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ON GROOVE INSTABILITY IN AN UNCOMPENSATED PLASMA

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Experiments carried out in recent years ([1]) indicate a very strong influence of the plasma' s own electric fields on its stability. By changing the configuration of the radial electric field with the aid of electrostatic screens in the "Ogra" installation, it proved possible to find regimes in which low-frequency instabilities of the groove type were partially or completely suppressed. The interest in a theoretical study of this problem is therefore understandable. In the present work we take as the starting point an equation obtained recently in ([2]). This equation is investigated numerically by the Galerkin method.

1. Let us consider a rarefied ion-electron plasma, inhomogeneous in the direction of the x-axis and placed in an external magnetic field $\mathbf{H} = (0, 0, H(x))$. We shall assume the magnetic field to be weakly inhomogeneous, so that the motion of the particles is described sufficiently well by the drift approximation. If the electron and ion densities do not coincide (the plasma is uncompensated), then a stationary electric field $\mathbf{E}(x) = (E(x), 0, 0)$ arises in the plasma. We shall assume that the stationary distribution functions of ions and electrons are Maxwellian distributions of particles located in the fields $\mathbf{E}$ and $\mathbf{H}$. In ([2]) it is shown that in such a plasma the potential Φ of longitudinal plasma oscillations of the groove type, $\Phi = \Phi(x)\exp[i(ky - \omega t)]$, in the case when the Larmor radii of the particles r_j are much smaller than the characteristic scale x_0 of the plasma inhomogeneity, satisfies a second-order differential equation:

$$L[\psi] = (T\psi')' - k^2(T - gn')\psi = 0, \tag{1}$$

where

$$T(x,\omega) = (\omega - kv_E)^2\left[n + \nu^2 - k\frac{v_{Ti}^2}{2\omega_i}\frac{n'}{\omega - kv_E}\right]; \qquad N_i(x) = N_0 n(x) —$$

the ion density (N_0 is a characteristic value of the density, $n(x) \sim 1$); $\nu = \omega_i/\omega_{0i}$; $\omega_{0i}^2 = 4\pi e^2 N_0/m_i$; $\omega_i = eH/m_{ic}$; v_{Ti} is the thermal velocity of the ions; $v_E(x) = -cE(x)/H$ is the velocity of the electric drift; $g = -\omega_i v_{Hi}$, $v_{Hi} = 3v_{Ti}^2 H'/2\omega_{iH}$ is the velocity of the magnetic drift of the ions; $\psi = \Phi(\omega - kv_E)^{-1}$. To complete the formulation of the stability problem, equation (1) must be supplemented by the boundary conditions:

$$|\psi(\mp\infty)| < \infty. \tag{2}$$

The plasma is unstable with respect to oscillations of the type under consideration if the problem (1)–(2) has eigenvalues in the upper half-plane $\operatorname{Im}\omega > 0$. In the region of ω of interest to us, equation (1) has no singular points on the x-axis. Only on the boundary of the region, for real ω, can singular points x_α appear on the x-axis; in considering the differential equation they must be bypassed from below if $kv_E'(x_\alpha) > 0$, and from above if $kv_E'(x_\alpha) < 0$.

2. The electric field enters equation (1) only in the combination $\omega - kv_E$, and therefore the usual crude method of qualitative investigation

the problem (1)–(2) with the aid of the "dispersion equation at a point" does not reveal the dependence of the eigenvalues on the electric field. In the present work, to solve the problem we apply the Galerkin method. To this end we shall seek the solution of problem (1)–(2) in the form of the sum $\psi = \sum_{\alpha=\alpha_1}^{\alpha_2} c_\alpha \psi_\alpha(x)$, where $\psi_\alpha(x)$ is some complete system of basis functions. To determine the coefficients c_α, we compose the system of equations:

$$\sum_{\alpha=\alpha_1}^{\alpha_2} c_\alpha L_{\alpha\beta} = 0, \qquad \text{where} \quad L_{\alpha\beta} = \int_{-\infty}^{+\infty} \psi_\beta L \psi_\alpha \, dx. \tag{3}$$

The requirement that the determinant of system (3) be equal to zero gives an approximate equation for determining the eigenvalues of problem (1)–(2)

$$D(\omega) = \det \|L_{\alpha\beta}\| = 0. \tag{4}$$

Equation (1) can be written in the form $L\psi = \omega^2 A\psi + \omega B\psi + C\psi$, where A, B, and C are self-adjoint operators independent of ω. It follows that the quantities $L_{\alpha\beta} = \omega^2 A_{\alpha\beta} + \omega B_{\alpha\beta} + C_{\alpha\beta}$ are polynomials of the second degree in ω, while the determinant $D(\omega)$ is a polynomial of degree $2(\alpha_2 - \alpha_1 + 1)$.

3. To carry out the calculations it is convenient to pass to dimensionless quantities. The state of the plasma in the case under consideration can be described by three dimensionless parameters: $\nu = \omega_i/\omega_i$, $e = v_E(0)/v_{Ti}^0$, and $h = -3H'(0)x_0/H(0)\rho^2$, where $\rho = r_i/x_0$, and by three functions of order unity: $n(x)$, $p(x) = v_E(x)/v_E(0)$, and $q(x) = H'(x)H(0)/H(x)H'(0)$, characterizing the density and the profiles of the electric and magnetic

Figure 1

Figure 1: Figure 1

fields. Instead of the wave number k and the frequency ω, we introduce the dimensionless variables $\varkappa = kx_0$ and $\Omega = \omega/\omega_i \rho^2 \varkappa$.

In carrying out the computations it was assumed that $n(x) = 2/(1+e^{2\xi})$, $\xi = x/x_0$, and $q(x) = 1$ (the results depend only weakly on the form of this function). Hermite functions were chosen as the basis functions. If the electric field is absent and the magnetic field decreases in the direction of decreasing plasma density ($g > 0$), then for sufficiently small $\varkappa$ (large wavelengths in the direction of the y axis) the oscillations are unstable. As $\varkappa$ increases, the oscillations are stabilized ([3]). The order of magnitude $\varkappa = \hat{\varkappa}$ at which stabilization occurs can be determined with the aid of the "dispersion equation at a point" : $\varkappa \sim \sqrt{8(1+\nu^2)h}$.

The calculations performed were aimed at clarifying the behavior of the quantity $\hat{\varkappa}$ when an electric field appears. As was to be expected, the first oscillation mode (in the direction of the x axis) proved to be the most unstable; therefore all subsequent results are given only for it.

A homogeneous electric field, which causes drift of the plasma as a whole, does not affect its stability. In the plane problem under consideration, slipping of plasma layers relative to one another arises only in the case of an inhomogeneous electric field and is determined by the derivative dE/dx, proportional to $dp/d\xi$. To study the influence of slipping of plasma layers on the behavior of the eigenvalues of problem (1)–(2), the function p was chosen in the form $p = 2e^{2\xi}/(1+e^{2\xi})$. For this profile, Fig. 1A shows the curves of the dependence of the quantity $\hat{\varkappa}$ on the parameter e for various values of the plasma density ($\nu^2 = 0.2;\ \nu^2 = 1;\ \nu^2 = 5$) and $h = 0.04$ (for $\varkappa > \hat{\varkappa}$ the oscillations are stable, for $\varkappa < \hat{\varkappa}$ they are unstable). Similar curves are obtained also for other values of the parameter h.

The graphs shown in Fig. 1A indicate that, for positive gradients of the electric field ($e > 0$), stabilization of the oscillations due to the finiteness of the Larmor radius is weakened and the quantity $\hat{\varkappa}$ sharply

increases. For small negative gradients the stabilization of oscillations of the denser plasma ($\nu \lesssim 1$) is somewhat improved. Since $dE/dx = 4\pi e(N_i(x) - N_e(x))$, this is possible in the case when, in the region

Fig. 1. Dependence of $\hat{\chi}$ on the parameter e for an electric-field profile defined by the functions $p = 2e^{2\xi}/(1+e^{2\xi})$ (A) and $p = \exp(-\xi^2)$ (B); $h = 0.04$

of maximum plasma-density gradients, the electron density slightly exceeds the ion density. With a further increase in $|dE/dx|$, the value of $\hat{\chi}$ again increases.

Fig. 2. Dependence of the increments on the wave number χ for the first

Figure 2

Figure 2: Figure 2

electric-field profile at $\nu = 0.2$, $h = 0.04$, and different values of the parameter e

If, in the region of maximum plasma-density gradients, the derivative dE/dx is small, then the second derivative of the electric field begins to play the main role. In Fig. 1B curves are shown for the dependence of the quantity $\hat{\chi}$ on e for an electric-field profile defined by the function $p = \exp(-\xi^2)$, for different values of the plasma density and $h = 0.04$. For $e > 0$ ($E'' < 0$) the plasma stability deteriorates; for $e < 0$ ($E'' > 0$), in a certain range of values of the parameter e, the stabilization of oscillations increases.

Figure 2 shows curves of the dependence of the increments $\mathrm{Im}(\omega/\omega_i\rho^2)$ on the wave number χ for the first electric-field profile at $\nu = 0.2$, $h = 0.04$, and three values of the parameter e ($e = -0.05,\ e = 0,\ e = 0.05$). In the presence of an electric field with a small negative gradient, the increments decrease somewhat.

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Note: Figure translations are in progress. See original paper for figures.