THE ENERGY STRUCTURE OF A COLLISION CASCADE OF IDENTICAL PARTICLES IN A RETARDING MEDIUM

PHYSICS

1967

SovietRxiv

Abstract

Full Text

UDC 539.2:539.16.04

PHYSICS

V. M. LEPCHENKO, Academician of the Academy of Sciences of the Uzbek SSR S. V. STARODUBTSEV

THE ENERGY STRUCTURE OF A COLLISION CASCADE OF IDENTICAL PARTICLES IN A RETARDING MEDIUM

The generation of excitations in solids, ionization of matter, defect formation, and other activation acts under the action of nuclear radiations occur both in primary collisions of penetrating particles with particles of the medium and in cascades of secondary electron–electron and atom–atom collisions initiated by fast electrons and atoms that have received their initial energy from the penetrating radiation. For a quantitative description of these secondary processes and for elucidating their role in radiation-physical and radiation-chemical effects, it is necessary to determine the energy spectrum of cascade particles as a function of the energy of the initial cascade particle and the stopping power of the medium. A problem of this kind for a cascade of atom–atom collisions was solved in works ([1]-[3]). However, the calculation methods used there had the following shortcomings: first, it was required that the total cross section for collision of two particles be finite, and, second, the slowing down of cascade particles due to inelastic and collective interactions with the medium was not taken into account, and it was unclear how to take it into account. The method proposed below is free of these shortcomings.

We shall characterize the energy spectrum by the function $F(E, \varepsilon)\, d\varepsilon$, which expresses the total path length of cascade particles of one kind having energy in the interval $(\varepsilon, \varepsilon + d\varepsilon)$; E is the energy of the initial cascade particle. Without allowance for multiplication, for $F(E, \varepsilon)$ one may write the expression

$$F(E, \varepsilon) = \varphi^{-1}(\varepsilon) + \lambda(\varepsilon)\delta(E - \varepsilon), \qquad (1)$$

where $\varphi(\varepsilon) = -(d\varepsilon/dx)$ is the total energy loss of the particle per unit path; $\lambda(\varepsilon)$ is the free path of the particle to the first collision.

Let us take multiplication into account. For this purpose, note that along a path dx the initial cascade particle undergoes $n_a\sigma(E', E_1)\cdot dE_1 dx$ collisions with transfer to struck particles of energy in the interval $E_1, E_1 + dE_1$, where n_a is the density of atoms; $\sigma(E', E_1)dE_1$ is the collision cross section, calculated per atom.

Each knocked-on particle, in turn, creates a cascade of the same particles, the total path length of which in the interval $(\varepsilon, \varepsilon+d\varepsilon)$ will be $F(E_1,\varepsilon)d\varepsilon$. Hence it is clear that the total contribution from cascade particles to the function $F(E,\varepsilon)$ will be given by the expression

$$\int n_a\,dx \int_{\varepsilon}^{E'} \sigma(E',E_1)F(E_1,\varepsilon)\,dE_1. \tag{2}$$

Noting that $dx=\varphi^{-1}(E')\,dE'$ and regarding equation (1) as an initial condition, we obtain for $F(E,\varepsilon)$ the integral equation:

$$F(E,\varepsilon)=\int_{\varepsilon}^{E} n_a\varphi^{-1}(E')\,dE' \int_{\varepsilon}^{E'} \sigma(E',E_1)F(E_1,\varepsilon)\,dE_1+\varphi^{-1}(\varepsilon)+\lambda(\varepsilon)\delta(E-\varepsilon). \tag{3}$$

This integral equation differs in principle from that used for similar purposes in works ([1]-[3]). Nevertheless, in the approximation

of the hard-sphere, for which

$$\sigma(E',E_1)=\sigma_0/E', \tag{4}$$

$$\varphi(\varepsilon)=\frac{1}{2}n_a\sigma_0\varepsilon, \tag{5}$$

$$\lambda(\varepsilon)=(n_a\sigma_0)^{-1}, \tag{6}$$

its solution gives results coinciding with those obtained earlier [1–3]. As was already noted, this approximation is insufficiently accurate, since it does not take into account the energy losses by cascade particles in distant collisions, which cannot be accounted for within the hard-sphere model. From very general considerations it follows that these additional losses depend on the energy as ε^{-1}, and therefore instead of expression (5) we must write the expression

$$\varphi(\varepsilon)=\frac{1}{2}n_a\sigma_0(\varepsilon+q^2/\varepsilon). \tag{7}$$

Here q is a quantity only weakly dependent on the energy, and we shall regard it as constant. Quantitatively it is determined by the choice of the hard-sphere cross section σ_0 and by the nature of the distant collisions.

It is not difficult to verify by direct substitution that the solution of equation (3), with allowance for (4), (6), and (7), has the form

$$F(E,\varepsilon) = \lambda(\varepsilon)S(E,\varepsilon), \tag{8}$$

where

$$S(E,\varepsilon) = 2E/(\varepsilon^2 + q^2) + \delta(E - \varepsilon) \tag{9}$$

is the number of collisions of particles in the cascade with transfer of energy ε to the struck particles in a unit interval.

For $q = 0$, expression (9) coincides with that obtained by another method in [1–3].

Thus, we are convinced that allowance for the additional slowing down substantially corrects the distribution function of cascade particles with respect to energies, and for $\varepsilon \ll q$ the distribution function changes mainly because of continuous slowing down, whereas for $\varepsilon \gg q$ its change occurs because of close displacement collisions.

Let us now apply equation (3) to the calculation of the energy distribution function of electrons in a cascade of electron–electron collisions in a slowing medium. Since for Coulomb collisions the cross section is not bounded, in the motion of electrons in a condensed medium, as was already noted, $\lambda = 0$. It is then convenient to represent the function $F(E,\varepsilon)$ in the form

$$F(E,\varepsilon) = \nu(E,\varepsilon)/\varphi(\varepsilon). \tag{10}$$

Here $\nu(E,\varepsilon)$ has the meaning of the number of particles with energy ε produced in the collision cascade by one primary particle with energy E. In the continuous-slowing-down approximation multiplication is neglected ($\nu = 1$), and then (10) for $\varepsilon \neq E$ coincides with equation (1).

Substituting (10) into equation (3), taking into account what has been said, we obtain the following equation for ν:

$$\nu(E,\varepsilon) = \int_\varepsilon^E \frac{n_a\,dE'}{\varphi(E')} \int_\varepsilon^{E'} \sigma(E',E_1)\nu(E,\varepsilon)\,dE_1 + 1. \tag{11}$$

In the nonrelativistic case a good approximation for the electron collision cross section is the expression [4]

$$\sigma(\varepsilon,\varepsilon') = \frac{2\pi Z e^4}{\varepsilon}\,\frac{1}{\varepsilon'^2}; \tag{12}$$

here Z is the atomic number.

Energy losses in this case go mainly to ionization and excitation of the atoms of the medium, as well as to the generation of collective excita-

of the medium. We write them in the form

$$\varphi(\varepsilon) = \frac{2\pi n_a Z e^4}{\varepsilon}\left(\ln\frac{\varepsilon}{\varepsilon_i} + a\right). \tag{13}$$

Here ε_i is the mean ionization potential; a is a quantity weakly dependent on energy, characterizing the energy losses of electrons to all possible excitations of the medium; we shall regard it as a constant quantity.

The solution of equation (11), in which $\sigma(\varepsilon, \varepsilon')$ and $\varphi(\varepsilon)$ are given by expressions (12) and (13), is readily determined by solving the differential equation obtained from (11) by differentiating twice with respect to the upper limit. Omitting the intermediate calculations, we write the final expression for $\nu(E, \varepsilon)$:

$$\nu(E,\varepsilon) = \frac{E}{\varepsilon}\left[1 - \frac{\varepsilon}{\varepsilon_1}\ln\frac{\varepsilon}{\varepsilon_1}\left(\mathrm{li}\,\frac{\varepsilon_1}{\varepsilon} - \mathrm{li}\,\frac{\varepsilon_1}{E}\right)\right], \tag{14}$$

where the notation $\varepsilon_1 = \varepsilon_i e^{-a}$ has been introduced.

If for the logarithmic integral one uses its asymptotic expansion

$$\mathrm{li}\left(\frac{1}{x}\right) = \frac{1}{x}\left(-\frac{1}{\ln x} + \frac{1}{\ln^2 x}\right) + R(x), \tag{15}$$

where $|R(x)| < 2/x \ln^3 x$, then for fast electrons, for which $\ln \varepsilon/\varepsilon_1 \gg 1$, instead of (14) one may write the more transparent expression

$$\nu(E,\varepsilon) \simeq \frac{E}{\varepsilon \ln \varepsilon/\varepsilon_1} + \frac{\ln \varepsilon/\varepsilon_1}{\ln E/\varepsilon_1} - \frac{\ln \varepsilon/\varepsilon_1}{\ln^2 E/\varepsilon_1}. \tag{16}$$

For illustration, let us note that one electron with energy $E = 5 \cdot 10^5$ eV in a medium for which $\varepsilon_1 \sim 1$ eV creates 1.2 electrons with energy 10^5 eV, 70 electrons with energy 10^3 eV, and $4 \cdot 10^3$ electrons with energy 10^2 eV. Since ε_1 enters equations (14) and (16) under the logarithm sign, the inaccuracy of its choice does not greatly affect the estimates given.

Received
1 VI 1966

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Note: Figure translations are in progress. See original paper for figures.