ON A PARTICULAR SOLUTION OF THE PROBLEM OF THE ROTATION OF A HEAVY RIGID BODY ABOUT A FIXED POINT

Mechanics

1966

SovietRxiv

Abstract

Full Text

UDC 531.38

Mechanics

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ON A PARTICULAR SOLUTION OF THE PROBLEM OF THE ROTATION OF A HEAVY RIGID BODY ABOUT A FIXED POINT

(Presented by Academician P. Ya. Kochina on 19 VII 1965)

The equations of the problem of the rotation of a heavy rigid body about a fixed point, in the notation of [1], have the form

$$\begin{gathered}\dot{x} = (a_2 - a_1)yz + (b_2 y - b_1 z)x,\\ \dot{y} = (a - a_2)zx + (b_1 y + b_2 z)z - b_2 x^2 - \gamma_3,\\ \dot{z} = (a_1 - a)xy - (b_1 y + b_2 z)y + b_1 x^2 + \gamma_2,\\ \dot{\gamma}_1 = \gamma_2\omega_3 - \gamma_3\omega_2, \qquad \dot{\gamma}_2 = \gamma_3\omega_1 - \gamma_1\omega_3, \qquad \dot{\gamma}_3 = \gamma_1\omega_2 - \gamma_2\omega_1,\end{gathered} \tag{1}$$

where

$$\omega_1 = ax + b_1 y + b_2 z, \qquad \omega_2 = a_1 y + b_1 x, \qquad \omega_3 = a_2 z + b_2 x.$$

The known integrals of these equations are

$$\begin{gathered}\tfrac{1}{2}(ax^2 + a_1 y^2 + a_2 z^2) + (b_1 y + b_2 z)x - \gamma_1 = h,\\ \gamma_1 x + \gamma_2 y + \gamma_3 z = m,\\ \gamma_1^2 + \gamma_2^2 + \gamma_3^2 = e^2.\end{gathered} \tag{2}$$

Let the center of gravity of the body lie on the perpendicular to the circular section of the gyration ellipsoid:

$$a_2 - a_1 = 0, \qquad b_2 = 0.$$

We assume that the initial value of the variable x is different from zero and that $b_1 \neq 0$, as a result of which the conditions under which the Hess and Lagrange solutions occur are not fulfilled. Under these assumptions, equations (1) and the integrals (2) can be written as follows:

$$
\begin{aligned}
dx/d\tau &= -zx,\\
dy/d\tau &= (a_0 - b_0)zx + zy - \gamma,\\
dz/d\tau &= (b_0 - a_0)xy - y^2 + x^2 + \beta,\\
d\alpha/d\tau &= -\gamma(b_0 y + x) + b_0\beta z,\\
d\beta/d\tau &= \gamma(a_0 x + y) - b_0\alpha z,\\
d\gamma/d\tau &= \alpha(b_0 y + x) - \beta(a_0 x + y);
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
\tfrac{1}{2}(ax^2 + a_1 y^2 + a_2 z^2) + (b_1 y + b_2 z)x - \gamma_1 = h_0,\\
\alpha x + \beta y + \gamma z = m_0, \qquad \alpha^2 + \beta^2 + \gamma^2 = l_0^2,
\end{aligned} \tag{4}
$$

where

$$
\tau = b_1 t, \qquad \alpha = \gamma_1/b_1, \qquad \beta = \gamma_2/b_1, \qquad \gamma = \gamma_3/b_1, \qquad a_0 = a/b_1,
$$

$$
b_0 = a_1/b_1 = a_2/b_1.
$$

The system (3) admits the particular solution

$$
y = y_1 x + y_2 x^{-1}, \qquad z^2 = r_1 x^2 + r_2 x^{-2} + r_0,
$$

$$
\alpha = \alpha_0 + \alpha_1 x^2, \qquad \beta = \beta_0 + \beta_1 x^2, \qquad \gamma = \gamma_0 xz.
$$

The dependence of the variables of the problem on time is determined by means of the equation

$$
dx/dt = -zx,
$$

which can be transformed to the form

$$
(dx/dt)^2 = -r_1 b_1^2 (x_1^2 - x^2)(x^2 - x_2^2).
$$

The constants $y_1, y_2, r_1, r_2, r_0, \alpha_0, \alpha_1, \beta_0, \beta_1, \gamma_0, x_1^2, x_2^2$ are expressed in terms of the parameters a_0, b_0 as follows:

$$
3y_1 = b_0 - 2a_0 + \delta, \qquad \delta = \pm\sqrt{a_0^2 - a_0 b_0 + b_0^2 + 3},
$$

$$
3\gamma_0 = -(a_0 + b_0) + 2\delta, \qquad \alpha_0 = \pm l_0, \qquad \beta_0 = b_0\alpha_0,
$$

$$(4 + b_0^2)\alpha_1 = \gamma_0(3b_0 y_1 + a_0 b_0 + 2),$$

$$(4 + b_0^2)\beta_1 = \gamma_0[(b_0^2 - 2)y_1 + b_0 - 2a_0],$$

$$y_2 = \frac{\beta_0}{\gamma_0}, \qquad r_1 = -\frac{1}{\gamma_0^2}(\alpha_1^2 + \beta_1^2), \qquad r_0 = -\frac{2}{\gamma_0^2}(\alpha_0\alpha_1 + \beta_0\beta_1), \qquad r_2 = -\frac{\beta_0^2}{\gamma_0^2},$$

$$x_1^2 = \frac{1}{2r_1}\left(-r_0 + \sqrt{r_0^2 - 4r_1 r_2}\right), \qquad x_2^2 = \frac{1}{2r_1}\left(-r_0 - \sqrt{r_0^2 - 4r_1 r_2}\right).$$

The signs of these constants, except for y_1, can be determined from the inequalities

$$\delta b_0 < 0, \qquad \delta\gamma_0 > 0, \qquad b_0\alpha_1 > 0, \qquad b_0\alpha_0 < 0,$$

$$\beta_0 < 0, \qquad \beta_1 > 0, \qquad r_1 < 0, \qquad r_2 < 0, \qquad r_0 > 0, \qquad b_0 y_2 > 0.$$

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Received
5 VII 1965

REFERENCES

1. P. V. Kharlamov, *Prikl. matem. i mekhanika*, **27**, 4 (1963).

Note: Figure translations are in progress. See original paper for figures.