COMBINATION SCATTERING OF ULTRAHIGH-FREQUENCY RADIATION BY ION-ACOUSTIC OSCILLATIONS OF A BOUNDED PLASMA

PHYSICS

1967

SovietRxiv

Abstract

Full Text

UDC 533.9+538.4

PHYSICS

D. D. RYUTOV

COMBINATION SCATTERING OF ULTRAHIGH-FREQUENCY RADIATION BY ION-ACOUSTIC OSCILLATIONS OF A BOUNDED PLASMA

(Presented by Academician E. K. Zavoisky, 27 IV 1966)

The purpose of the present work is to investigate certain effects associated with the influence of the finite dimensions of a plasma on the combination scattering of electromagnetic waves by ion-acoustic oscillations.

First of all, let us point out those features of the scattering process that are characteristic of an infinite plasma, restricting ourselves for simplicity to the case when the frequency of the incident electromagnetic wave ω_1 considerably exceeds the electron plasma frequency ω_{pe}. If in an infinite plasma an electromagnetic wave with wave vector $\mathbf{k}_1$ is scattered by an ion-acoustic wave with wave vector $\mathbf{K}$, then the wave vector of the scattered electromagnetic wave $\mathbf{k}_2$ is related to $\mathbf{k}_1$ and $\mathbf{K}$ by the relation

$$\mathbf{k}_2 = \mathbf{k}_1 \pm \mathbf{K}. \tag{1}$$

This relation may be interpreted as the law of conservation of momentum in scattering. We note that the conservation law (1) is a consequence of the homogeneity of the system and therefore, strictly speaking, is applicable only to an infinite homogeneous plasma. The frequency of the scattered wave ω_2 is determined by the formula

$$\omega_2 = \omega_1 \pm \Omega, \tag{2}$$

where $\Omega = c_s K = \sqrt{m/M}\, v_{Te} K$ is the frequency of ion sound.

Thus, two satellites are present in the spectrum of the scattered radiation. Formulas (1) and (2) make it possible to express the satellite frequencies in terms of the scattering angle θ (i.e., the angle between the vectors $\mathbf{k}_1$ and $\mathbf{k}_2$):

$$\omega_2^{(\pm)} = \omega_1 \left(1 \pm 2\,\frac{c_s}{c} \sin\frac{\theta}{2}\right). \tag{3}$$

Relation (3) is analogous to the well-known Mandelstam–Brillouin result for the scattering of electromagnetic waves by acoustic oscillations of a medium (see ([1])). Estimates of the intensity of the scattered radiation obtained under conditions in which the conservation laws (1), (2) are valid may be found, for example, in papers ([2],[3]).

Let us now consider the question of the influence of the finite dimensions of the plasma on the scattering process. We shall examine this problem on the simplest possible model. Namely, we shall assume that the plasma fills a plane layer of thickness d, and outside this layer the plasma density is zero. It is clear that if the wavelength of the incident wave $\lambda_1 = c/\omega_1$ is small in comparison with d, then the influence of the plasma boundaries on the scattering process may be neglected and the conservation law (1) may be used. In this case, as was indicated above, the frequency of the scattered radiation is related to the scattering angle θ by relation (3). It follows from (3) that the frequency shift in scattering is very small:

$$\left|\omega_2^{(\pm)} - \omega_1\right| / \omega_1 \leqslant 2c_s/c$$

(for a hydrogen plasma with electron temperature ~ 100 eV,

$$\left|\omega_2^{(\pm)} - \omega_1\right| / \omega_1 \lesssim 10^{-3}$$

). Under such conditions, separation of the small signal from

against the large signal with the fundamental frequency is a very difficult experimental problem (especially if one takes into account that in many experiments the lifetime of intense ion-sound oscillations is $10^{-6} \div 10^{-7}$ sec.).

The situation changes substantially if the length of the incident electromagnetic wave is chosen so that the inequality* $\lambda_1 \gtrsim d$ is satisfied. Let us find the combination frequencies for this case. Under the condition $\lambda_1 \gtrsim d$, the conservation law (1) is valid only for the tangential component of the momentum, $\mathbf{k}_{2t} = \mathbf{k}_{1t} \pm \mathbf{K}_t$. The nonconservation of the normal component of the momentum is due to the fact that, with respect to waves with $\lambda_1 \gtrsim d$, the system is strongly inhomogeneous in the normal direction: the scale of the inhomogeneity d is small compared with the wavelength λ_1.

From the equality $K_t = |\mathbf{k}_{2t} - \mathbf{k}_{1t}|$ it follows that scattering occurs on ion-sound oscillations with $K_t \lesssim \omega_1/c \lesssim d^{-1}$. The eigenvalues of the normal component of the wave vector $K_\perp$ for such oscillations are determined by the relation $K_{\perp p} = p\pi/2d,\ p = 1, 2, ...$. The corresponding values of the eigenfrequencies are $\Omega_p = c_s p\pi/2d$. Since the conservation laws impose no restrictions on the choice of the normal component of the ion-sound wave vector, ion-sound oscillations with arbitrary values of p can take part in the scattering of an electromagnetic wave with a definite frequency. Therefore, in the spectrum of the scattered radiation there may be frequencies of the form

$$\omega_{2p}^{(\pm)} = \omega_1 \pm c_s \pi p / 2d. \tag{4}$$

When formulas (3) and (4) are compared, three circumstances attract attention. First, the spectrum of the scattered radiation contains not two lines, as was the case for an unbounded plasma, but an entire set of lines. Second, the position of these lines does not depend on the scattering angle θ. Third, the distance between neighboring lines is equal to $c_s\pi/2d$, which considerably exceeds (under the condition $\omega_1/c \ll d^{-1}$) the separation between the lines arising in an unbounded plasma. This facilitates the isolation of satellites (especially with $p \gg 1$) against the background of the signal with the fundamental frequency.

To find the intensity of the scattered radiation, we shall use the equation

$$\operatorname{rot}\operatorname{rot}\mathbf{E} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\hat{\varepsilon}\mathbf{E} = -\frac{4\pi}{c^2}\frac{\partial \delta\mathbf{j}}{\partial t}, \tag{5}$$

where $\hat{\varepsilon}$ is the linear operator of the dielectric permittivity, and $\delta\mathbf{j}$ is the nonlinear current, expressed through $\mathbf{E}$ by means of the material equations, for which we shall take the equations of two-fluid hydrodynamics. The general method for calculating the intensity of the scattered radiation is based on applying perturbation theory and remains essentially the same as in the case of an unbounded plasma. The only difference consists in taking into account the influence of the finite plasma dimensions on the form of the operator $\hat{\varepsilon}$ and the nonlinear current $\delta\mathbf{j}$.

Calculations carried out by us for the case of normal incidence of a wave on a layer show that scattering occurs only on those oscillations which correspond to odd values of p. Therefore, in the spectrum of the scattered radiation there are only frequencies of the form $\omega_{2q}^{(\mp)} = \omega_1 \pm c_s\pi(q-1/2)/d,\ q = 1, 2, \ldots$. The power of the scattered radiation at frequencies $\omega_{2q}^{(\pm)}$, referred to unit area of the layer and to a unit interval

* Such a formulation of the problem can be easily realized in experiments in which the transverse size of the plasma does not exceed several centimeters (for example, (4)).

per unit solid angle, is equal to

$$P_q^{(\pm)} = \frac{32}{\pi^4} S_0 \left(\frac{\omega_{pe} d}{c}\right)^3 \frac{\omega_1^2 \omega_{pe}}{c^3} \left(1 - \frac{(\mathbf{e}\xi)^2}{1 + (\mathbf{n}\xi)^2}\right) \frac{(\mathbf{n}\xi)^2 (1 + \mathbf{n}\xi)^2}{(q - 1/2)^4} \frac{W_{K_t,\,2q-1}}{n m v_{Te}^2}, \tag{6}$$

$$\mathbf{K}_t = \pm \mathbf{k}_{2t} = \pm \frac{\omega_1}{c} \left(\xi - \mathbf{n}\,\xi\mathbf{n}\right),$$

where $\mathbf{n}$ is the vector normal to the layer, ξ is a unit vector in the direction of propagation of the scattered wave, $\mathbf{e}$ is the polarization vector of the incident wave, and S_0 is the energy-flux density of the incident wave. The quantity $W_{K_t,\,2q-1}$ entering formula (6) is determined as follows: $W_{K_t,\,2q-1}d^2K_t$ is the energy of oscillations of the mode $p = 2q-1$, per unit area of the layer and lying in the wave-vector interval $(\mathbf{K}_t, \mathbf{K}_t + d^2\mathbf{K}_t)$. Formula (6) refers to scattered radiation with E-polarization. In the scattered radiation, generally speaking, waves with H-polarization are also present. The formula for the intensity of the H-polarized radiation is analogous in structure to formula (6), and we shall not present it. Estimates carried out using formula (6) show that, in the experiments of Ref. (4), Raman scattering can be observed if the ratio of the energy of the ion-acoustic oscillations to the thermal energy of the particles exceeds 10^{-4}.

Let us summarize the main results of the work. We have shown that, under conditions where the wavelength of the incident electromagnetic wave exceeds the transverse size of the plasma, the spectrum of the scattered radiation is substantially changed in comparison with the case of an unbounded plasma. The separation between the satellites increases, which facilitates the experimental conditions. The estimate obtained for the intensity of the scattered radiation gives reason to hope that the effect considered can be detected in experiments of the type of Ref. (4).

The author expresses his gratitude to L. I. Rudakov for his attention to the work.

Received
13 IV 1966

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Note: Figure translations are in progress. See original paper for figures.