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Estimates of the domain of solvability for the system of equations of a moment-free reticulated cylindrical shell in the case of the first boundary value problem

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Abstract

The paper investigates a system of three equations of strongly elliptic type, supplemented by a boundary condition for one variable and a periodicity condition for the second. This system corresponds to the problem of determining the displacements of a momentless lattice cylindrical shell from an initial state prestressed by internal pressure. The paper provides sufficient conditions on the dimensions of the domain under which the boundary value problem under consideration is well-posed. These conditions are refined for the case where the solution to the original problem is assumed to be symmetric a priori. 2 illustrations. 7 references.

Full Text

Preamble

This work, published in 1967 (Vol. III, No. 8), addresses problems in mathematical physics related to boundary value problems for partial differential equations, specifically focusing on the methods developed by E. G. D' yakonov and I. K. Larin [**?**, **?**]. The study considers systems of equations arising in the theory of elasticity and shell theory (including plates and shallow shells), which are characterized by specific boundary conditions. The approach builds upon the foundational work in [**?**], which utilized iterative methods for solving elliptic systems, including those related to the Lamé equations and various problems in the bending of plates.

The present investigation extends the results of E. G. D' yakonov and I. K. Larin regarding the convergence of iterative processes for these systems. In particular, we examine the spectral properties and the choice of optimal parameters for

the iterative schemes, building on the estimates provided in [**?**, **?**]. The analysis involves the use of functional spaces and energy estimates to establish the stability and convergence of the numerical solutions.

Problem Formulation and Operator Estimates

We consider the operator equation of the form:

$$Lu(x) + Ru(x) + Qu(x) = f(x)$$

where $u(x) = (u, v, w)^T$ is the vector of unknown functions representing displacements in the x_1, x_2 directions and the normal deflection. The operators L, R, and Q are defined based on the geometric and physical properties of the shell, involving differential operators D_1 and D_2 with respect to the spatial coordinates. The parameter p characterizes the geometry, with $0 < p < \pi/2$.

The domain of interest is defined as $\Omega = \{-\infty < x_1 < +\infty, -l_2 < x_2 < l_2\}$, assuming periodicity in the x_1 direction with period $2l_1$. The boundary conditions at $x_2 = \pm l_2$ are taken to be homogeneous, $u|_{x_2=\pm l_2} = 0$. We work within the Sobolev space W_2^1, equipped with the norm:

$$||u||_{W_2^1} = \left(\sum_{|\alpha| \leq 1} ||D^\alpha u||^2 \right)^{1/2}$$

where $(\cdot, \cdot)$ denotes the standard L_2 inner product.

Energy Estimates and Convergence

To establish the convergence of the iterative method, we derive lower bounds for the operator L. Specifically, we seek a constant $\gamma > 0$ such that for any $u \in W_2^1$:

$$[Lu, u] \geq \gamma ||u||^2_{W_2^1}$$

The derivation involves several lemmas. Lemma 1 and Lemma 2 provide estimates for the bilinear forms $[Ru, u]$ and $[Qu, u]$ in terms of the integrals J_1 and J_2, which involve the derivatives of the displacement components. By combining these results, Lemma 3 establishes the positive definiteness of the total operator under certain conditions on the parameters p, q, and r.

Specifically, we analyze the expression:

$$[Lu, u] = (1 - s^2)(J_1 + J_2) + J$$

where $s = \sin p$. By introducing auxiliary parameters and applying Cauchy-Schwarz inequalities, we can bound the terms involving cross-derivatives. The goal is to find the supremum of the lower bound $\gamma(t, p)$ over the admissible range of parameters.

Numerical Results and Parameter Optimization

The effectiveness of the iterative scheme depends heavily on the spectral radius of the transition operator. We investigate the behavior of the convergence factor γ as a function of the geometric parameter $t = \tan p$. Our analysis shows that for $0 < p < 1$, the operator L remains strongly elliptic, and the iterative process converges at a rate determined by the ratio of the minimum and maximum eigenvalues of the associated operator.

As shown in the figures and tables provided in the full text, the optimal parameters q and r can be determined to maximize the convergence rate. For the case $p = 0$, our results coincide with the classical estimates for plates provided in [**?**]. The extension to $p > 0$ allows for the treatment of curved shells, where the coupling between the membrane and bending states becomes significant.

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Figures

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DIFFERENTIAL EQUATIONS

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ESTIMATES OF THE SOLVABILITY REGION FOR THE SYSTEM OF EQUATIONS OF A MOMENTLESS NET CYLINDRICAL SHELL IN THE CASE OF THE FIRST BOUNDARY VALUE PROBLEM

E. G. D'YAKONOV, I. K. NIKOLAEV

Shells formed by a system of mutually intersecting layers consisting of absolutely flexible elastic threads have numerous applications in mechanical engineering.

A net shell, pre-stressed by air pressure, located in its internal cavity, serves as the force base of the frame of a pneumatic tire. In works [1, 2], a method for solving problems of calculating net shells is developed, consiting in splitting the total stressed state in a thin momentless shell into a stressed state caused by a symmetrical load (internal air pressure) — the monontsaıcı for internal air pressure) — the initial state, and a stressed state caused by additional loards, and for a stressed state caused by addoctional loads, while in a linear approximation differential equations of equilibirrium for small deformations from the initial state under the action of additional loads are obtained.

Equations for determining displacements and forces in threads for small deformations of the shell from the initial state, containing in the geeral case of a shell of revolution variable coeffficients depeninding on the shame of the shell in the initial state, are significently simplificrd when moving to a cylindrical shell. Such a transition can be completed if one puts one of the curvatures in the shell of revolution equal to nero, in this case the initial shapee of the shell section in a cirycl, and the coeffficients of the system of equillirrium equations become constant.

The results obtained in the study of a cylindrical shell can be used as a first approximation to a shell of revolution in the case when the curvature of the shell in one of the directions is susfictievtly small.

At the same time, since the basic proertries of the problem, such as the type of equations, brondary conditions, the right side, are pre-served during the transition to a cylindrical shell, the study of this particular case has a certain methodological value.

Initially, the equilibcium equations of a net shell were obtained in work [1] using the assumption of the invariability of the length of threads in the process of deformation (conditions of inextensibility of threads). This essential assnumption leads to the fact that the system of equations of deformations turns out to be of non-elliptic type, moreover, the directions along whiich the threads are located are the characteristics of the system. When solving a boundary value problem for a system of differential equationss of non-elliptic type, serious difficulties arise in substantiating the legitimacy of applying known methods of solution (methods of expansion in trigonometric series, various variational methods), since a priori the existence of a solution is not guaranteed.

Figure 1: Figure 1

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Subsequently, in work [2], a generalization to the case of an arbitrary characteristic of the thread (dependence of the force in it on the deformation) was carried out and equilibrium equations for a net shell with extensible threads were obtained.

Using the example of cylindrical shell equations, we have shown that the introduction of thread extensibility significantly facilitates the study of the correctness of the boundary value problem formulation (the concept of correctness, as is known, means the existence and uniqueness of at least a ie-

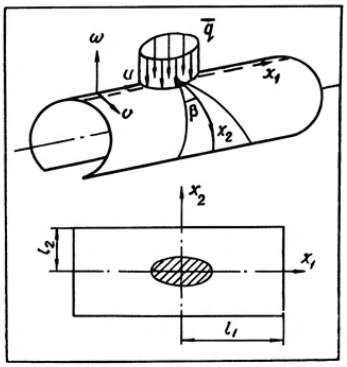

Fig. 1.

generalized in some sense solution that continuously depends on the

The system of equilibrium equations for a cylindrical shell turns out to be a system of strongly elliptic type, for which the results obtained in [3] can be used. For such systems of equations, it also becomes possible to apply approximate differencon methods developed in [5 and 6], as well as a wide class of variational methods [7]. Partaclepating attention is paid to obtainem..

Particular attention is paid to obtaining, if possible, weaker restrictions on the size of the domain, upon fulfillment of which the correctness of the studied boundary value problem is guaranteed (these restrictions will be called sufficient conditions for correctness in what follows).

Calculation results are presented, showing the influence of parameterering into the coefficients of the system, namely the angle between threads β and the parameter μ characterizing the extensibility of threads, on the sufficient conditions for correctness. At the same time, it should be borne in mind that although the conditions on the domain dimensions obtained in the work, apparently, can be refined, nevertheless in the most important for practice range of change of β and μ [$0<\mu<0.05$; $45°<\beta<60°$] they are sufficient to guarantee the correctness of the posed boundary value problems and the possibility of applying approximate methods [6, 7] for the case of symmettrical loads.

The equations, onuscribing malse deforminacon uuliundronveciral cetvatoй equations describing small deformations of a cylindrical net shell (Fig. 1), obtained in [2], can be written in vector form as follows:

$$L\overline{u}(x) \equiv R\overline{u}(x) + \mu Q\overline{u}(x) = \overline{f}(x). \tag{1}$$

Here $(x) = (x_1, x_2)$ are dimensionless coordinates on the shell surface; $\overline{u}(x_1, x_2)$ is the total displacement vector to be determined,

$$\overline{u}(x_1, x_2) = \begin{pmatrix} u_1(x_1, x_2) \\ u_2(x_1, x_2) \\ u_3(x_1, x_2) \end{pmatrix} = \begin{pmatrix} u(x_1, x_2) \\ v(x_1, x_2) \\ w(x_1, x_2) \end{pmatrix};$$

Figure 2: Figure 2

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For the determination of $\psi_k(t)$ $(k = 0, 1, 2, \ldots)$ let us pass from the problem (5_k) to an equivalent problem

$$\psi_k(t) = \lambda_0 \int_0^t \int_0^1 2A_3(u, s)\psi_{-1}(s)\psi_k(s)\,ds\,du + \\ + \int_0^t F_k(\lambda_0, \psi_{-1}, \psi_0, \psi_1, \ldots, \psi_{k-1})\,du + \varphi_k(0), \qquad (6_k)$$

$$\psi_k(t) = \varphi_k(t) \text{ on } E_0.$$

Having changed the order of integration in the double integral (6_k), we obtain the Fredholm equation

$$\psi_k(t) = \lambda_0 \int_0^1 \Phi(t, s)\psi_k(s)\,ds + \Psi_k(t), \qquad \psi_k(t) = \varphi_k(t) \text{ on } E_0, \qquad (7_k)$$

where $\Psi_k(t) = \varphi_k(0) + \int_0^t F_k(\lambda_0, \psi_{-1}, \psi_0, \ldots, \psi_{k-1})\,du$ — is a continuous function; $\Phi(t, s) = 2\psi_{-1}(s)\int_0^t A_3(u, s)\,du$ — is a kernel, continuous in D.

1. Let λ_0 not be a characteristic number of the integral equation ypabation

$$y(t) = \lambda_0 \int_0^1 \Phi(t, s)\,y(s)\,ds. \qquad (8)$$

Then, on the basis of a known Fredholm theorem, each of the problems (7_k) has a unique non-zero solution belonging to class C. Sequentially integrating the equations (7_k), we will find

$$\psi_k(t) = \Psi_k(t) + \lambda_0 \int_0^1 R_{\lambda_0}(t, s)\,\Psi_k(s)\,ds, \qquad \psi_k(t) = \varphi_k(t) \text{ on } E_0,$$

where $R_{\lambda_0}(t, s)$ — is the resolvent of the kernel $\Phi(t, s)$. Thus, the coefficients of the series (4) are determined sequentially and uniquely.

Let us denote by R such a number that

$$\left\{\sup_D |R_{\lambda_0}(t, s)|;\ \sup_D \left|\frac{\partial R_{\lambda_0}(t, s)}{\partial t}\right|\right\} \leqslant R.$$

For the proof of the convergence of the series (4) and its first derivative with respect to t, it is sufficient to prove the convergence of the series

$$\sum_{k=0}^{\infty} \lambda^k \psi_k(t), \qquad \sum_{k=0}^{\infty} \lambda^k \dot{\psi}_k(t). \qquad (9)$$

For this purpose, let us consider a system of two algebraic equations

$$U_1(X, Y, \lambda) \equiv X - Q\left[\frac{1}{|\lambda_0|\,Ma}\sum_{k=0}^{\infty} \lambda^k |\varphi_k(0)| + 2 + 2\lambda + |\lambda_0|\,a + \right. \\ \left. + \lambda(X + Y) + \frac{\lambda(X + Y + X^2) + \lambda^2(2X + XY)}{|\lambda_0|\,a}\right] = 0, \qquad (10)$$

Figure 3: Figure 3

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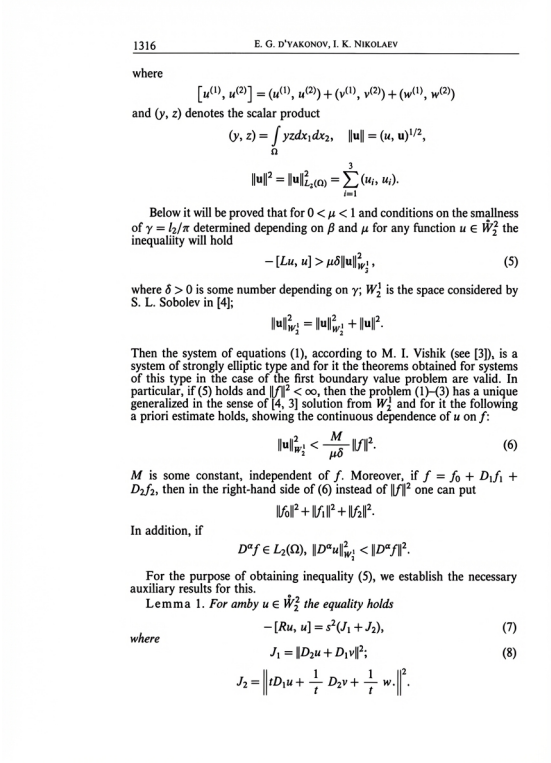

where

$$\left[u^{(1)}, u^{(2)}\right] = (u^{(1)}, u^{(2)}) + (v^{(1)}, v^{(2)}) + (w^{(1)}, w^{(2)})$$

and (y, z) denotes the scalar product

$$(y, z) = \int_{\Omega} yz dx_1 dx_2, \quad \|\mathbf{u}\| = (u, \mathbf{u})^{1/2},$$

$$\|\mathbf{u}\|^2 = \|\mathbf{u}\|^2_{L_2(\Omega)} = \sum_{i=1}^{3} (u_i, u_i).$$

Below it will be proved that for $0 < \mu < 1$ and conditions on the smallness of $\gamma = l_2/\pi$ determined depending on β and μ for any function $u \in \mathring{W}_2^2$ the inequaliity will hold

$$-[Lu, u] > \mu\delta \|\mathbf{u}\|^2_{W_2^1}, \tag{5}$$

where $\delta > 0$ is some number depending on γ; W_2^1 is the space considered by S. L. Sobolev in [4];

$$\|\mathbf{u}\|^2_{W_2^1} = \|\mathbf{u}\|^2_{W_2^1} + \|\mathbf{u}\|^2.$$

Then the system of equations (1), according to M. I. Vishik (see [3]), is a system of strongly elliptic type and for it the theorems obtained for systems of this type in the case of the first boundary value problem are valid. In particular, if (5) holds and $\|f\|^2 < \infty$, then the problem (1)–(3) has a unique generalized in the sense of [4, 3] solution from W_2^1 and for it the following a priori estimate holds, showing the continuous dependence of u on f:

$$\|\mathbf{u}\|^2_{W_2^1} < \frac{M}{\mu\delta} \|f\|^2. \tag{6}$$

M is some constant, independent of f. Moreover, if $f = f_0 + D_1 f_1 + D_2 f_2$, then in the right-hand side of (6) instead of $\|f\|^2$ one can put

$$\|f_0\|^2 + \|f_1\|^2 + \|f_2\|^2.$$

In addition, if

$$D^\alpha f \in L_2(\Omega), \ \|D^\alpha u\|^2_{W_2^1} < \|D^\alpha f\|^2.$$

For the purpose of obtaining inequality (5), we establish the necessary auxiliary results for this.

Lemma 1. *For amby $u \in \mathring{W}_2^2$ the equality holds*

$$-[Ru, u] = s^2(J_1 + J_2), \tag{7}$$

where

$$J_1 = \|D_2 u + D_1 v\|^2; \tag{8}$$

$$J_2 = \left\| tD_1 u + \frac{1}{t} D_2 v + \frac{1}{t} w. \right\|^2.$$

Figure 4: Figure 4

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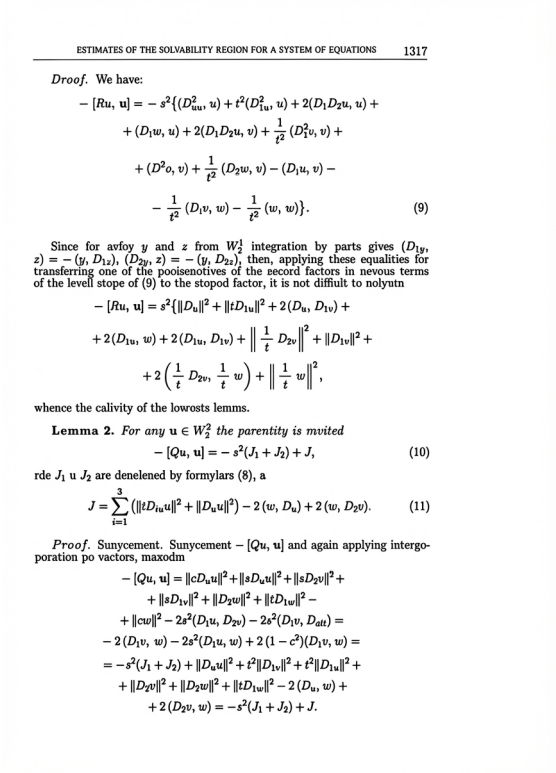

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Droof. We have:

$$- [Ru,\ \mathbf{u}] = - s^2\{(D_{uu}^2,\ u) + t^2(D_{1u}^2,\ u) + 2(D_1D_2u,\ u) +$$
$$+ (D_1w,\ u) + 2(D_1D_2u,\ v) + \frac{1}{t^2}\,(D_1^2v,\ v) +$$
$$+ (D^2o,\ v) + \frac{1}{t^2}\,(D_2w,\ v) - (D_1u,\ v) -$$
$$- \frac{1}{t^2}\,(D_1v,\ w) - \frac{1}{t^2}\,(w,\ w)\}. \tag{9}$$

Since for avfoy y and z from W_2^1 integration by parts gives $(D_{1y},\ z) = -(y,\ D_{1z})$, $(D_{2y},\ z) = -(y,\ D_{2z})$, then, applying these equalities for transferring one of the pooisenotives of the aecord factors in nevous terms of the levell stope of (9) to the stopod factor, it is not diffiult to nolyutn

$$- [Ru,\ \mathbf{u}] = s^2\{\|D_u\|^2 + \|tD_{1u}\|^2 + 2(D_u,\ D_{1v}) +$$
$$+ 2(D_{1u},\ w) + 2(D_{1u},\ D_{1v}) + \left\|\frac{1}{t}\,D_{2v}\right\|^2 + \|D_{1v}\|^2 +$$
$$+ 2\left(\frac{1}{t}\,D_{2v},\ \frac{1}{t}\,w\right) + \left\|\frac{1}{t}\,w\right\|^2,$$

whence the calivity of the lowrosts lemms.

Lemma 2. *For any* $\mathbf{u} \in W_2^2$ *the parentity is mvited*

$$- [Qu,\ \mathbf{u}] = - s^2(J_1 + J_2) + J, \tag{10}$$

rde J_1 u J_2 are denelened by formylars (8), a

$$J = \sum_{i=1}^{3} (\|tD_{iu}u\|^2 + \|D_uu\|^2) - 2(w,\ D_u) + 2(w,\ D_{2v}). \tag{11}$$

Proof. Sunycement. Sunycement $-[Qu,\ \mathbf{u}]$ and again applying intergoporation po vactors, maxodm

$$- [Qu,\ \mathbf{u}] = \|cD_uu\|^2 + \|sD_uu\|^2 + \|sD_{2v}\|^2 +$$
$$+ \|sD_{1v}\|^2 + \|D_{2w}\|^2 + \|tD_{1w}\|^2 -$$
$$+ \|cw\|^2 - 2s^2(D_1u,\ D_{2v}) - 2s^2(D_1v,\ D_{att}) =$$
$$- 2(D_1v,\ w) - 2s^2(D_1u,\ w) + 2(1 - c^2)(D_1v,\ w) =$$
$$= - s^2(J_1 + J_2) + \|D_uu\|^2 + t^2\|D_{1v}\|^2 + t^2\|D_{1u}\|^2 +$$
$$+ \|D_{2v}\|^2 + \|D_{2w}\|^2 + \|tD_{1w}\|^2 - 2(D_u,\ w) +$$
$$+ 2(D_{2v},\ w) = - s^2(J_1 + J_2) + J.$$

Figure 5: Figure 5

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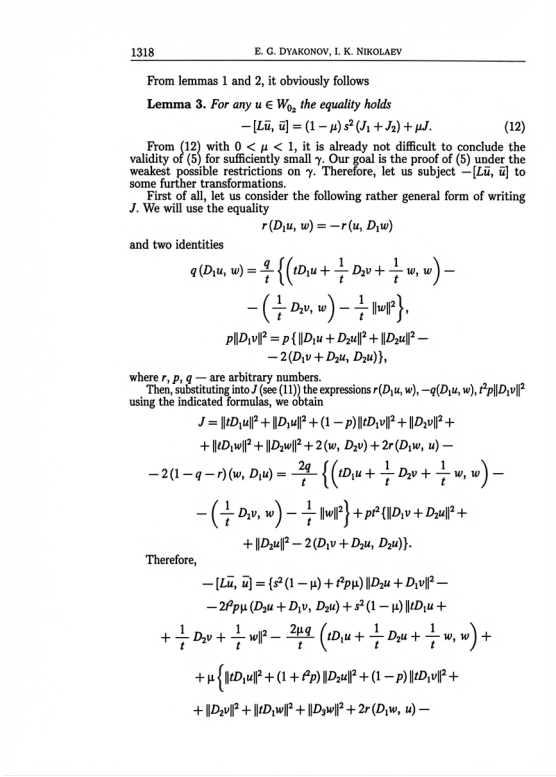

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From lemmas 1 and 2, it obviously follows

Lemma 3. *For any $u \in W_{0_2}$ the equality holds*

$$-[L\vec{u},\ \vec{u}] = (1-\mu)\, s^2 (J_1 + J_2) + \mu J. \tag{12}$$

From (12) with $0 < \mu < 1$, it is already not difficult to conclude the validity of (5) for sufficiently small γ. Our goal is the proof of (5) under the weakest possible restrictions on γ. Therefore, let us subject $-[L\vec{u},\ \vec{u}]$ to some further transformations.

First of all, let us consider the following rather general form of writing J. We will use the equality

$$r(D_1 u,\ w) = -r(u,\ D_1 w)$$

and two identities

$$q(D_1 u,\ w) = \frac{q}{t}\left\{\left(tD_1 u + \frac{1}{t} D_2 v + \frac{1}{t} w,\ w\right) - \right.$$
$$\left. - \left(\frac{1}{t} D_2 v,\ w\right) - \frac{1}{t}\|w\|^2\right\},$$
$$p\|D_1 v\|^2 = p\{\|D_1 u + D_2 u\|^2 + \|D_2 u\|^2 -$$
$$- 2(D_1 v + D_2 u,\ D_2 u)\},$$

where r, p, q — are arbitrary numbers.

Then, substituting into J (see (11)) the expressions $r(D_1 u, w)$, $-q(D_1 u, w)$, $t^2 p\|D_1 v\|^2$ using the indicated formulas, we obtain

$$J = \|tD_1 u\|^2 + \|D_1 u\|^2 + (1-p)\|tD_1 v\|^2 + \|D_2 v\|^2 +$$
$$+ \|tD_1 w\|^2 + \|D_2 w\|^2 + 2(w,\ D_2 v) + 2r(D_1 w,\ u) -$$
$$- 2(1-q-r)(w,\ D_1 u) = \frac{2q}{t}\left\{\left(tD_1 u + \frac{1}{t} D_2 v + \frac{1}{t} w,\ w\right) - \right.$$
$$\left. - \left(\frac{1}{t} D_1 v,\ w\right) - \frac{1}{t}\|w\|^2\right\} + pt^2\{\|D_1 v + D_2 u\|^2 +$$
$$+ \|D_2 u\|^2 - 2(D_1 v + D_2 u,\ D_2 u)\}.$$

Therefore,

$$-[L\vec{u},\ \vec{u}] = \{s^2(1-\mu) + t^2 p\mu)\|D_2 u + D_1 v\|^2 -$$
$$- 2t^2 p\mu (D_2 u + D_1 v,\ D_2 u) + s^2(1-\mu)\|tD_1 u +$$
$$+ \frac{1}{t} D_2 v + \frac{1}{t} w\|^2 - \frac{2\mu q}{t}\left(tD_1 u + \frac{1}{t} D_2 u + \frac{1}{t} w,\ w\right) +$$
$$+ \mu\left\{\|tD_1 u\|^2 + (1 + t^2 p)\|D_2 u\|^2 + (1-p)\|tD_1 v\|^2 + \right.$$
$$\left. + \|D_2 v\|^2 + \|tD_1 w\|^2 + \|D_3 w\|^2 + 2r(D_1 w,\ u) -\right.$$

Figure 6: Figure 6

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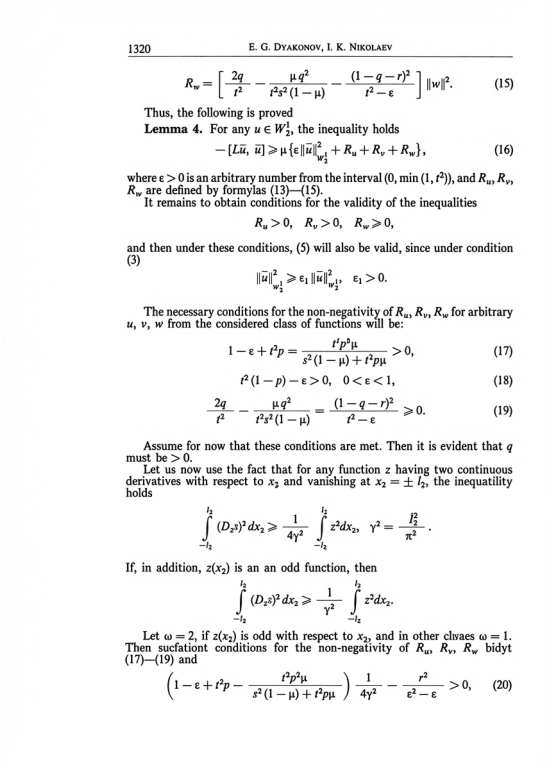

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$$R_w = \left[\frac{2q}{t^2} - \frac{\mu q^2}{t^2 s^2 (1-\mu)} - \frac{(1-q-r)^2}{t^2-\varepsilon} \right] \|w\|^2. \tag{15}$$

Thus, the following is proved

Lemma 4. For any $u \in \mathring{W}_2^1$, the inequality holds

$$-[L\bar{u},\ \bar{u}] \geqslant \mu \{\varepsilon \|\bar{u}\|^2_{\mathring{W}_2^1} + R_u + R_v + R_w\}, \tag{16}$$

where $\varepsilon > 0$ is an arbitrary number from the interval $(0, \min(1, t^2))$, and R_u, R_v, R_w are defined by formylas (13)—(15).

It remains to obtain conditions for the validity of the inequalities

$$R_u > 0, \quad R_v > 0, \quad R_w \geqslant 0,$$

and then under these conditions, (5) will also be valid, since under condition (3)

$$\|\bar{u}\|^2_{\mathring{W}_2^1} \geqslant \varepsilon_1 \|\bar{u}\|^2_{\mathring{W}_2^1}, \quad \varepsilon_1 > 0.$$

The necessary conditions for the non-negativity of R_u, R_v, R_w for arbitrary u, v, w from the considered class of functions will be:

$$1 - \varepsilon + t^2 p = \frac{t^4 p^2 \mu}{s^2 (1-\mu) + t^2 p \mu} > 0, \tag{17}$$

$$t^2 (1-p) - \varepsilon > 0, \quad 0 < \varepsilon < 1, \tag{18}$$

$$\frac{2q}{t^2} - \frac{\mu q^2}{t^2 s^2 (1-\mu)} = \frac{(1-q-r)^2}{t^2-\varepsilon} \geqslant 0. \tag{19}$$

Assume for now that these conditions are met. Then it is evident that q must be > 0.

Let us now use the fact that for any function z having two continuous derivatives with respect to x_2 and vanishing at $x_2 = \pm l_2$, the inequatility holds

$$\int_{-l_2}^{l_2} (D_{x_2} z)^2 \, dx_2 \geqslant \frac{1}{4\gamma^2} \int_{-l_2}^{l_2} z^2 dx_2, \quad \gamma^2 = \frac{l_2^2}{\pi^2}.$$

If, in addition, $z(x_2)$ is an an odd function, then

$$\int_{-l_2}^{l_2} (D_{x_2} z)^2 \, dx_2 \geqslant \frac{1}{\gamma^2} \int_{-l_2}^{l_2} z^2 dx_2.$$

Let $\omega = 2$, if $z(x_2)$ is odd with respect to x_2, and in other clwaes $\omega = 1$. Then sucfationt conditions for the non-negativity of R_u, R_v, R_w bidyt (17)—(19) and

$$\left(1 - \varepsilon + t^2 p - \frac{t^4 p^2 \mu}{s^2 (1-\mu) + t^2 p \mu}\right) \frac{1}{4\gamma^2} - \frac{r^2}{\varepsilon^2 - \varepsilon} > 0, \tag{20}$$

Figure 7: Figure 7

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$$(1-\varepsilon)\frac{\omega^2}{4\gamma^2}-\frac{\left(1+\frac{q}{t^2}\right)^2}{1-\varepsilon}\geqslant 0. \tag{21}$$

Since we are interested in the maximum value of γ for which (20), (21) will hold when (17)—(19) are satisfied for any $\mu \geqslant 0$ sufficiently small, it is clear that we must take ρ as large as possible.

Therefore, let $p = 1-1-\frac{\varepsilon}{t^2}$. Then (18) is satisfied as an equality, and (17) transforms into

$$\rho(t,\varepsilon,\mu)=1+t^2-2\varepsilon-\frac{\mu(t^2-\varepsilon)^2}{(1-\mu)s^2+\mu(t^2-\varepsilon)}>0. \tag{22}$$

Instead of (20), (21), we easily obtain equivalent imporribilities

$$r^2\gamma^2\leqslant\frac{t^2-\varepsilon}{4}\rho(t,\varepsilon,\mu), \tag{23}$$

$$\gamma\leqslant\frac{(1-\varepsilon)\omega t^2}{2(t^2+q)}. \tag{24}$$

It is obvious that

$$\rho(t,\varepsilon,\mu)>1+t^2-2\varepsilon-(t^2-\varepsilon)=1-\varepsilon$$

and (17), (22) will be satisfied, sir since $\varepsilon<1$.

The set of (ε, q, r) for which $q>0$ and (19) are satisfied, we donate by E_t, and by $\gamma(\varepsilon, q, r)$ we denate the eupxer bound of the values of γ satisfelying (23), (24) for fixed ε, q, r from E_t.

Let us find

$$\gamma(t)=\sup_{(\varepsilon,q,r)\in E_t}\gamma(\varepsilon,q,r)^*.$$

It is easy to verefy, that

$$\gamma(t)=\sup_{(0,q,r)\in E_t}\gamma(0,q,r).$$

Is (19), (23), (24) with $\varepsilon=0$ we obtain

$$(1+\alpha)q^2-2(2-r)q+(1-r)^2\leqslant 0, \tag{25}$$

$$r^2\gamma^2\leqslant\frac{t^2}{4}\rho(\beta,\mu), \tag{26}$$

$$\gamma\leqslant\frac{\omega t^2}{2(t^2+q)}, \tag{27}$$

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$$\alpha=\frac{\mu}{s^2(1-\mu)};$$

$$\rho(\beta,\mu)=1+t^2-\frac{\mu t^4}{(1-\mu)s^2+\mu t^2}>1.$$

*) The dependence of γ on μ is omitted in a number of cases for brevity.

Figure 8: Figure 8

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It is also useful to note that in case 2), of course, it will be

$$b = 1 - \frac{1}{a} > 0.$$

Thus, for fixed μ and β, $\gamma(t, \mu)$ can be easily calculated using the formulas indicated above.

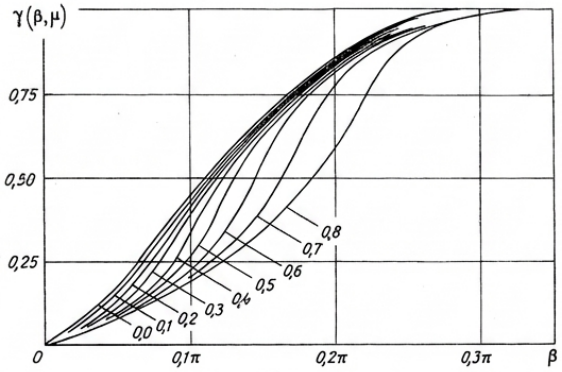

Fig. 2.

Now it is not difficult to verify that the following is true

Theorem 1. *If $\mu < 1$ and $\frac{l_2}{\pi} < \gamma(t, \mu)$, where $\gamma(t, \mu)$ is defined by formulas* (30), (32), *then there exists a sufficiently small $\delta > 0$ such that for any functions u from W_2^2 the inequality will hold*

$$-[Lu, u] \geqslant \mu\delta \|u\|^2_{W_2^1}.$$

In conclusion, we present several graphs of $\gamma(t, \mu)$ for fixed μ (Fig. 2), from which it can be concluded that for $\mu > 0$ there is some deterioration of the sufficient conditions for well-posedness compared to the graph of $\gamma(t, 0)$, but it should be noted that, firstly, this deterioration refers to the case of those $\mu > 0$ which have very little practical significance, and, secondly, this is a deterioration only of the sufficient conditions obtained by the methodology adopted in the work, which, generally speaking, can be refined.

We also note the following: although the system of equations for determining displacements and forces in threads for the case $\mu = 0$, given in [1], cannot be obtained from system (1) by substituting $\mu = 0$, the graph of $\gamma(t, 0)$ gives sufficient conditions on the dimensions of the region, guaranteeing the uniqueness of the solution to the boundary value problem for system [1] also in the case of inextensible threads.

Figure 9: Figure 9

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Indeed, if system [1] is written in the form

$$\overline{P}\overline{u} = \begin{pmatrix} p_1\overline{u} \\ p_2\overline{u} \\ \cdots\cdots \\ p_5\overline{u} \end{pmatrix} = \overline{f},$$

where $\overline{u}$ is an unknown five-dimensional vector, three components of which are displacements, and the others are forces; $\overline{P}$ is a matrix differential operator; $\overline{f}$ is a known five-dimensional vector, then, calculating $[\overline{P}\overline{u}, \overline{u}]_0 \equiv$ $= \sum_{i=1}^{5} (p_i, \overline{u}, \overline{u})$, it is possible to verify that the forces in the threads will drop out of the functional $[\overline{P}\overline{u}, \overline{u}]_0$, and this functional itself will have a form exactly coinciding with that obtained from the right side of formula (12) after substituting $\mu = 0$.

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Figure 10: Figure 10