ON DIVERGENCES OF CURRENTS

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Abstract

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ON DIVERGENCES OF CURRENTS

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Recently the possibility of testing the equal-time commutation relations ([1]) of vector and axial currents $\mathfrak{B}_\mu^i$ and $\mathfrak{A}_\mu^i$, where i is the index of the unitary octet, $i = 1, \ldots, 8$, has been widely discussed. For simplicity let us assume that the strong interactions do not violate unitary symmetry. If electromagnetic and weak interactions are neglected, then the vector currents are conserved,

$$\partial_\mu \mathfrak{B}_\mu^i = 0. \tag{1}$$

As for the axial currents, for convenience we shall assume, following Gell-Mann and Lévy ([2]), that their divergences are proportional to pseudoscalar fields,

$$\partial_\mu \mathfrak{A}_\mu^i = ia\varphi^i, \tag{2}$$

although this is not necessary for deriving the consequences of current algebra ([3]). When corrections due to weak and electromagnetic interactions are taken into account, relations (1) and (2) must be modified. If one assumes minimal electromagnetic interactions (replace ∂_μ by $\partial_\mu \pm ieA_\mu$, or by ∂_μ for particles with charges $+1, -1, 0$), then, for example, for the charged currents

$$\mathfrak{B}^\pm = \frac{1}{\sqrt{2}} (\mathfrak{B}^1 \pm i\mathfrak{B}^2)$$

instead of relation (1) we obtain ([4])

$$\partial_\mu \mathfrak{B}_\mu^\pm = ieA_\mu \mathfrak{B}_\mu^\pm. \tag{3}$$

In a recent paper by Weltman ([5]), relations were proposed for the divergences of vector and axial currents which are a generalization of relations (3) to the case in which electromagnetic and semileptonic weak interactions are taken into account to first order. In that paper it was also shown that, from the hypothesis proposed there concerning the divergences of currents, many sum rules follow which had previously been obtained on the basis of equal-time commutation relations.

In the present paper we shall show that the equal-time commutation relations of Gell-Mann and Weltman' s hypothesis concerning current divergences are consequences of one and the same dynamical model—the quark model. We shall also give a generalization of Weltman' s results to the case in which nonleptonic weak interactions are taken into account. From our generalized hypothesis concerning current divergences there also follow directly all the relations between amplitudes of nonleptonic decays of baryons and mesons that were obtained earlier by means of current algebra.

In deriving the equal-time commutation relations, Gell-Mann assumed that the currents $\mathfrak{B}^i_\mu$ and $\mathfrak{A}^i_\mu$ have the form

$$\mathfrak{B}^i_\mu = \bar{\psi}\gamma_\mu\lambda^i\psi, \qquad \mathfrak{A}^i_\mu = \bar{\psi}\gamma_\mu\gamma_5\lambda^i\psi, \tag{4}$$

where ψ are field operators for quarks. For quarks we shall assume mini-

minimal electromagnetic and universal $V - A$ weak interactions of Feynman—Gell-Mann ([6]), Sudershan and Marshak ([7]), Cabibbo ([8]). If neutral hadron currents exist, then we shall assume that these currents are the neutral components ([9]) of the currents (4). Then the Lagrangian of the electromagnetic and weak interactions of hadrons can be written in the form (summation over repeated indices is implied)

$$\mathcal{L}_e = ieA^i_\mu\mathfrak{B}^i_\mu, \tag{5}$$

$$\mathcal{L}_W = \frac{iG}{\sqrt{2}}(\mathfrak{B}^i_\mu + \mathfrak{A}^i_\mu)W^i_\mu + \frac{G}{\sqrt{2}}\,a_{ij}(\mathfrak{B}^i_\mu + \mathfrak{A}^i_\mu)(\mathfrak{B}^j_\mu + \mathfrak{A}^j_\mu). \tag{6}$$

Here, for convenience, we represent the electromagnetic field in the form of an octet A^i_μ. The quantities W^i_μ denote the products of lepton currents by the corresponding constants (for example, products of the sine and cosine of the Cabibbo angle), a_{ij} are constants. We shall further assume that the strong interactions of quarks are such that the vector current of the form (4) is conserved:

$$\partial_\mu(\overline{\psi}\gamma_\mu\lambda^i\psi) = 0.$$

We shall denote the divergence of the axial currents in the absence of electromagnetic and weak interactions by $(\operatorname{div}\mathfrak{A}^i)_0$. The PCAC hypothesis means that

$$(\operatorname{div}\mathfrak{A}^i)_0 = ia\varphi^i. \tag{7}$$

Using the Lagrange equation with account taken of the electromagnetic and weak interactions from the Lagrangians (5) and (6), and taking account of relation (7), we obtain expressions for the divergences of the vector and axial currents

$$\partial_\mu \mathfrak{B}^i_\mu = ief_{ijk}A^j_\mu \mathfrak{B}^k_\mu + \frac{iG}{\sqrt{2}} f_{ijk} W^j_\mu (\mathfrak{B}^k_\mu + \mathfrak{A}^k_\mu) +$$

$$+ \frac{G}{\sqrt{2}} a_{jk} \left[f_{ijl}(\mathfrak{B}^l_\mu + \mathfrak{A}^l_\mu)(\mathfrak{B}^k_\mu + \mathfrak{A}^k_\mu) + f_{ikl}(\mathfrak{B}^l_\mu + \mathfrak{A}^l_\mu)(\mathfrak{B}^j_\mu + \mathfrak{A}^j_\mu) \right]; \qquad (8)$$

$$\partial_\mu \mathfrak{A}^i_\mu = ia\varphi^i + ief_{ijk}A^j_\mu \mathfrak{A}^k_\mu +$$

$$+ \frac{G}{\sqrt{2}} a_{jk} \left[f_{ijl}(\mathfrak{B}^l_\mu + \mathfrak{A}^l_\mu)(\mathfrak{B}^k_\mu + \mathfrak{A}^k_\mu) + f_{ikl}(\mathfrak{B}^l_\mu + \mathfrak{A}^l_\mu)(\mathfrak{B}^j_\mu + \mathfrak{A}^j_\mu) \right]. \qquad (9)$$

These formulas are a generalization of the Veltman relations. They contain terms corresponding to nonleptonic weak interactions.

Thus, Veltman' s hypothesis on current divergences is a consequence of the quark model with minimal electromagnetic interactions (5) and with universal weak interactions (6). The equal-time commutation relations of Gell-Mann were obtained within the framework of the same model. Therefore it is not surprising that many consequences of Gell-Mann' s equal-time commutation relations can also be obtained from Veltman' s hypothesis on current divergences. Veltman' s method and the current algebra constitute two different approaches to the study of one and the same model.

Above we assumed that the strong interactions do not violate unitary symmetry. When violation of unitary symmetry is taken into account, relations (8) and (9) change inessentially: for example, on the right-hand sides of these relations we must add terms $(\operatorname{div} \mathfrak{B}^i)_0$, equal to the divergence of the vector current in the absence of electromagnetic and weak interactions. As in all other works, we shall assume that the vector and axial currents have the form (4) also when violation of unitary symmetry is taken into account.

Let us note that from expression (9) for the divergence of the axial current one can obtain all known results of current algebra concerning non-

leptonic decays of baryons and mesons. Indeed, the amplitude of the process of nonleptonic decay

$$B_1 \to B_2 + \varphi^i$$

(φ^i is the octet of pseudoscalar mesons) off the mass shell of the meson φ^i has the form

$$M(B_1 \to B_2\varphi^i) = (m_\pi^2 + k^2)\langle B_2|\varphi^i(0)|B_1\rangle, \tag{10}$$

where k is the momentum of the virtual meson, and φ^i is the Heisenberg meson field with weak interactions taken into account to first order. Since

$$\lim_{k\to 0}\langle B_2|\partial_\mu A_\mu^i|B_1\rangle = 0,$$

from formula (9) we obtain

$$\lim_{k\to 0}\langle B_2|\varphi^i|B_1\rangle = \frac{1}{c}\frac{G}{\sqrt{2}}\,\alpha_{jk}\lim_{k\to 0}\langle B_2|[f_{ijl}(\mathfrak{B}_\mu^l + \mathfrak{A}_\mu^l)(\mathfrak{B}_\mu^k + \mathfrak{A}_\mu^k) + f_{ikl}(\mathfrak{B}_\mu^l + \mathfrak{A}_\mu^l)(\mathfrak{B}_\mu^j + \mathfrak{A}_\mu^j)]|B_1\rangle. \tag{11}$$

Such relations were also obtained on the basis of current algebra [10].

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Note: Figure translations are in progress. See original paper for figures.