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Abstract

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GEOPHYSICS

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SEISMIC TURBIDITY OF REAL MEDIA AND THE POSSIBILITY OF STUDYING IT

(Presented by Academician M. A. Sadovskii, March 13, 1967)

The seismic parameters of real media—density, velocities, and absorption coefficients of longitudinal and transverse waves—vary from point to point. Usually a medium is considered homogeneous if the average values of these parameters, determined for some volume, lie within an admissible interval. Fluctuations of the average values of the parameters that are small in magnitude gradually change the shape of the wave; these changes accumulate as the wave propagates and over a large portion of the path may become quite substantial. Taking into account the distortion of the wave shape caused by the scattering action of a turbid medium will make it possible to quantitatively estimate the nature of the inhomogeneity and thus to obtain a new seismic characteristic of the medium.

Let $v(x,y,z)$ be the velocity of the longitudinal wave, $v_0 = \bar{v}(x,y,z)$, $\delta v = v - v_0$, with $\delta v \ll v_0$. In real media the velocities of longitudinal and transverse waves and the density are strongly correlated, while the relative change in density is much smaller than that in velocity. Therefore the function δv gives a sufficiently complete description of the inhomogeneity. We shall regard the random field $\delta v(x,y,z)$ as homogeneous and isotropic. This means that the autocorrelation function depends only on the distance between points ([1]). Suppose that for $x < x_1$, $\delta v = 0$, and for $x > x_1$, $\delta v \neq 0$, and the mean value of the effective attenuation coefficient is equal to k. In the x-direction there propagates a plane monochromatic wave of frequency ν, having in the plane $x = x_1$ the amplitude F_{x_1}. In the plane $x_1 + \Delta x$ the wave amplitude is

$$F_{x_1+\Delta x} = F_{x+\Delta x} + \delta F_{x+\Delta x},$$

where F is the deterministic component of the amplitude field, which is the mean value of F in the plane $x_1 + \Delta x$, and δF is the random component; its mean over the front plane is zero. Denote the square of the amplitude by U and call this function the *wave energy*. The change in the energy of the deterministic component due to the absorbing action of the medium is $\Delta U = U_{x_1} - U_{x_1+\Delta x} = kU_{x_1}\Delta x$; $k = k_\mathrm{p} + k_\mathrm{n}$ is the effective attenuation coefficient, and k_p and k_n are coefficients characterizing the decrease in energy due to scattering

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and absorption. Accordingly, ΔU consists of two terms, $\Delta U = \Delta U_\mathrm{n} + \Delta U_\mathrm{p} = k_\mathrm{n} U_{x_1} \Delta x + k_\mathrm{p} U_{x_1} \Delta x$, where ΔU_p is a quantity proportional to the energy of the wave scattered by the layer $(x_1, x_1 + \Delta x)$. Hence, passing to the limit as $\Delta x \to 0$ and integrating from x_1 to x $(x > x_1)$, we obtain the well-known relation $U_x = U_{x_1} \exp[-(k_\mathrm{n}+k_\mathrm{p})(x-x_1)]$. In the plane $x_1+\Delta x$, $\Delta U_\mathrm{p} < \delta U$, since part of the scattered energy goes backward; therefore one may write $\overline{\delta U} = p\Delta U$, $1 > p > 1/2$. The coefficient p characterizes the directionality of scattering: $p = 1/2$ if scattering forward and backward is the same; $p = 1$ if backward scattering is absent. In the plane $x + \Delta x$ the energy of the random component generated by the layer $(x, x + \Delta x)$ is

$$\overline{\delta U}_{x+\Delta x}(x, x + \Delta x) = g\Delta x \cdot U_{x_1} \exp\left[(-k_\mathrm{n} + k_\mathrm{p})(x - x_1)\right],$$

here $g = k_p p$. During propagation to the plane x_2, part of the energy of the random component is absorbed by the medium, and part is scattered backward; therefore

$$\overline{\delta U}_{x_2}(x, x+\Delta x) = g\Delta x \cdot U_{x_1} \exp - [(k\ + k\ )(x - x_1) + (k\ + k\ - g)(x_2 - x)] = g\Delta x \cdot U_{x_2} \exp g(x_2 - x).$$

It is evident from this that the energy of the random component decreases with distance more slowly than that of the deterministic component: the scattered part of the energy of the deterministic component is partially transformed into the random component of the field, whereas the forward-scattered energy of the random component remains random and is therefore preserved. The energy of the random part of the field in the plane x_2 is equal to the sum of the energies of the components produced by the individual thin layers; therefore

$$\overline{\delta U}_{x_2}(x_1, x_2) = \int_{x_1}^{x_2} g U_{x_2} \exp g(x_2 - x_1)\, dx = -U_{x_2}[1 - \exp g(x_2 - x_1) + C].$$

From the condition $\overline{\delta U}_{x_1} = 0$ as $x_2 \to x_1$, it follows that $C = 0$. In the case when $g(x_2 - x_1) \ll 1$, this equality is simplified:

$$\overline{\delta U}_{x_2}(x_1, x_2) = g(x_2 - x_1) U_{x_2}, \tag{*}$$

in a homogeneous medium, the energy of the random part of the wave field is proportional to the thickness of the layer (x_1, x_2) and to the energy of the deterministic part of the field that generated it at the boundary $x = x_2$. The coefficient g characterizes the seismic turbidity of the medium; we shall call it the **turbidity coefficient**. In limiting cases it may be equal to the energetic scattering coefficient k_p $(p = 1)$, or to the amplitude coefficient, $k_p/2$ $(p = 1/2)$.

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Equality (*) is generalized to the case when the medium consists of thick layers and the parameters k and g depend on the coordinates:

$$\overline{\delta U}_{M_2}(M_1, M_2) = U_{M_2} \int_{M_1}^{M_2} g(x, y, z)\, dl = \overline{\delta U}_{M_2}/U_{M_2} - \overline{\delta U}_{M_1}/U_{M_1},$$

M_1 and M_2 are points lying on the ray, and dl is an element of ray length. This equation is completely analogous in structure to the equation for the travel time of a wave from M_1 to M_2; therefore the method of determining the turbidity coefficient g from wave amplitudes is the same as that for determining the velocity v from phase hodographs. In this case there is the following correspondence: the turbidity coefficient g is the quantity $1/v$; the ratio of the energies of the random and deterministic components of the field at a certain point is the arrival time of the wave at that point; the graph of $\overline{\delta U}_M/U_M$ along some profile is the hodograph of the wave along this profile. This correspondence helps in choosing methodological procedures for determining g from waves of different types—transmitted, reflected, head, and refracted.

The separation of the observed field into the deterministic component F_M and the random component δF_M can be carried out under two basic assumptions. The first is that some hypothesis is adopted regarding the general form of F_M corresponding to the chosen model of the medium; then the concrete form of F_M is determined from the condition that, in some sense, it must differ least from F_M; this function is regarded as the deterministic part of the field, and the difference of the two functions as the random part δF_M. The second assumption is more general: F_M varies more smoothly than δF_M, and their separation is reduced to spatial filtering of F_M. The first assumption makes it possible to describe the discrepancy between the adopted model and nature in terms of turbidity; the second, to estimate the turbidity of the medium associated with the existence of inhomogeneities whose dimensions do not exceed some specified level (determined by the parameters of the spatial filter). To determine U_M and δU_M, it is much more convenient to work not with the waveform

as a whole, but with some of its parameters, in particular, the amplitudes F. In this case the ratio of the energies of the random and deterministic components is determined by the equality $\delta U_M/U_M = D \ln F$, where D is the variance sign, and the bar denotes averaging over a certain region containing the point M.

Real seismic signals have the form of a pulse with several (3-6) extrema; therefore, to determine g one may use the amplitudes of individual phases, and the result obtained must be referred to the predominant frequency of the oscillations.

The turbidity coefficient g has been determined for the Earth' s crust as a whole and for the upper layer of the mantle from amplitude plots of reflected longitudinal waves recorded during deep seismic sounding in Turkmenia ([2]). The plots of the logarithms of the amplitudes of waves reflected from the Mohorovičić

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boundary (P_M^{ref}), from the boundary at a depth of 75-80 km (P_2^{ref}), and at a depth of 150-120 km (P_4^{ref}), have been averaged (averaging is spatial filtering) by smooth lines. The quantity $D \ln F$ depends on the nonidentity of seismogeological conditions and of the installation of seismographs, which give a certain constant value $D \ln F_0$ entering as a term in $D \ln F$ for each wave. Therefore, when determining g for a layer from the amplitude plots of waves reflected from its top and bottom, the result proves to be independent of the conditions of instrument installation and of the properties of the medium above the top of the layer. The mean values of $D \ln F$ have been referred to the mean paths of the waves in the layers; the wave velocity in the crust was taken as 6.0 km/sec, and in the two upper mantle layers as 8.0 and 8.5 km/sec, respectively ([3]). Nonidentity in instrument installation and surface conditions gives an average scatter of amplitudes of about 30%, which corresponds to $D \ln F_0 \approx 0.050$. In our case the turbidity coefficient for the crust and for the 120-kilometer upper part of the section as a whole depends on this quantity. The data on reflected waves needed to determine g are presented in Table 1.

Table 1

Wave	Predominant frequency, Hz	Mean path in the crust, km	Mean path in the layer 40–80 km	Mean path in the layer 80–120 km	$D \ln F - D \ln F_0$
P_M^{ref}	10-12	210	–	–	0.060-0.077
P_2^{ref}	10-12	145	205	–	0.132-0.160
P_4^{ref}	6-8	145	205	225	0.086-0.108

The last column gives the limits of the 80% confidence interval. The turbidity coefficient for the Earth' s crust as a whole is 0.0003 km^{-1}, and for the upper mantle layer (40–80 km) 0.0005 km^{-1}; these values refer to the frequency 10-12 Hz; the mean value for the upper 120-kilometer layer at a frequency of 6–8 Hz is 0.0002 km^{-1}. It is significant that the Earth' s crust as a whole is more transparent than the upper 40-kilometer layer of the mantle. The obtained values of g are much smaller than the effective absorption coefficients, which in the Earth' s crust and upper mantle are of the order of 0.01–0.04 for frequencies of 10-12 Hz ([4]).

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