AUTONOMY IN MULTIVARIABLE AUTOMATIC CONTROL SYSTEMS WITH VARIABLE STRUCTURE

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Abstract

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Certain problems are considered concerning the synthesis of control for multivariable objects whose internal properties determine the presence of finite or differential couplings among the controlled coordinates. It is assumed that the parameters of the controlled object vary within fairly wide limits.

As is known, the problem of synthesizing control for a certain class of multivariable objects consists in choosing such an algorithm for the controlling device that ensures independence of the motion in each of the controlled quantities from changes in the other controlled coordinates, i.e., fulfillment of the autonomy conditions ([1]).

The problem of autonomy is closely connected with the problem of invariance, and in a number of cases, when constructing an autonomous system, the principle of two-channel control ([2]) can be applied, making it possible to determine the form of compensating couplings that ensure autonomy in a multivariable automatic control system.

In the present work it is proposed to solve the autonomy problem by methods of systems with variable structure, i.e., such systems in which the structure and parameters of the regulator change discontinuously during the transient process in accordance with a chosen logical law and depending on the state of the system.

Using the properties of systems with variable structure ([3–5]), we shall construct a multivariable control system that is autonomous in a certain subspace of the phase space of its coordinates, in such a way that the autonomy conditions are insensitive to changes, within fairly wide limits, of the characteristics of the controlled object. Let the free motions in a system consisting of m control loops of the n-th order, interconnected through the controlled object, be described by the following system of differential equations:

$$d\bar{x}_i/dt = \bar{f}_i(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_m), \qquad i = 1, 2, \dots, m, \tag{1}$$

where

$$\bar{x}_i = (x_{1i}, x_{2i}, \dots, x_{ni}); \qquad \bar{f}_i = (f_{1i}, f_{2i}, \dots, f_{ni});$$

$$f_{ji} = x_{(j+1)i}, \quad j = 1, 2, \dots, n-1;$$

$$f_{ni} = -\sum_{j=1}^{n} a_{ji} x_{ji} + \sum_{\substack{r=1 \\ r \neq i}}^{m} \sum_{j=1}^{l_r} b_{jr} x_{jr} - \Phi_i(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_m);$$

$x_{11}, x_{12}, \dots, x_{1m}$ are the controlled quantities; a_{ji}, b_{jr} are the intrinsic parameters of the loops and the coupling parameters, varying in the range

$$a_{ji\,\min} \le a_{ji}(t) \le a_{ji\,\max}, \qquad b_{jr\,\min} \le b_{jr}(t) \le b_{jr\,\max}; \tag{2}$$

Φ_i is the control in the i-th loop. It is assumed that $l_r \le n$.

The phase state of the system under consideration (1) is uniquely characterized by the variables $x_{11}, \dots, x_{n1}, \dots, x_{1i}, \dots, x_{ni}, \dots, x_{1m}, \dots, x_{nm}$. For the subsequent exposition it is convenient to regard them as the coordinates of the vector $\bar{R}$

$$\bar{R} = (x_{11}, \dots, x_{n1}, \dots, x_{1i}, \dots, x_{ni}, \dots, x_{1m}, \dots, x_{nm}).$$

We shall assume that the control functions of each loop undergo discontinuities on certain $(mn-1)$-dimensional linear manifolds (hyperplanes S_i), specified in the mn-dimensional space of the vector $\bar{R}$ by the equations

$$g_i = \sum_{j=1}^{n} c_{ji} x_{ji} = 0, \qquad i = 1, \dots, m. \tag{3}$$

It is obvious that if, in a multiloop system, the chosen control algorithm ensures, in the presence of couplings with respect to the regulated quantities, that the representative point reaches the discontinuity boundary and that the conditions for the existence of a sliding mode are satisfied at any point of the discontinuity boundary, then the subsequent motion will occur along trajectories belonging to the hyperplane S_i and will not depend on the parameters of the plant, on the disturbing forces acting on the plant, or on the internal couplings with respect to the regulated quantities. Therefore, in the case under consideration, it is proposed to create a separate control loop for each of the m quantities and to synthesize the control in such a way that the system always exhibits motion in a sliding mode at any point of the discontinuity boundary of the control functions Φ_i of each of the loops.

If the representative point successively reaches all m hyperplanes S_i, then the transient process terminates on the $m(n-1)$-dimensional manifold

$$\Gamma = \bigcap_{i=1}^{m} S_i,$$

and the motion is described by a system of linear homogeneous differential equations with constant coefficients

$$d\bar{x}_i/dt = \bar{f}_i^0(\bar{x}_i), \qquad i = 1, \dots, m, \tag{4}$$

where

$$\bar{x}_i = (x_{1i}, x_{2i}, \dots, x_{(n-1)i}); \qquad \bar{f}^0 = (f_{1i}^0, f_{2i}^0, \dots, f_{(n-1)i}^0);$$

$$f_{ji}^0 = x_{(j+1)i}, \quad j = 1, 2, \dots, n-2;$$

$$f_{(n-1)i}^0 = -\sum_{j=1}^{n-1} c_{ji} x_{ji}.$$

It is obvious that, from the moment the representative point reaches the manifold Γ, the system becomes autonomous and its dynamic properties are determined only by the previously specified coefficients c_{ji} of the discontinuity boundary.

The motion of the system, beginning at some instant of time, will enter a sliding mode if, in an arbitrarily small neighborhood of each of the discontinuity boundaries S_i, the necessary and sufficient conditions for its existence [6] are satisfied:

$$\bar{c}_i \, d\bar{R}/dt < 0 \quad \text{for } g_i > 0; \qquad \bar{c}_i \, d\bar{R}/dt > 0 \quad \text{for } g_i < 0, \tag{5}$$

where $\bar{c}_i = (0, \dots, \dots, c_{1i}, \dots, c_{ni}, 0, \dots, 0)$ is the normal vector to the hyperplane S_i.

Conditions (5) for the system under consideration have the form

$$\sum_{j=1}^{n-1} (c_{(j-1)i} - a_{ji} - c_{(n-1)i} c_{ji} + a_{ni} c_{ji}) x_{ji} + \sum_{\substack{r=1 \\ r \neq i}}^{m} \sum_{j=1}^{l_r} b_{jr} x_{jr} - \Phi_i(\bar{x}_1, \dots, \bar{x}_m) > 0 \quad \text{for } g_i < 0;$$

(6)

$$\sum_{j=1}^{n-1}(c_{(j-1)i}-a_{ji}-c_{(n-1)i}c_{ji}+a_{ni}c_{ji})x_{ji}+\sum_{\substack{r=1\\r\neq i}}^{m}\sum_{j=1}^{l_r}b_{jr}x_{jr}-\Phi_i(\bar{x}_1,\dots,\bar{x}_m)<0 \quad \text{for } g_i>0.$$

It is obvious from relations (6) that the conditions for the occurrence of a sliding mode depend not only on the controlled quantities and their derivatives, but also on the coupling signals with other loops. Therefore, we shall seek the control function Φ_i in the form of a linear combination of the indicated coordinates with discontinuously varying coefficients

$$\Phi_i(\bar{x}_1,\dots,\bar{x}_m)=\sum_{j=1}^{n-1}\psi_{ji}x_{ji}+\sum_{\substack{r=1\\r\neq i}}^{m}\sum_{j=1}^{l_r}\varphi_{jr}x_{jr}, \tag{7}$$

where

$$\psi_{ji}=\begin{cases}\omega_{ji}, & \text{for } g_i x_{ji}>0,\\ \lambda_{ji}, & \text{for } g_i x_{ji}<0;\end{cases}$$

$$\varphi_{jr}=\begin{cases}\xi_{jr}, & \text{for } g_i x_{jr}>0,\\ \eta_{jr}, & \text{for } g_i x_{jr}<0;\end{cases}$$

$\lambda_{ji},\omega_{ji},\xi_{jr},\eta_{jr}$ are constant quantities.*

It follows from (6) and (7) that, for the existence at any point of the discontinuity boundary S_i $(i=1,\dots,m)$ of a sliding mode, it is necessary and sufficient that

$$\omega_{ji}\geq\max_{a_{ji},a_{ni}}\left(c_{(j-1)i}-a_{ji}-c_{(n-1)i}c_{ji}+a_{ni}c_{ji}\right),$$

$$\lambda_{ji}\leq\min_{a_{ji},a_{ni}}\left(c_{(j-1)i}-a_{ji}-c_{(n-1)i}c_{ji}+a_{ni}c_{ji}\right), \tag{8}$$

$$\xi_{jr}\geq b_{jr\,\max},\qquad \eta_{jr}\leq b_{jr\,\min}.$$

If conditions (8) are satisfied, then from the moment the representative point reaches the discontinuity boundary S_i, the motion in the i-th loop becomes independent of the motions in the other controlled coordinates, and consequently the system of inequalities (8), for $i=1,2,\dots,m$, constitutes the conditions of autonomy for a multiply connected system with variable structure. Since these conditions are based on inequalities, the property of autonomy is preserved when the parameters of the plant vary over the wide range (2).

It should be noted that autonomy in system (1) is achieved not during the entire transient process, but only from the moment sliding motions arise on the manifold T. The discontinuous variation of the coefficients of action through the cross-couplings φ_{jr} (7), until autonomous variation of each of the regulated quantities $x_{11}, \dots, x_{1m}$ is ensured, may in a number of cases lead to an increase in the speed of response of the control process in a multiply connected system with variable structure, as compared with a completely autonomous linear system. It should also be noted that the dynamic properties of the independent (autonomous) processes are completely determined by the coefficients c_{ji} of the system of differential equations (4), which should be selected on the basis of the specified requirements imposed on the control system.

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* Motion along the discontinuity boundaries is determined in accordance with (7).

Note: Figure translations are in progress. See original paper for figures.