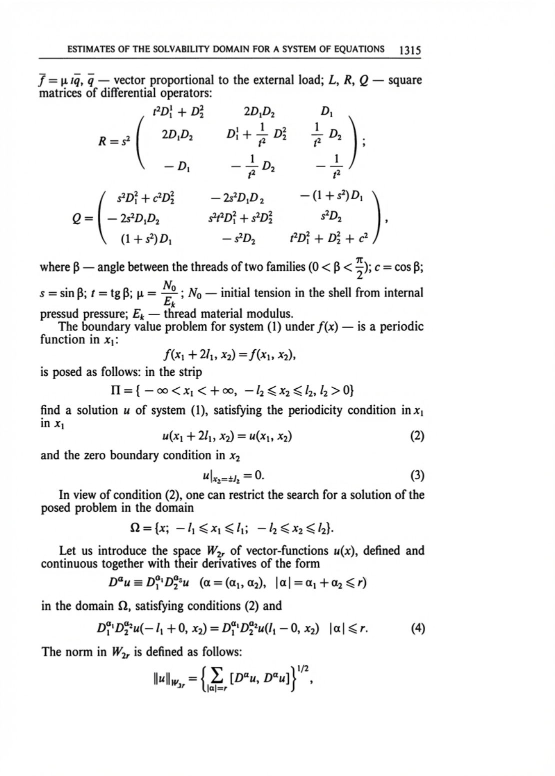
The equations describing small deformations of a cylindrical net shell (Fig. 1), obtained in [2], can be written in vector form as follows:

$$L\bar{u}(x) \equiv R\bar{u}(x) + \mu Q\bar{u}(x) = \bar{f}(x). \tag{1}$$

Where $(x) = (x_1,\ x_2)$ — dimensionless coordinates on the shell surface; $\bar{u}'(x_1, x_2)$ — the total displacement vector to be determined,

$$\bar{u}(x_1,\ x_2) = \begin{pmatrix} u_1(x_1, x_2) \\ u_2(x_1, x_2) \\ u_3(x_1, x_2) \end{pmatrix} = \begin{pmatrix} u(x_1, x_2) \\ v(x_1, x_2) \\ w(x_1, x_2) \end{pmatrix}.$$

Figure 2: Figure 2



$\bar{f} = \mu \imath \bar{q}$, $\bar{q}$ — vector proportional to the external load; L, R, Q — square matrices of differential operators:

$$R = s^2 \begin{pmatrix} t^2D_1^1 + D_2^2 & 2D_1D_2 & D_1 \\ 2D_1D_2 & D_1^1 + \frac{1}{t^2} D_2^2 & \frac{1}{t^2} D_2 \\ -D_1 & -\frac{1}{t^2} D_2 & -\frac{1}{t^2} \end{pmatrix};$$

$$Q = \begin{pmatrix} s^2D_1^2 + c^2D_2^2 & -2s^2D_1D_2 & -(1+s^2)D_1 \\ -2s^2D_1D_2 & s^2t^2D_1^2 + s^2D_2^2 & s^2D_2 \\ (1+s^2)D_1 & -s^2D_2 & t^2D_1^2 + D_2^2 + c^2 \end{pmatrix},$$

where β — angle between the threads of two families $(0 < \beta < \frac{\pi}{2})$; $c = \cos\beta$; $s = \sin\beta$; $t = \mathrm{tg}\,\beta$; $\mu = \frac{N_0}{E_k}$; N_0 — initial tension in the shell from internal pressud pressure; E_k — thread material modulus.

The boundary value problem for system (1) under $f(x)$ — is a periodic function in x_1:

$$f(x_1 + 2l_1, x_2) = f(x_1, x_2),$$

is posed as follows: in the strip

$$\Pi = \{-\infty < x_1 < +\infty,\ -l_2 \leqslant x_2 \leqslant l_2,\ l_2 > 0\}$$

find a solution u of system (1), satisfying the periodicity condition in x_1 in x_1

$$u(x_1 + 2l_1, x_2) = u(x_1, x_2) \tag{2}$$

and the zero boundary condition in x_2

$$u|_{x_2 = \pm l_2} = 0. \tag{3}$$

In view of condition (2), one can restrict the search for a solution of the posed problem in the domain

$$\Omega = \{x;\ -l_1 \leqslant x_1 \leqslant l_1;\ -l_2 \leqslant x_2 \leqslant l_2\}.$$

Let us introduce the space W_{2r} of vector-functions $u(x)$, defined and continuous together with their derivatives of the form

$$D^\alpha u \equiv D_1^{\alpha_1} D_2^{\alpha_2} u \quad (\alpha = (\alpha_1, \alpha_2),\ |\alpha| = \alpha_1 + \alpha_2 \leqslant r)$$

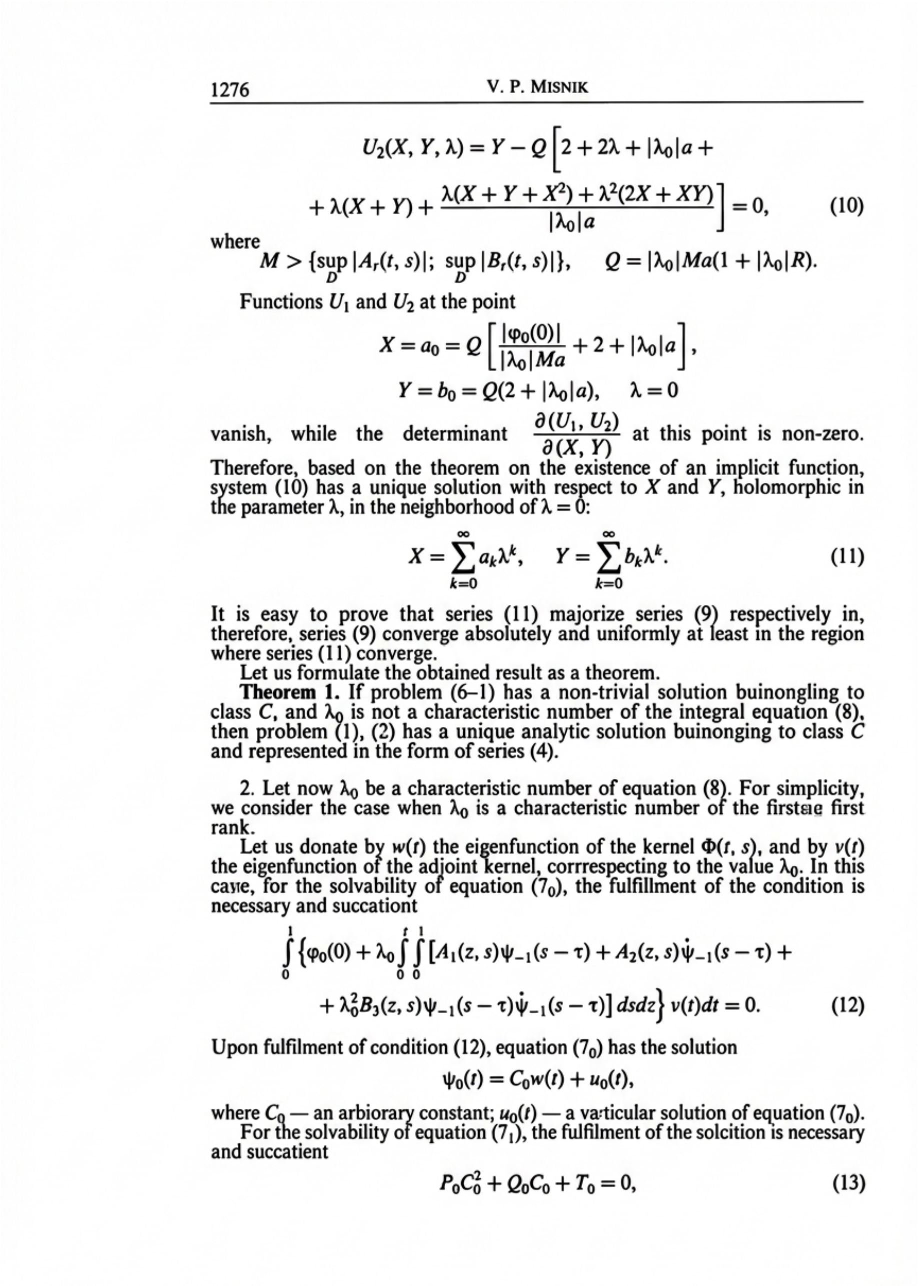
in the domain Ω, satisfying conditions (2) and

$$D_1^{\alpha_1} D_2^{\alpha_2} u(-l_1 + 0, x_2) = D_1^{\alpha_1} D_2^{\alpha_2} u(l_1 - 0, x_2) \quad |\alpha| \leqslant r. \tag{4}$$

The norm in W_{2r} is defined as follows:

$$\|u\|_{W_{2r}} = \Big\{ \sum_{|\alpha| = r} [D^\alpha u, D^\alpha u] \Big\}^{1/2},$$

Figure 3: Figure 3



$$U_2(X, Y, \lambda) = Y - Q\left[2 + 2\lambda + |\lambda_0|a + \right.$$

$$\left. + \lambda(X + Y) + \frac{\lambda(X + Y + X^2) + \lambda^2(2X + XY)}{|\lambda_0|a}\right] = 0, \tag{10}$$

where

$$M > \{\sup_D |A_r(t, s)|;\ \sup_D |B_r(t, s)|\}, \qquad Q = |\lambda_0|Ma(1 + |\lambda_0|R).$$

Functions U_1 and U_2 at the point

$$X = a_0 = Q\left[\frac{|\varphi_0(0)|}{|\lambda_0|Ma} + 2 + |\lambda_0|a\right],$$

$$Y = b_0 = Q(2 + |\lambda_0|a), \qquad \lambda = 0$$

vanish, while the determinant $\dfrac{\partial(U_1, U_2)}{\partial(X, Y)}$ at this point is non-zero. Therefore, based on the theorem on the existence of an implicit function, system (10) has a unique solution with respect to X and Y, holomorphic in the parameter λ, in the neighborhood of $\lambda = 0$:

$$X = \sum_{k=0}^{\infty} a_k\lambda^k, \qquad Y = \sum_{k=0}^{\infty} b_k\lambda^k. \tag{11}$$

It is easy to prove that series (11) majorize series (9) respectively in, therefore, series (9) converge absolutely and uniformly at least in the region where series (11) converge.

Let us formulate the obtained result as a theorem.

Theorem 1. If problem (6–1) has a non-trivial solution buinongling to class C, and λ_0 is not a characteristic number of the integral equation (8), then problem (1), (2) has a unique analytic solution buinonging to class C and represented in the form of series (4).

2. Let now λ_0 be a characteristic number of equation (8). For simplicity, we consider the case when λ_0 is a characteristic number of the firstæe first rank.

Let us donate by $w(t)$ the eigenfunction of the kernel $\Phi(t, s)$, and by $v(t)$ the eigenfunction of the adjoint kernel, corrrespecting to the value λ_0. In this cayıe, for the solvability of equation (7_0), the fulfillment of the condition is necessary and succationt

$$\int_0^1 \left\{\varphi_0(0) + \lambda_0 \int_0^t \int_0^1 \left[A_1(z, s)\psi_{-1}(s - \tau) + A_2(z, s)\dot{\psi}_{-1}(s - \tau) + \right.\right.$$

$$\left.\left. + \lambda_0^2 B_3(z, s)\psi_{-1}(s - \tau)\dot{\psi}_{-1}(s - \tau)\right] dsdz\right\} v(t)dt = 0. \tag{12}$$

Upon fulfilment of condition (12), equation (7_0) has the solution

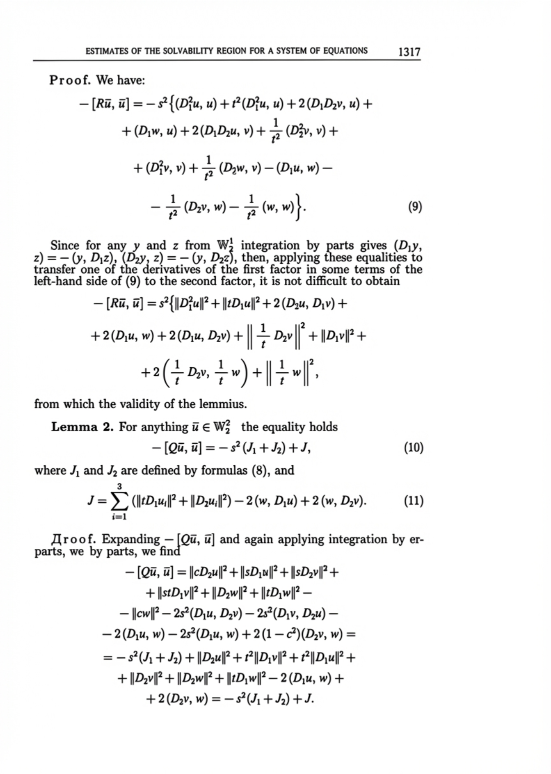
$$\psi_0(t) = C_0 w(t) + u_0(t),$$

where C_0 — an arbiorary constant; $u_0(t)$ — a va:ticular solution of equation (7_0).

For the solvability of equation (7_1), the fulfilment of the solcition is necessary and succatient

$$P_0C_0^2 + Q_0C_0 + T_0 = 0, \tag{13}$$

Figure 4: Figure 4



Proof. We have:

$$-[R\bar{u},\bar{u}]=-s^2\Big\{(D_1^2u,u)+t^2(D_1^2u,u)+2(D_1D_2v,u)+$$

$$+(D_1w,u)+2(D_1D_2u,v)+\frac{1}{t^2}(D_2^2v,v)+$$

$$+(D_1^2v,v)+\frac{1}{t^2}(D_2w,v)-(D_1u,w)-$$

$$-\frac{1}{t^2}(D_2v,w)-\frac{1}{t^2}(w,w)\Big\}. \tag{9}$$

Since for any y and z from $\mathbb{W}_2^1$ integration by parts gives $(D_1y, z)=-(y,D_1z)$, $(D_2y,z)=-(y,D_2z)$, then, applying these equalities to transfer one of the derivatives of the first factor in some terms of the left-hand side of (9) to the second factor, it is not difficult to obtain

$$-[R\bar{u},\bar{u}]=s^2\Big\{\|D_1^2u\|^2+\|tD_1u\|^2+2(D_2u,D_1v)+$$

$$+2(D_1u,w)+2(D_1u,D_2v)+\left\|\frac{1}{t}D_2v\right\|^2+\|D_1v\|^2+$$

$$+2\left(\frac{1}{t}D_2v,\frac{1}{t}w\right)+\left\|\frac{1}{t}w\right\|^2,$$

from which the validity of the lemmius.

Lemma 2. For anything $\bar{u}\in\mathbb{W}_2^2$ the equality holds

$$-[Q\bar{u},\bar{u}]=-s^2(J_1+J_2)+J, \tag{10}$$

where J_1 and J_2 are defined by formulas (8), and

$$J=\sum_{i=1}^{3}(\|tD_1u_i\|^2+\|D_2u_i\|^2)-2(w,D_1u)+2(w,D_2v). \tag{11}$$

Дroof. Expanding $-[Q\bar{u},\bar{u}]$ and again applying integration by er-parts, we by parts, we find

$$-[Q\bar{u},\bar{u}]=\|cD_2u\|^2+\|sD_1u\|^2+\|sD_2v\|^2+$$

$$+\|stD_1v\|^2+\|D_2w\|^2+\|tD_1w\|^2-$$

$$-\|cw\|^2-2s^2(D_1u,D_2v)-2s^2(D_1v,D_2u)-$$

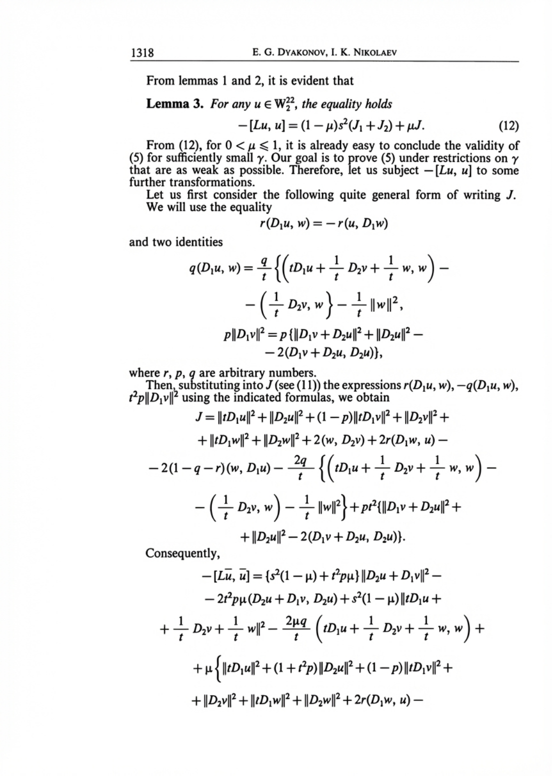
$$-2(D_1u,w)-2s^2(D_1u,w)+2(1-c^2)(D_2v,w)=$$

$$=-s^2(J_1+J_2)+\|D_2u\|^2+t^2\|D_1v\|^2+t^2\|D_1u\|^2+$$

$$+\|D_2v\|^2+\|D_2w\|^2+\|tD_1w\|^2-2(D_1u,w)+$$

$$+2(D_2v,w)=-s^2(J_1+J_2)+J.$$

Figure 5: Figure 5



From lemmas 1 and 2, it is evident that

Lemma 3. *For any $u \in \mathbb{W}_2^{22}$, the equality holds*

$$-[Lu, u] = (1-\mu)s^2(J_1 + J_2) + \mu J. \tag{12}$$

From (12), for $0 < \mu \leqslant 1$, it is already easy to conclude the validity of (5) for sufficiently small γ. Our goal is to prove (5) under restrictions on γ that are as weak as possible. Therefore, let us subject $-[Lu, u]$ to some further transformations.

Let us first consider the following quite general form of writing J. We will use the equality

$$r(D_1u, w) = -r(u, D_1w)$$

and two identities

$$q(D_1u, w) = \frac{q}{t}\left\{\left(tD_1u + \frac{1}{t}D_2v + \frac{1}{t}w, w\right) - \right.$$

$$\left. - \left(\frac{1}{t}D_2v, w\right\} - \frac{1}{t}\|w\|^2,\right.$$

$$p\|D_1v\|^2 = p\{\|D_1v + D_2u\|^2 + \|D_2u\|^2 -$$

$$-2(D_1v + D_2u, D_2u)\},$$

where r, p, q are arbitrary numbers.

Then, substituting into J (see (11)) the expressions $r(D_1u, w)$, $-q(D_1u, w)$, $t^2p\|D_1v\|^2$ using the indicated formulas, we obtain

$$J = \|tD_1u\|^2 + \|D_2u\|^2 + (1-p)\|tD_1v\|^2 + \|D_2v\|^2 +$$

$$+ \|tD_1w\|^2 + \|D_2w\|^2 + 2(w, D_2v) + 2r(D_1w, u) -$$

$$-2(1-q-r)(w, D_1u) - \frac{2q}{t}\left\{\left(tD_1u + \frac{1}{t}D_2v + \frac{1}{t}w, w\right) - \right.$$

$$\left. - \left(\frac{1}{t}D_2v, w\right) - \frac{1}{t}\|w\|^2\right\} + pt^2\{\|D_1v + D_2u\|^2 +$$

$$+ \|D_2u\|^2 - 2(D_1v + D_2u, D_2u)\}.$$

Consequently,

$$-[L\bar{u}, \bar{u}] = \{s^2(1-\mu) + t^2p\mu\}\|D_2u + D_1v\|^2 -$$

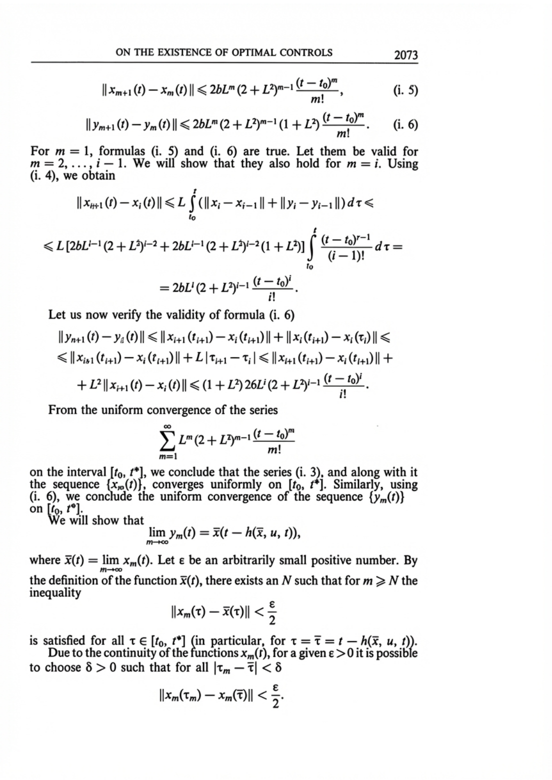
$$-2t^2p\mu(D_2u + D_1v, D_2u) + s^2(1-\mu)\|tD_1u +$$

$$+ \frac{1}{t}D_2v + \frac{1}{t}w\|^2 - \frac{2\mu q}{t}\left(tD_1u + \frac{1}{t}D_2v + \frac{1}{t}w, w\right) +$$

$$+ \mu\left\{\|tD_1u\|^2 + (1+t^2p)\|D_2u\|^2 + (1-p)\|tD_1v\|^2 + \right.$$

$$+ \|D_2v\|^2 + \|tD_1w\|^2 + \|D_2w\|^2 + 2r(D_1w, u) -$$

Figure 6: Figure 6



$$\| x_{m+1}(t) - x_m(t) \| \leqslant 2bL^m (2 + L^2)^{m-1} \frac{(t - t_0)^m}{m!}, \qquad \text{(i. 5)}$$

$$\| y_{m+1}(t) - y_m(t) \| \leqslant 2bL^m (2 + L^2)^{m-1} (1 + L^2) \frac{(t - t_0)^m}{m!}. \qquad \text{(i. 6)}$$

For $m = 1$, formulas (i. 5) and (i. 6) are true. Let them be valid for $m = 2, \ldots, i - 1$. We will show that they also hold for $m = i$. Using (i. 4), we obtain

$$\| x_{i+1}(t) - x_i(t) \| \leqslant L \int_{t_0}^{t} (\| x_i - x_{i-1} \| + \| y_i - y_{i-1} \|) \, d\tau \leqslant$$

$$\leqslant L [2bL^{i-1} (2 + L^2)^{i-2} + 2bL^{i-1} (2 + L^2)^{i-2} (1 + L^2)] \int_{t_0}^{t} \frac{(t - t_0)^{r-1}}{(i-1)!} \, d\tau =$$

$$= 2bL^i (2 + L^2)^{i-1} \frac{(t - t_0)^i}{i!}.$$

Let us now verify the validity of formula (i. 6)

$$\| y_{n+1}(t) - y_i(t) \| \leqslant \| x_{i+1}(t_{i+1}) - x_i(t_{i+1}) \| + \| x_i(t_{i+1}) - x_i(\tau_i) \| \leqslant$$

$$\leqslant \| x_{i+1}(t_{i+1}) - x_i(t_{i+1}) \| + L |\tau_{i+1} - \tau_i| \leqslant \| x_{i+1}(t_{i+1}) - x_i(t_{i+1}) \| +$$

$$+ L^2 \| x_{i+1}(t) - x_i(t) \| \leqslant (1 + L^2) 26L^i (2 + L^2)^{i-1} \frac{(t - t_0)^i}{i!}.$$

From the uniform convergence of the series

$$\sum_{m=1}^{\infty} L^m (2 + L^2)^{m-1} \frac{(t - t_0)^m}{m!}$$

on the interval $[t_0, t^*]$, we conclude that the series (i. 3), and along with it the sequence $\{x_m(t)\}$, converges uniformly on $[t_0, t^*]$. Similarly, using (i. 6), we conclude the uniform convergence of the sequence $\{y_m(t)\}$ on $[t_0, t^*]$.

We will show that

$$\lim_{m \to \infty} y_m(t) = \bar{x}(t - h(\bar{x}, u, t)),$$

where $\bar{x}(t) = \lim_{m \to \infty} x_m(t)$. Let ε be an arbitrarily small positive number. By the definition of the function $\bar{x}(t)$, there exists an N such that for $m \geqslant N$ the inequality

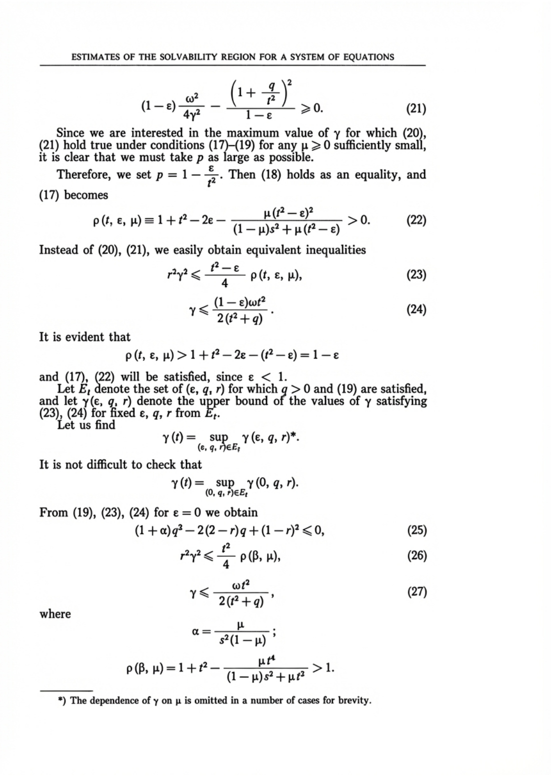
$$\|x_m(\tau) - \bar{x}(\tau)\| < \frac{\varepsilon}{2}$$

is satisfied for all $\tau \in [t_0, t^*]$ (in particular, for $\tau = \bar{\tau} = t - h(\bar{x}, u, t)$).

Due to the continuity of the functions $x_m(t)$, for a given $\varepsilon > 0$ it is possible to choose $\delta > 0$ such that for all $|\tau_m - \bar{\tau}| < \delta$

$$\|x_m(\tau_m) - x_m(\bar{\tau})\| < \frac{\varepsilon}{2}.$$

Figure 7: Figure 7



$$(1-\varepsilon)\frac{\omega^2}{4\gamma^2}-\frac{\left(1+\frac{q}{t^2}\right)^2}{1-\varepsilon}\geqslant 0. \tag{21}$$

Since we are interested in the maximum value of γ for which (20), (21) hold true under conditions (17)–(19) for any $\mu \geqslant 0$ sufficiently small, it is clear that we must take p as large as possible.

Therefore, we set $p = 1-\frac{\varepsilon}{t^2}$. Then (18) holds as an equality, and (17) becomes

$$\rho(t,\varepsilon,\mu)\equiv 1+t^2-2\varepsilon-\frac{\mu(t^2-\varepsilon)^2}{(1-\mu)s^2+\mu(t^2-\varepsilon)}>0. \tag{22}$$

Instead of (20), (21), we easily obtain equivalent inequalities

$$r^2\gamma^2\leqslant\frac{t^2-\varepsilon}{4}\,\rho(t,\varepsilon,\mu), \tag{23}$$

$$\gamma\leqslant\frac{(1-\varepsilon)\omega t^2}{2(t^2+q)}. \tag{24}$$

It is evident that

$$\rho(t,\varepsilon,\mu)>1+t^2-2\varepsilon-(t^2-\varepsilon)=1-\varepsilon$$

and (17), (22) will be satisfied, since $\varepsilon < 1$.

Let E_t denote the set of (ε, q, r) for which $q>0$ and (19) are satisfied, and let $\gamma(\varepsilon, q, r)$ denote the upper bound of the values of γ satisfying (23), (24) for fixed ε, q, r from E_t.

Let us find

$$\gamma(t)=\sup_{(\varepsilon,q,r)\in E_t}\gamma(\varepsilon,q,r)^*.$$

It is not difficult to check that

$$\gamma(t)=\sup_{(0,q,r)\in E_t}\gamma(0,q,r).$$

From (19), (23), (24) for $\varepsilon=0$ we obtain

$$(1+\alpha)q^3-2(2-r)q+(1-r)^2\leqslant 0, \tag{25}$$

$$r^2\gamma^2\leqslant\frac{t^2}{4}\,\rho(\beta,\mu), \tag{26}$$

$$\gamma\leqslant\frac{\omega t^2}{2(t^2+q)}, \tag{27}$$

where

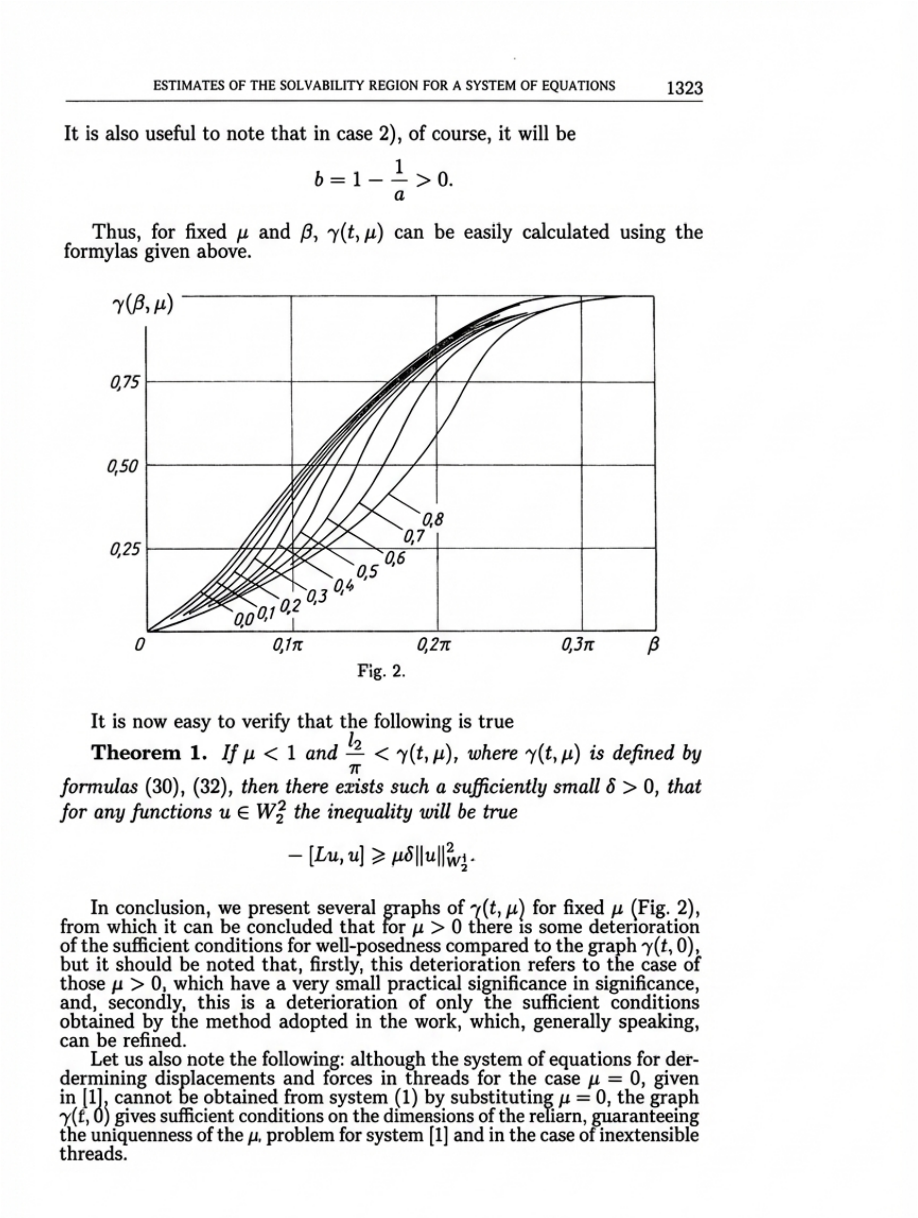
$$\alpha=\frac{\mu}{s^2(1-\mu)};$$

$$\rho(\beta,\mu)=1+t^2-\frac{\mu t^4}{(1-\mu)s^2+\mu t^2}>1.$$

---

*) The dependence of γ on μ is omitted in a number of cases for brevity.

Figure 8: Figure 8



It is also useful to note that in case 2), of course, it will be

$$b = 1 - \frac{1}{a} > 0.$$

Thus, for fixed μ and β, $\gamma(t,\mu)$ can be easily calculated using the formylas given above.

Fig. 2.

It is now easy to verify that the following is true

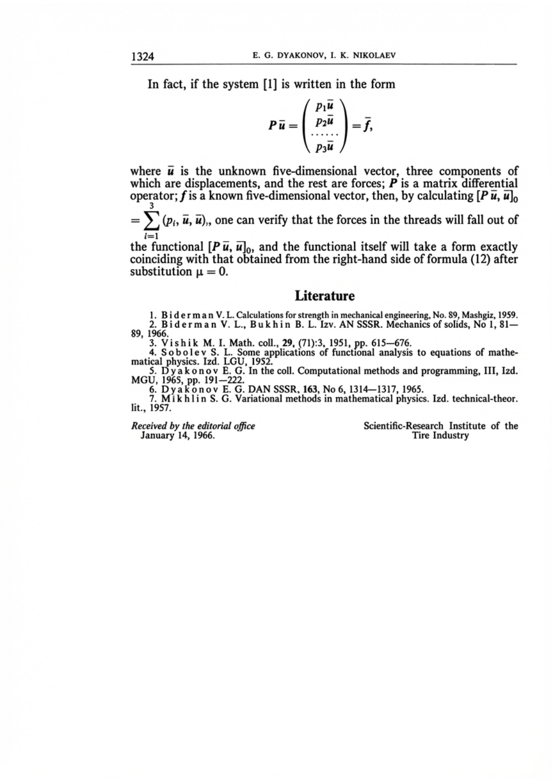
Theorem 1. *If $\mu < 1$ and $\frac{l_2}{\pi} < \gamma(t,\mu)$, where $\gamma(t,\mu)$ is defined by formulas* (30), (32), *then there exists such a sufficiently small $\delta > 0$, that for any functions $u \in W_2^2$ the inequality will be true*

$$-[Lu, u] \geqslant \mu\delta\|u\|^2_{W_2^1}.$$

In conclusion, we present several graphs of $\gamma(t,\mu)$ for fixed μ (Fig. 2), from which it can be concluded that for $\mu > 0$ there is some deterioration of the sufficient conditions for well-posedness compared to the graph $\gamma(t,0)$, but it should be noted that, firstly, this deterioration refers to the case of those $\mu > 0$, which have a very small practical significance in significance, and, secondly, this is a deterioration of only the sufficient conditions obtained by the method adopted in the work, which, generally speaking, can be refined.

Let us also note the following: although the system of equations for derdermining displacements and forces in threads for the case $\mu = 0$, given in [1], cannot be obtained from system (1) by substituting $\mu = 0$, the graph $\gamma(t,0)$ gives sufficient conditions on the dimensions of the reliarn, guaranteeing the uniquenness of the μ, problem for system [1] and in the case of inextensible threads.

Figure 9: Figure 9



In fact, if the system [1] is written in the form

$$P\bar{\boldsymbol{u}} = \begin{pmatrix} p_1\bar{\boldsymbol{u}} \\ p_2\bar{\boldsymbol{u}} \\ \cdots\cdots \\ p_3\bar{\boldsymbol{u}} \end{pmatrix} = \bar{f},$$

where $\bar{\boldsymbol{u}}$ is the unknown five-dimensional vector, three components of which are displacements, and the rest are forces; $\boldsymbol{P}$ is a matrix differential operator; $\boldsymbol{f}$ is a known five-dimensional vector, then, by calculating $[P\bar{\boldsymbol{u}}, \bar{\boldsymbol{u}}]_0 = \sum_{i=1}^{3} (p_i, \bar{\boldsymbol{u}}, \bar{\boldsymbol{u}})_i$, one can verify that the forces in the threads will fall out of the functional $[P\bar{\boldsymbol{u}}, \bar{\boldsymbol{u}}]_0$, and the functional itself will take a form exactly coinciding with that obtained from the right-hand side of formula (12) after substitution $\mu = 0$.

Literature

1. Biderman V. L. Calculations for strength in mechanical engineering, No. 89, Mashgiz, 1959.
2. Biderman V. L., Bukhin B. L. Izv. AN SSSR. Mechanics of solids, No 1, 81—89, 1966.
3. Vishik M. I. Math. coll., **29**, (71):3, 1951, pp. 615—676.
4. Sobolev S. L. Some applications of functional analysis to equations of mathematical physics. Izd. LGU, 1952.
5. Dyakonov E. G. In the coll. Computational methods and programming, III, Izd. MGU, 1965, pp. 191—222.
6. Dyakonov E. G. DAN SSSR, **163**, No 6, 1314—1317, 1965.
7. Mikhlin S. G. Variational methods in mathematical physics. Izd. technical-theor. lit., 1957.

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January 14, 1966.

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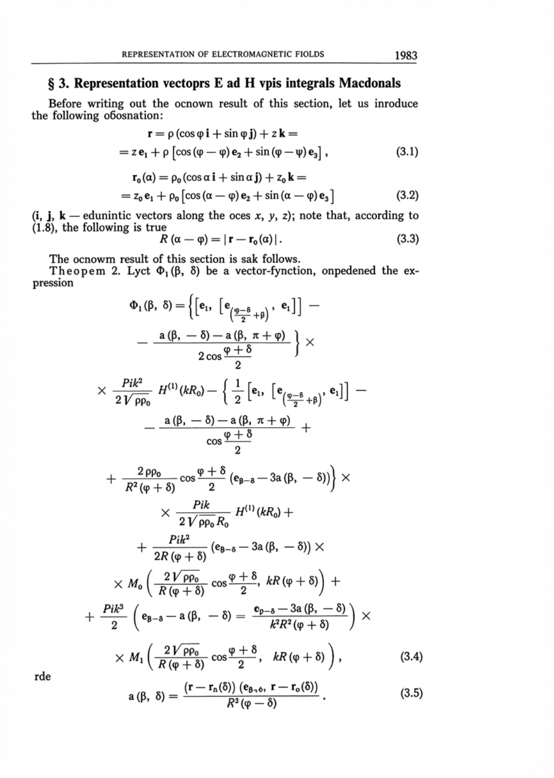
Figure 10: Figure 10



REPRESENTATION OF ELECTROMAGNETIC FIOLDS 1983

§ 3. Representation vectoprs E ad H vpis integrals Macdonals

Before writing out the ocnown result of this section, let us inroduce the following oбosnation:

$$\mathbf{r} = \rho\,(\cos\varphi\,\mathbf{i} + \sin\varphi\,\mathbf{j}) + z\,\mathbf{k} = \\ = z\,\mathbf{e}_1 + \rho\left[\cos(\varphi-\varphi)\,\mathbf{e}_2 + \sin(\varphi-\psi)\,\mathbf{e}_3\right], \tag{3.1}$$

$$\mathbf{r}_0(\alpha) = \rho_0\,(\cos\alpha\,\mathbf{i} + \sin\alpha\,\mathbf{j}) + z_0\,\mathbf{k} = \\ = z_0\,\mathbf{e}_1 + \rho_0\left[\cos(\alpha-\varphi)\,\mathbf{e}_2 + \sin(\alpha-\varphi)\,\mathbf{e}_3\right] \tag{3.2}$$

($\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ — edunintic vectors along the oces x, y, z); note that, according to (1.8), the following is true

$$R(\alpha-\varphi) = |\mathbf{r} - \mathbf{r}_0(\alpha)|. \tag{3.3}$$

The ocnowm result of this section is sak follows.

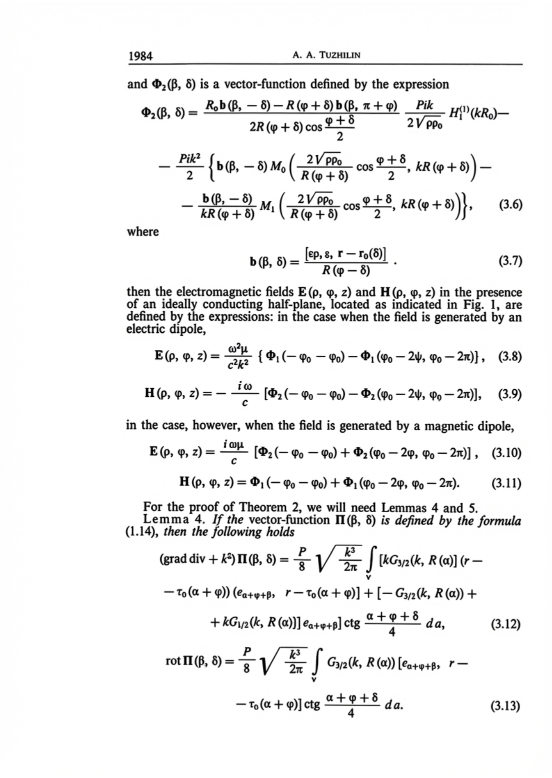
Theopem 2. Lyct $\Phi_1(\beta, \delta)$ be a vector-fynction, onpedened the expression

$$\Phi_1(\beta, \delta) = \left\{\left[\mathbf{e}_1, \left[\mathbf{e}_{\left(\frac{\varphi-\delta}{2}+\beta\right)}, \mathbf{e}_1\right]\right] - \right. \\ \left. - \frac{\mathrm{a}(\beta, -\delta) - \mathrm{a}(\beta, \pi+\varphi)}{2\cos\frac{\varphi+\delta}{2}}\right\} \times \\ \times \frac{Pik^2}{2\sqrt{\rho\rho_0}}\,H^{(1)}(kR_0) - \left\{\frac{1}{2}\left[\mathbf{e}_1, \left[\mathbf{e}_{\left(\frac{\varphi-\delta}{2}+\beta\right)}, \mathbf{e}_1\right]\right] - \right. \\ - \frac{\mathrm{a}(\beta, -\delta) - \mathrm{a}(\beta, \pi+\varphi)}{\cos\frac{\varphi+\delta}{2}} + \\ \left. + \frac{2\rho\rho_0}{R^2(\varphi+\delta)}\cos\frac{\varphi+\delta}{2}\left(\mathbf{e}_{\beta-\delta} - 3\mathrm{a}(\beta, -\delta)\right)\right\} \times \\ \times \frac{Pik}{2\sqrt{\rho\rho_0}\,R_0}\,H^{(1)}(kR_0) + \\ + \frac{Pik^2}{2R(\varphi+\delta)}\left(\mathbf{e}_{\beta-\delta} - 3\mathrm{a}(\beta, -\delta)\right) \times \\ \times M_0\left(\frac{2\sqrt{\rho\rho_0}}{R(\varphi+\delta)}\cos\frac{\varphi+\delta}{2},\ kR(\varphi+\delta)\right) + \\ + \frac{Pik^3}{2}\left(\mathbf{e}_{\beta-\delta} - \mathrm{a}(\beta, -\delta) = \frac{\mathbf{e}_{\rho-\delta} - 3\mathrm{a}(\beta, -\delta)}{k^2R^2(\varphi+\delta)}\right) \times \\ \times M_1\left(\frac{2\sqrt{\rho\rho_0}}{R(\varphi+\delta)}\cos\frac{\varphi+\delta}{2},\ kR(\varphi+\delta)\right), \tag{3.4}$$

rde

$$\mathrm{a}(\beta, \delta) = \frac{(\mathbf{r} - \mathbf{r}_n(\delta))\,(\mathbf{e}_{\beta,\delta}, \mathbf{r} - \mathbf{r}_o(\delta))}{R^3(\varphi-\delta)}. \tag{3.5}$$

Figure 11: Figure 11



and $\mathbf{\Phi}_2(\beta, \delta)$ is a vector-function defined by the expression

$$\mathbf{\Phi}_2(\beta, \delta) = \frac{R_0\mathbf{b}(\beta, -\delta) - R(\varphi+\delta)\mathbf{b}(\beta, \pi+\varphi)}{2R(\varphi+\delta)\cos\frac{\varphi+\delta}{2}} \frac{Pik}{2\sqrt{\rho\rho_0}} H_1^{(1)}(kR_0) -$$

$$-\frac{Pik^2}{2}\left\{\mathbf{b}(\beta, -\delta) M_0\left(\frac{2\sqrt{\rho\rho_0}}{R(\varphi+\delta)}\cos\frac{\varphi+\delta}{2}, kR(\varphi+\delta)\right) -\right.$$

$$\left. -\frac{\mathbf{b}(\beta, -\delta)}{kR(\varphi+\delta)} M_1\left(\frac{2\sqrt{\rho\rho_0}}{R(\varphi+\delta)}\cos\frac{\varphi+\delta}{2}, kR(\varphi+\delta)\right)\right\}, \quad (3.6)$$

where

$$\mathbf{b}(\beta, \delta) = \frac{[\varepsilon\rho, \varepsilon, \mathbf{r} - \mathbf{r}_0(\delta)]}{R(\varphi-\delta)}. \quad (3.7)$$

then the electromagnetic fields $\mathbf{E}(\rho, \varphi, z)$ and $\mathbf{H}(\rho, \varphi, z)$ in the presence of an ideally conducting half-plane, located as indicated in Fig. 1, are defined by the expressions: in the case when the field is generated by an electric dipole,

$$\mathbf{E}(\rho, \varphi, z) = \frac{\omega^2\mu}{c^2k^2}\{\mathbf{\Phi}_1(-\varphi_0 - \varphi_0) - \mathbf{\Phi}_1(\varphi_0 - 2\psi, \varphi_0 - 2\pi)\}, \quad (3.8)$$

$$\mathbf{H}(\rho, \varphi, z) = -\frac{i\omega}{c}[\mathbf{\Phi}_2(-\varphi_0 - \varphi_0) - \mathbf{\Phi}_2(\varphi_0 - 2\psi, \varphi_9 - 2\pi)], \quad (3.9)$$

in the case, however, when the field is generated by a magnetic dipole,

$$\mathbf{E}(\rho, \varphi, z) = \frac{i\omega\mu}{c}[\mathbf{\Phi}_2(-\varphi_0 - \varphi_0) + \mathbf{\Phi}_2(\varphi_0 - 2\varphi, \varphi_0 - 2\pi)], \quad (3.10)$$

$$\mathbf{H}(\rho, \varphi, z) = \mathbf{\Phi}_1(-\varphi_0 - \varphi_0) + \mathbf{\Phi}_1(\varphi_0 - 2\varphi, \varphi_0 - 2\pi). \quad (3.11)$$

For the proof of Theorem 2, we will need Lemmas 4 and 5.

Lemma 4. *If the* vector-function $\mathbf{\Pi}(\beta, \delta)$ *is defined by the formula* (1.14), *then the following holds*

$$(\text{grad div} + k^2)\mathbf{\Pi}(\beta, \delta) = \frac{P}{8}\sqrt{\frac{k^3}{2\pi}}\int\limits_{V}[kG_{3/2}(k, R(\alpha)](r -$$

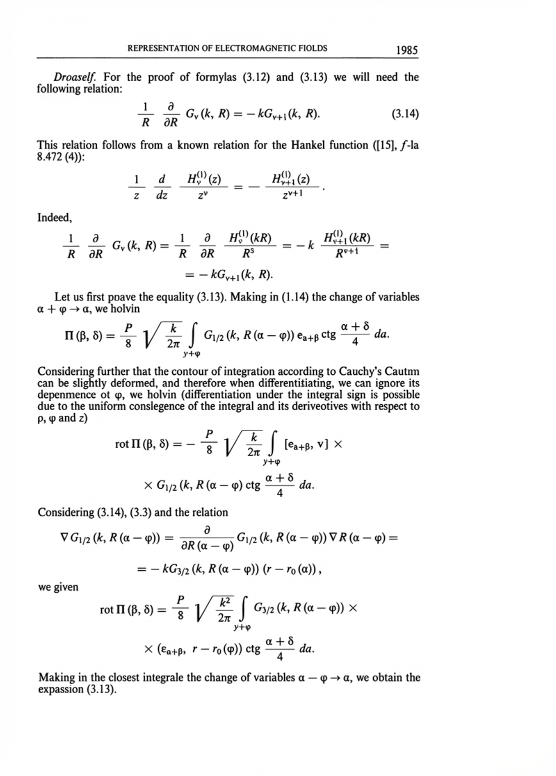
$$-\tau_0(\alpha+\varphi))(e_{\alpha+\varphi+\beta}, r - \tau_0(\alpha+\varphi)] + [-G_{3/2}(k, R(\alpha)) +$$

$$+kG_{1/2}(k, R(\alpha))]e_{\alpha+\varphi+\beta}]\operatorname{ctg}\frac{\alpha+\varphi+\delta}{4}\,da, \quad (3.12)$$

$$\operatorname{rot}\mathbf{\Pi}(\beta, \delta) = \frac{P}{8}\sqrt{\frac{k^3}{2\pi}}\int\limits_{V} G_{3/2}(k, R(\alpha))[e_{\alpha+\varphi+\beta}, r -$$

$$-\tau_0(\alpha+\varphi)]\operatorname{ctg}\frac{\alpha+\varphi+\delta}{4}\,da. \quad (3.13)$$

Figure 12: Figure 12



Droaself. For the proof of formylas (3.12) and (3.13) we will need the following relation:

$$\frac{1}{R}\ \frac{\partial}{\partial R}\ G_{\nu}(k,\ R) = -\,kG_{\nu+1}(k,\ R). \qquad (3.14)$$

This relation follows from a known relation for the Hankel function ([15], *f*-la 8.472 (4)):

$$\frac{1}{z}\ \frac{d}{dz}\ \frac{H_{\nu}^{(1)}(z)}{z^{\nu}} = -\ \frac{H_{\nu+1}^{(1)}(z)}{z^{\nu+1}}\,.$$

Indeed,

$$\frac{1}{R}\ \frac{\partial}{\partial R}\ G_{\nu}(k,\ R) = \frac{1}{R}\ \frac{\partial}{\partial R}\ \frac{H_{\nu}^{(1)}(kR)}{R^{s}} = -\,k\ \frac{H_{\nu+1}^{(1)}(kR)}{R^{\nu+1}} =$$

$$= -\,kG_{\nu+1}(k,\ R).$$

Let us first poave the equality (3.13). Making in (1.14) the change of variables $\alpha+\varphi\rightarrow\alpha$, we holvin

$$\Pi(\beta,\ \delta) = \frac{P}{8}\sqrt{\frac{k}{2\pi}}\int\limits_{y+\varphi} G_{1/2}(k,\ R(\alpha-\varphi))\ \mathrm{e}_{a+\beta}\operatorname{ctg}\frac{\alpha+\delta}{4}\ da.$$

Considering further that the contour of integration according to Cauchy's Cautm can be slightly deformed, and therefore when differentitiating, we can ignore its depenmence ot φ, we holvin (differentiation under the integral sign is possible due to the uniform conslegence of the integral and its deriveotives with respect to ρ, φ and z)

$$\operatorname{rot}\Pi(\beta,\ \delta) = -\ \frac{P}{8}\sqrt{\frac{k}{2\pi}}\int\limits_{y+\varphi}[\mathrm{e}_{a+\beta},\ \mathrm{v}]\ \times$$

$$\times\ G_{1/2}(k,\ R(\alpha-\varphi)\operatorname{ctg}\frac{\alpha+\delta}{4}\ da.$$

Considering (3.14), (3.3) and the relation

$$\nabla G_{1/2}(k,\ R(\alpha-\varphi)) = \frac{\partial}{\partial R(\alpha-\varphi)}G_{1/2}(k,\ R(\alpha-\varphi))\,\nabla R(\alpha-\varphi) =$$

$$= -\,kG_{3/2}(k,\ R(\alpha-\varphi))\ (r-r_0(\alpha))\,,$$

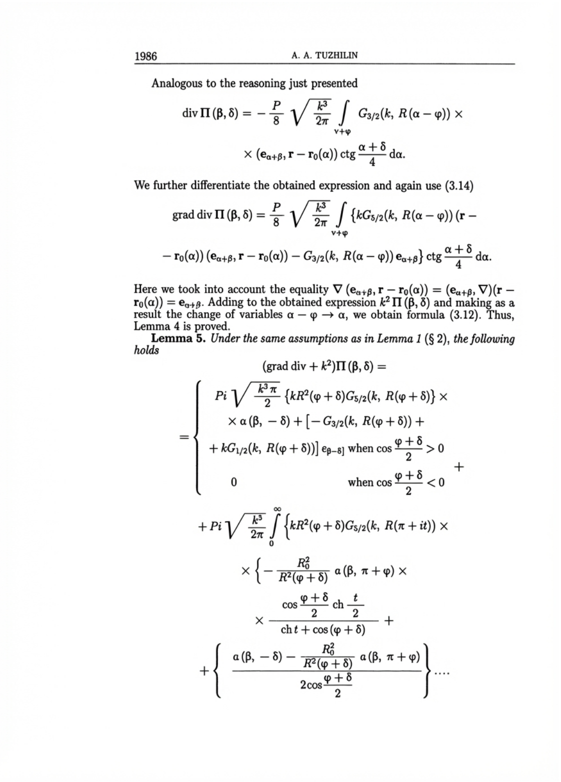
we given

$$\operatorname{rot}\Pi(\beta,\ \delta) = \frac{P}{8}\sqrt{\frac{k^2}{2\pi}}\int\limits_{y+\varphi} G_{3/2}(k,\ R(\alpha-\varphi))\ \times$$

$$\times\ (\varepsilon_{a+\beta},\ r-r_0(\varphi))\operatorname{ctg}\frac{\alpha+\delta}{4}\ da.$$

Making in the closest integrale the change of variables $\alpha-\varphi\rightarrow\alpha$, we obtain the expassion (3.13).

Figure 13: Figure 13



Analogous to the reasoning just presented

$$\operatorname{div}\mathbf{\Pi}(\beta,\delta) = -\frac{P}{8}\sqrt{\frac{k^3}{2\pi}}\int\limits_{\nu+\varphi} G_{3/2}(k,\ R(\alpha-\varphi))\times$$

$$\times\,(\mathbf{e}_{\alpha+\beta}, \mathbf{r}-\mathbf{r}_0(\alpha))\operatorname{ctg}\frac{\alpha+\delta}{4}\,d\alpha.$$

We further differentiate the obtained expression and again use (3.14)

$$\operatorname{grad}\operatorname{div}\mathbf{\Pi}(\beta,\delta) = \frac{P}{8}\sqrt{\frac{k^3}{2\pi}}\int\limits_{\nu+\varphi}\{kG_{5/2}(k,\ R(\alpha-\varphi))\,(\mathbf{r}-$$

$$-\,\mathbf{r}_0(\alpha))\,(\mathbf{e}_{\alpha+\beta}, \mathbf{r}-\mathbf{r}_0(\alpha)) - G_{3/2}(k,\ R(\alpha-\varphi))\,\mathbf{e}_{\alpha+\beta}\}\operatorname{ctg}\frac{\alpha+\delta}{4}\,d\alpha.$$

Here we took into account the equality $\nabla\,(\mathbf{e}_{\alpha+\beta}, \mathbf{r}-\mathbf{r}_0(\alpha)) = (\mathbf{e}_{\alpha+\beta}, \nabla)(\mathbf{r}-\mathbf{r}_0(\alpha)) = \mathbf{e}_{\alpha+\beta}$. Adding to the obtained expression $k^2\,\mathbf{\Pi}(\beta,\delta)$ and making as a result the change of variables $\alpha-\varphi\to\alpha$, we obtain formula (3.12). Thus, Lemma 4 is proved.

Lemma 5. *Under the same assumptions as in Lemma 1* (§ 2), *the following holds*

$$(\operatorname{grad}\operatorname{div} + k^2)\mathbf{\Pi}(\beta,\delta) =$$

$$= \begin{cases} Pi\sqrt{\dfrac{k^3\pi}{2}}\,\{kR^2(\varphi+\delta)G_{5/2}(k,\ R(\varphi+\delta)\}\times \\ \quad\times\,\alpha(\beta,\ -\delta) + [-G_{3/2}(k,\ R(\varphi+\delta)) + \\ \quad + kG_{1/2}(k,\ R(\varphi+\delta))]\,\mathbf{e}_{\beta-\delta]} & \text{when } \cos\dfrac{\varphi+\delta}{2} > 0 \\ 0 & \text{when } \cos\dfrac{\varphi+\delta}{2} < 0 \end{cases} +$$

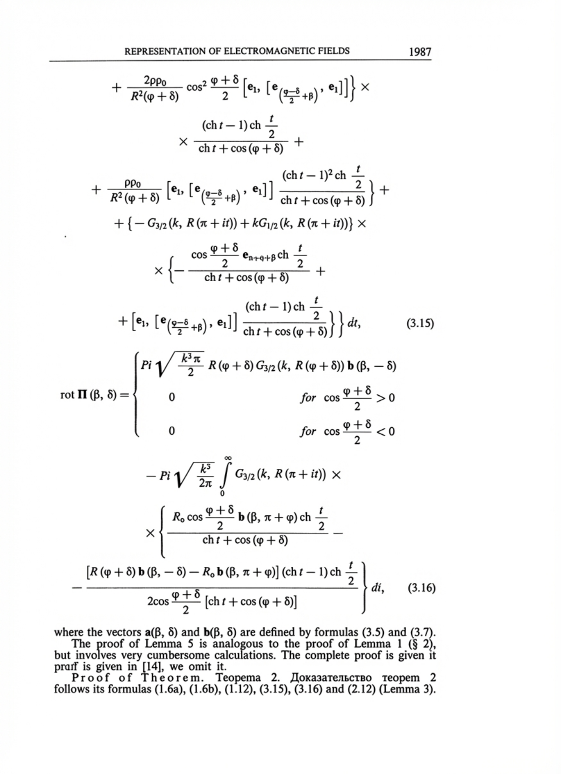
$$+\,Pi\sqrt{\frac{k^3}{2\pi}}\int\limits_0^\infty\Big\{kR^2(\varphi+\delta)G_{5/2}(k,\ R(\pi+it))\times$$

$$\times\Big\{-\frac{R_0^2}{R^2(\varphi+\delta)}\,a(\beta,\ \pi+\varphi)\times$$

$$\times\,\frac{\cos\dfrac{\varphi+\delta}{2}\operatorname{ch}\dfrac{t}{2}}{\operatorname{ch}t+\cos(\varphi+\delta)} +$$

$$+\left\{\frac{a(\beta,\ -\delta) - \dfrac{R_0^2}{R^2(\varphi+\delta)}\,a(\beta,\ \pi+\varphi)}{2\cos\dfrac{\varphi+\delta}{2}}\right\}\cdots.$$

Figure 14: Figure 14



$$+\frac{2\rho\rho_0}{R^2(\varphi+\delta)}\cos^2\frac{\varphi+\delta}{2}\left[\mathbf{e}_1,\left[\mathbf{e}_{\left(\frac{\varphi-\delta}{2}+\beta\right)},\mathbf{e}_1\right]\right]\Bigg\}\times$$

$$\times\frac{(\operatorname{ch}t-1)\operatorname{ch}\frac{t}{2}}{\operatorname{ch}t+\cos(\varphi+\delta)}+$$

$$+\frac{\rho\rho_0}{R^2(\varphi+\delta)}\left[\mathbf{e}_1,\left[\mathbf{e}_{\left(\frac{\varphi-\delta}{2}+\beta\right)},\mathbf{e}_1\right]\right]\frac{(\operatorname{ch}t-1)^2\operatorname{ch}\frac{t}{2}}{\operatorname{ch}t+\cos(\varphi+\delta)}\Bigg\}+$$

$$+\left\{-G_{3/2}(k,R(\pi+it))+kG_{1/2}(k,R(\pi+it))\right\}\times$$

$$\times\left\{-\frac{\cos\frac{\varphi+\delta}{2}\mathbf{e}_{\pi+\varphi+\beta}\operatorname{ch}\frac{t}{2}}{\operatorname{ch}t+\cos(\varphi+\delta)}+\right.$$

$$\left.+\left[\mathbf{e}_1,\left[\mathbf{e}_{\left(\frac{\varphi-\delta}{2}+\beta\right)},\mathbf{e}_1\right]\right]\frac{(\operatorname{ch}t-1)\operatorname{ch}\frac{t}{2}}{\operatorname{ch}t+\cos(\varphi+\delta)}\right\}\Bigg\}dt, \qquad (3.15)$$

$$\operatorname{rot}\mathbf{\Pi}(\beta,\delta)=\begin{cases}Pi\sqrt{\frac{k^3\pi}{2}}R(\varphi+\delta)G_{3/2}(k,R(\varphi+\delta))\mathbf{b}(\beta,-\delta)\\ 0 & for\ \cos\frac{\varphi+\delta}{2}>0\\ 0 & for\ \cos\frac{\varphi+\delta}{2}<0\end{cases}$$

$$-Pi\sqrt{\frac{k^3}{2\pi}}\int_0^\infty G_{3/2}(k,R(\pi+it))\times$$

$$\times\left\{\frac{R_o\cos\frac{\varphi+\delta}{2}\mathbf{b}(\beta,\pi+\varphi)\operatorname{ch}\frac{t}{2}}{\operatorname{ch}t+\cos(\varphi+\delta)}-\right.$$

$$\left.-\frac{[R(\varphi+\delta)\mathbf{b}(\beta,-\delta)-R_o\mathbf{b}(\beta,\pi+\varphi)](\operatorname{ch}t-1)\operatorname{ch}\frac{t}{2}}{2\cos\frac{\varphi+\delta}{2}[\operatorname{ch}t+\cos(\varphi+\delta)]}\right\}di, \qquad (3.16)$$

where the vectors **a**(β, δ) and **b**(β, δ) are defined by formulas (3.5) and (3.7).

The proof of Lemma 5 is analogous to the proof of Lemma 1 (§ 2), but involves very cumbersome calculations. The complete proof is given it prαf is given in [14], we omit it.

Proof of Theorem. Теорема 2. Доказательство теорем 2 follows its formulas (1.6a), (1.6b), (1.12), (3.15), (3.16) and (2.12) (Lemma 3).

Figure 15: Figure 15