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Abstract

Full Text

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PHYSICS

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CHANGE OF THE ANGULAR AND ENERGY DISTRIBUTION IN A FLUX OF CHARGED PARTICLES UPON PASSAGE THROUGH A MAGNETIC FIELD

(Presented by Academician V. A. Fock on 14 V 1966)

In ([1]) the influence of radiation damping on the motion of a charged particle in a homogeneous magnetic field was considered. In the present paper we shall determine how radiation damping affects the distribution over angles and energies in a flux of charged particles that have passed through a layer of magnetic field of finite thickness. The results obtained may be of significance in astrophysics when considering the motion of fluxes of fast electrons through regions of stellar atmospheres and nebulae with strong magnetic fields.

We shall assume that the magnetic field is concentrated in a plane-parallel layer of thickness L, has within the layer a constant intensity H, and is directed along the normal to the surface of the layer. We take the direction of $\mathbf{H}$ as the OZ axis of the coordinate system. Suppose that a flux of particles with a definite energy and with density $j_0(\theta_0)$, $0 \leq \theta_0 \leq \pi/2$, falls on the surface of the layer, where θ_0 is the angle between the direction of the particle velocity and the OZ axis upon entry into the layer. It is easy to imagine the character of the distribution over angles and energies of the particles that have emerged from the layer.

Let θ denote the angle between the direction of the velocity and the OZ axis upon exit from the layer of magnetic field. Obviously, for $\theta_0 = 0$ one will also have $\theta = 0$. As θ_0 increases, θ also increases. On the other hand, for $\theta_0 \approx \pi/2$ the particle will move for a long time in the layer, will lose almost all its transverse velocity, so that $\theta \ll 1$. Thus one may expect—and this is confirmed by direct calculation—that all exit angles θ are contained in the cone $0 \leq \theta \leq \theta_m$, whose aperture θ_m depends on the thickness of the layer and the intensity of the magnetic field. To each θ inside the cone there correspond two values of θ_0, and since the radiative energy losses depend on θ_0, in the flux of

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particles emerging from the layer along each direction θ there move particles with two values of the energy.

Turning to the calculation, let us note that, according to ([1]), in the ultrarelativistic limiting case the transverse component $v_\perp$ decreases with time according to the law

$$v_\perp = v_\perp(0)/\operatorname{ch}\left(\frac{\delta t}{c} v_0 \sin\theta_0\right), \tag{1}$$

where

$$\delta = {}^2/_3\, e^4 H^2/m^3c^5. \tag{2}$$

The longitudinal velocity $v_z = v_0 \cos\theta_0$ remains unchanged. Therefore the time t of motion through the layer can be expressed in terms of v_z and the layer thickness L:

$$t = L/v_z = L/v_0 \cos\theta_0. \tag{3}$$

Introduce $\operatorname{tg}\theta = v_\perp/v_z$ and denote

$$k = \frac{\delta L}{c} = {}^2/_3\, e^2/m^3c^5\, H^2 L; \tag{4}$$

then

$$\operatorname{tg}\theta = \operatorname{tg}\theta_0 \frac{1}{\operatorname{ch}(k \operatorname{tg}\theta_0)}. \tag{5}$$

It follows from this formula that two values of θ_0 correspond to one value of θ, if θ does not exceed the value θ_m ($\theta_0' < \overline{\theta}_0$ and $\theta_0'' > \overline{\theta}_0$). The maximum value θ_m is attained at $\theta_0 = \overline{\theta}_0$, where $k \operatorname{tg}\overline{\theta}_0 \approx 1.2$, and is determined by the relation $\operatorname{tg}\theta_m = 2/3k$.

The energy of the particle is determined by the formula

$$\frac{1}{E} - \frac{1}{E_0} = \frac{\sin\theta_0}{mc^2} \operatorname{th}(k \operatorname{tg}\theta_0), \tag{6}$$

which is obtained from formula (12) of paper (1) by substituting expression (3) for the time t in terms of the angle θ_0.*

Formulas (6) and (5) express, in parametric form (through the angle θ_0), the dependence of $1/E - 1/E_0$ on θ. Plots of this dependence for several values of the quantity k are given in Fig. 1. The flux of particles leaving the layer

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Fig. 1

Figure 1: Fig. 1

through an element of solid angle, $dN = 2\pi j(\theta) \sin\theta \, d\theta$, is equal to the sum of the fluxes of particles entering the layer at angles θ_0' and θ_0''. We have

$$j(\theta) = j_0(\theta_0') \, d\cos\theta / d\cos\theta_0' + j_0(\theta_0'') \, d\cos\theta / d\cos\theta_0''. \quad (7)$$

Using (5), one can obtain

$$d\cos\theta / d\cos\theta_0 = \cos^3\theta_0 / \cos^3\theta \times$$

$$\times \operatorname{ch}^2(k \operatorname{tg}\theta_0) /$$

$$/[1 - k \operatorname{tg}\theta_0 \operatorname{th}(k \operatorname{tg}\theta_0)]. \quad (8)$$

Fig. 1

In conclusion we give an estimate of the magnitude of the coefficient k for some cosmic objects. For the Sun, taking $L \sim 10^{12}$ cm, $H \sim 1$ G, we have $k \sim 5 \cdot 10^{-8}$. For a magnetic sunspot, $L \sim 10^{10}$ cm, $H \sim 10^3$ G, $k \sim 5 \cdot 10^{-4}$. For some giant stars of late spectral classes [3], $L \sim 10^{13}$ cm, $H \sim 10^3$ G, $k \sim 0.5$.

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* From (6) follows the result obtained as early as 1939 by I. Ya. Pomeranchuk [2], that as $E_0 \to \infty$ the final energy tends to a constant limit independent of E_0. However, in contrast to (2), we do not replace the true trajectory of the particle by a straight line.

Note: Figure translations are in progress. See original paper for figures.

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