NUMERICAL SOLUTION OF THE PROBLEM OF THE PROPAGATION OF SEMIDIURNAL TIDAL WAVES (M_2 and S_2) IN THE WORLD OCEAN

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Abstract

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NUMERICAL SOLUTION OF THE PROBLEM OF THE PROPAGATION OF SEMIDIURNAL TIDAL WAVES (M_2 and S_2) IN THE WORLD OCEAN

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The work is an attempt at a numerical solution of the global problem of the propagation of tidal waves. A similar problem for the component of the semidiurnal wave M_2 was attempted by Ueno ([8]). However, the results obtained by him, by the author' s own admission, are so inconsistent with field data that they do not make it possible to construct a satisfactory tidal chart. The solution of such a problem, in addition to the integrity of the resulting picture of the propagation of tidal waves in the World Ocean, makes it possible almost completely to avoid prescribing initial data along the open (water) boundaries of the basin and thereby to increase the accuracy of the solution. In solving a similar problem for individual seas and oceans, it is inevitable that one must prescribe initial values on open boundaries, which, as a rule, are not accurate.

The propagation of periodic tidal waves in the World Ocean is described by the equation

$$-\frac{1}{g}(a\tau\cos\varphi)^2\zeta + \operatorname{div}(hA^{-1})\ \operatorname{grad}(\eta-\zeta) = 0. \tag{1}$$

Here the following notation is adopted:

$$\operatorname{div}\{z_1; z_2\} = \frac{\partial z_1}{\partial\lambda} + \cos\varphi\,\frac{\partial z_2}{\partial\varphi};$$

$$\operatorname{grad} z = \left\{\frac{\partial z}{\partial\lambda}; \cos\frac{\partial z}{\partial\varphi}\right\}; \qquad A = \begin{Vmatrix} 1+i\frac{\varrho}{\tau} & -i\frac{2\omega}{\tau}\sin\varphi \\ i\frac{2\omega}{\tau}\sin\varphi & 1+i\frac{\varrho}{\tau} \end{Vmatrix};$$

φ, λ are the northern latitude and western longitude of the point, ζ is the unknown harmonic component of the dynamic tide, η is the harmonic component of the static tide, $h = h(\varphi, \lambda)$ is the depth at the point, a is the Earth' s radius, τ

Fig. 1. Tidal chart of the semidiurnal wave M_2

Figure 1: Fig. 1. Tidal chart of the semidiurnal wave M_2

is the angular velocity of propagation of the tidal wave, ω is the angular velocity of the Earth' s rotation, and ρ is the friction coefficient.

Equation (1) is a consequence of the well-known Laplace equations. For non-singular A it is equivalent to a strongly elliptic system of differential equations in the sense of Vishik. Therefore, for the boundary-value problem defined by equation (1) and the boundary condition

$$a(\varphi,\lambda)(A\,\mathrm{grad}\,\zeta, n) + b(\varphi,\lambda)\zeta = f(\varphi,\lambda)\big|_\Gamma \tag{2}$$

($a+b=1$; $ab=0$; Γ is the boundary of the basin, n is the vector normal to the boundary of the basin), all three Fredholm alternatives are valid. In particular, for $\rho \neq 0$ the boundary-value problem (1)–(2) is well posed ([2]).

In the present case, the first boundary-value problem was solved, i.e., ζ was prescribed on the boundary. It was assumed that $\rho = 0$. In this case equation (1) is self-adjoint, and $\det A \neq 0$ for the waves M_2 and S_2.

A grid with a step of 5° in longitude and latitude was introduced, and the first boundary-value problem was reduced to a self-adjoint system of algebraic equations of the form:

$$\zeta + B\zeta = F, \tag{3}$$

where B is a Young matrix with zero diagonal.

System (3) was solved by Seidel' s method with optimal relaxation. A convergence criterion is known [4]: the largest eigenvalue λ of the matrix B must be less than unity. The value of λ depends substantially on the bottom topography and the outlines of the basin. For example, for the North Atlantic $\lambda = 0.97$, while for the World Ocean $\lambda = 1.04$. Therefore, after ten–

Fig. 1. Tidal chart of the semidiurnal wave M_2 (solid lines are isophases in degrees. Dashed lines are isoamplitudes in centimeters. Black circles are boundary points of the contour of the World Ocean)

two initial iterations necessary for determining λ, system (3) was replaced by the nearby system:

$$(1 - i\delta)\zeta + B\zeta = F, \qquad \delta \ll 1, \tag{4}$$

for which Seidel' s method with optimal relaxation converges. The behavior of the spectrum of the matrix B near $\lambda = 1$ was not known during the computation,

Fig. 2. Tidal map of the semidiurnal wave S_2. The notation is the same

Figure 2: Fig. 2. Tidal map of the semidiurnal wave S_2. The notation is the same

and the closeness of the solutions of systems (3) and (4) could be judged by comparing the solutions of system (4) for various δ both with one another and with the vector obtained from system (3) after a certain number of iterations. In the latter case we relied on the asymptotic property of the iterative process for system (3) with a relaxation coefficient equal to unity.

Figures 1 and 2 present tidal charts of the semidiurnal waves M_2 and S_2, constructed from the computed data. Comparison of the obtained computed values of the harmonic tidal constants for the waves M_2 and S_2 with the actual values of the harmonic constants at numerous islands in the open parts of the oceans indicates quite satisfactory agreement for the areas of the Atlantic and Indian Oceans, as well as for the overwhelming part of the Pacific Ocean. In the tropical region of the Pacific Ocean between the Marquesas Islands and New Zealand, where the amplitudes of tidal oscillations of sea level due to the tidal wave M_2 are small, this agreement is violated, and in this region of the Pacific Ocean the position of the isophase lines was adjusted in accordance with the actual values of the harmonic tidal constants. This procedure does not reduce the overall value of the results obtained and is

justified, since differences in the positions of cotidal lines on maps at small tidal amplitudes, despite the apparent contradiction, are not of fundamental significance [6].

A comparison of the computed tidal maps of the constituent waves M_2 and S_2 for the World Ocean (with the exception of the polar basin) with the corresponding maps constructed by computational and empirical methods, both for separate parts of the World Ocean [6,7] and for the entire World Ocean [5,9],

Fig. 2. Tidal map of the semidiurnal wave S_2. The notation is the same.

shows that the results obtained have a number of fundamental differences from the known schemes of the propagation of semidiurnal waves over the World Ocean. A number of features of the propagation of semidiurnal waves that were previously unknown have been revealed; in particular, several new amphidromic systems have been found that were not marked on the tidal maps of other authors. At the same time, many features of the propagation of tidal waves in different parts of the oceans, noted in recent years, are confirmed [1–3].

The values of semidiurnal tidal currents obtained in the course of the calculation are not presented in this article.

At present, the calculation of the propagation of diurnal tidal waves over the World Ocean is being completed on an electronic computer.

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Note: Figure translations are in progress. See original paper for figures.