ON THE QUESTION OF THE UNITARITY OF THE S-MATRIX IN BROKEN $SL(6)$ SYMMETRY

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Abstract

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ON THE QUESTION OF THE UNITARITY OF THE S-MATRIX IN BROKEN $SL(6)$ SYMMETRY

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In papers ([1], [2]) it was noted that the invariance of the S-matrix with respect to the group $SL(6)$ ([3–9]) (or the group $\tilde{U}(12)$ ([10], [11])) is incompatible with the unitarity condition. However, since the wave equations even for free particles violate $SL(6)$ symmetry, we have no grounds to require invariance of the S-matrix with respect to the group $SL(6)$. We shall show that if the breaking of $SL(6)$ symmetry is taken into account, then the S-matrix is unitary. Moreover, the unitarity condition itself is invariant with respect to the group $SL(6)$ if it contains 36-dimensional momenta instead of the usual 4-momenta.

As was shown in a number of papers, invariance with respect to the group $SL(6)$ requires the existence of 36-dimensional momenta $(P)^{\dot{A}}_{B}$ and $(P)^{A}_{\dot{B}}$, $A = (a\alpha)$, $a = 1, 2, 3$, $\alpha = 1, 2$, which transform as the corresponding spinors of the group $SL(6)$. An invariant, with respect to the group $SL(6)$, wave equation for quarks, for example, has the form

$$(iP)^{\dot{A}}_{B}\varphi^{B} + m\chi^{\dot{A}} = 0, \qquad (iP)^{A}_{\dot{B}}\chi^{\dot{B}} + m\varphi^{A} = 0. \tag{1}$$

If in this equation one makes the replacement

$$(P)^{\dot{A}}_{B} \to (p)^{\dot{a}}_{b}\delta^{\alpha}_{\beta}, \qquad (P)^{A}_{\dot{B}} \to (p)^{a}_{\dot{b}}\delta^{\alpha}_{\beta}, \tag{2}$$

then we obtain the Dirac equation. The matrix elements of scattering processes and vertex functions explicitly contain not only the wave functions of the initial and final particles, but also their momenta*. If 36-dimensional momenta are introduced instead of the usual 4-momenta, then the matrix elements are invariant with respect to the group $SL(6)$, while after the replacement (2) the $SL(6)$ symmetry is broken.

As an example let us consider the scattering of a neutral scalar meson (singlet) on a quark. According to the proposed method, we first consider the 4-momenta of the mesons and quarks in the initial and final states k_1, p_1, k_2, p_2, respectively, as components of tensors $(K_i)^{\dot{A}}_{B}$, $(P_i)^{\dot{A}}_{B}$ and $(K_i)^{A}_{\dot{B}}$, $(P_i)^{A}_{\dot{B}}$. The pair of tensors K_i is equivalent to a system of nine 4-vectors $(k_i)^j_\mu$ and nine 4-pseudovectors $(l_i)^j_\mu$, $j = 0, 1, \dots, 8$, $\mu = 1, 2, 3, 4$, while the pair of tensors P_i is a system of 4-vectors $(p_i)^j_\mu$ and 4-pseudovectors $(q_i)^j_\mu$:

$$(K_i)^{\dot{A}}_{B} = \frac{1}{\sqrt{2}} \sum_{\mu=1}^{4} \sum_{j=0}^{8} (\lambda_j)^{\alpha}_{\beta} \left[(k_i)^j_\mu + (l_i)^j_\mu \right] (\sigma_\mu)^{\dot{a}}_{b}, \tag{3}$$

$$(K_i)^{A}_{\dot{B}} = \frac{1}{\sqrt{2}} \sum_{\mu=1}^{4} \sum_{j=0}^{8} (\lambda_j)^{\alpha}_{\beta} \left[(k_i)^j_\mu - (l_i)^j_\mu \right] (\sigma_\mu)^{a}_{\dot{b}},$$

* Terms explicitly containing momenta are called in paper ([12]) nonregular. The existence of such nonregular structures in matrix elements was noted for the first time in paper ([7]) by one of the authors, where the so-called spurion formalism of the theory of broken $SL(6)$ symmetry was proposed, and then independently in paper ([12]). In paper ([8]), devoted to the study of the structure of vector and axial currents, contributions from all nonregular structures are taken into account.

and analogously for the tensors P_i. The quadratic invariant for the momentum K, for example, is equal to

$$\frac{1}{3} (K)^{A}_{\dot{B}} (K)^{\dot{B}}_{A} = \sum_j \left[(k^j)^2 - (l^j)^2 \right] = \sum_{j,\mu} \{ k^j_\mu k^j_\mu - l^j_\mu l^j_\mu \}, \tag{4}$$

the invariant volume element is

$$\frac{1}{(2)^{36}} \prod_{A,\dot{B}} d(K)^{A}_{\dot{B}} \prod_{\dot{C},D} d(K)^{\dot{C}}_{\dot{D}} = \prod_{j=0}^{8} d^4 k^j \, d^4 l^j, \tag{5}$$

and the invariant δ-function has the form

$$(2)^{36} \delta^{36}(K^{A}_{\dot{B}}) \delta^{36}(K^{\dot{A}}_{B}) = \delta^{36}(k^j) \delta^{36}(l^j). \tag{6}$$

The matrix element of the process under consideration, containing the 36-momenta K_i and P_i and invariant with respect to the group $SL(6)$, may be written as follows:

$$M_{fi} = (2\pi)^{72}\delta^{36}(p_1^j + k_1^j - p_2^j - k_2^j)\delta^{36}(q_1^j + l_1^j - q_2^j - l_2^j)\times$$
$$\times\Big\{A(P_2, K_2; P_1, K_1)\left[\bar\varphi(P_2)_{\dot A}\chi(P_1)^{\dot A} + \bar\chi(P_2)_A\varphi(P_1)^A\right] +$$
$$+ B(P_2, K_2; P_1, K_1)\left[\bar\varphi(P_2)_{\dot A}\left(i\frac{K_1 + K_2}{2m}\right)^{\dot A}_{B}\varphi(P_1)^B + \right. \tag{7}$$
$$\left.+\bar\chi(P_2)_A\left(i\frac{K_1 + K_2}{2m}\right)^{A}_{\dot B}\chi(P_1)^{\dot B}\right]\Big\}.$$

The phase volume, invariant with respect to the group $SL(6)$, for example, for a meson with mass μ, is equal to

$$\frac{1}{(2\pi)^{72}}\sum_{j=0}^{8} d^4k^j\, d^4l^j\, 2\pi\delta\left(\sum_j (k^j)^2 - \sum_j (l^j)^2 + \mu^2\right). \tag{8}$$

The unitarity condition in the two-particle approximation has the form:

$$\operatorname{Im} A(P_2, K_2; P_1, K_1)\left[\bar\varphi(P_2)_{\dot A}\chi(P_1)^{\dot A} + \bar\chi(P_2)_A\varphi(P_1)^A\right] +$$
$$+ \operatorname{Im} B(P_2, K_2, P_1, K_1)\left[\bar\varphi(P_2)_{\dot A}\left(i\frac{K_1 + K_2}{2m}\right)^{\dot A}_{B}\varphi(P_1)^B + \right.$$
$$\left.+\chi(P_2)_A\left(i\frac{K_1 + K_2}{2m}\right)^{A}_{\dot B}\chi(P_1)^{\dot B}\right]$$
$$= \frac{m}{(2\pi)^{72}}\int d^4p^j\, d^4q^j\, d^4k^j\, d^4l^j\, \delta^{36}(p^j + k^j - p_1^j - k_1^j)\times$$
$$\times\,\delta^{36}(q^j + l^j - q_1^j - l_1^j)(2\pi)^2\delta\left(\sum(p^j)^2 - \sum(q^j)^2 + m^2\right)\times$$
$$\times\,\delta\left(\sum(k^j)^2 - \sum(l^j)^2 + \mu^2\right)\sum_r\Big\{A^*(P, K; P_2, K_2)\left[\bar\varphi(P_2)_{\dot A}\chi^r(p)^A + \right.$$
$$\left.+\bar\chi(P_2)_A\varphi^r(P)^A\right] + B^*(P, K; P_2, K_2)[...]\Big\}\Big\{A(P, K; P_1, K_1)\times$$
$$\times\left[\bar\varphi^r(P)_{\dot A}\chi(P_1)^{\dot A} + \bar\chi^r(P)_A\varphi(P_1)^A\right] + B(P, K; P_1, K_1)[...]\Big\}. \tag{9}$$

For summation over the polarizations of the intermediate states we use the formula

$$\sum_r \bar\varphi^r_{\dot A}\varphi^{rB} = \frac{1}{2}\left(-\frac{iP}{m}\right)^{B}_{\dot A}, \qquad \sum_r \bar\chi^r_{A}\chi^{r\dot B} = \frac{1}{2}\left(-\frac{iP}{m}\right)^{\dot B}_{A},$$

$$\sum_r \bar{\varphi}^r_{\dot{A}} \chi^{r\dot{B}} = \frac{1}{2}\delta^{\dot{B}}_{\dot{A}}, \qquad \sum_r \bar{\chi}^r_A \varphi^{rB} = \frac{1}{2}\delta^B_A. \tag{10}$$

It is obvious that the unitarity condition (9) is invariant with respect to the group $SL(6)$.

The physical scattering amplitude is obtained from the invariant amplitude (7) by the replacement

$$A(P_2, K_2; P_1, K_1) \to \prod_{j=1}^{8}(2\pi)^4\delta^4(p_1^j - p_2^j)\prod_{j=0}^{8}(2\pi)^4\delta^4(q_1^j - q_2^j)\theta[(p_i)_0^0]\, A(s,t) \tag{11}$$

and similarly for B, while for the momenta of the initial particles we make the replacement (2), i.e., we retain only the physical 4-momenta $(p_i)^0_\mu$ and $(k_i)^0_\mu$. Then the unitarity condition (9), together with formula (10), gives the usual unitarity condition

$$\begin{aligned} \operatorname{Im} A(s,t) + \operatorname{Im} B(s,t)\, i\,\frac{\hat{k}_1 + \hat{k}_2}{2m} = \frac{1}{(2\pi)^2}\int d^4p\, d^4k\, \theta(p^0)\theta(k^0)\delta(p^2+m^2)\delta(k^2+\mu^2)\times \\ \times\left[A^+(s,t'') + B^+(s,t'')\, i\,\frac{\hat{k}_2 + \hat{k}}{2m}\right]\frac{m - i\hat{p}}{2} \\ \times\left[A(s,t') + B(s,t')\, i\,\frac{\hat{k}_1 + \hat{k}}{2m}\right]. \end{aligned} \tag{12}$$

Thus, if 36-dimensional momenta are used instead of the usual 4-momenta, the theory is completely invariant with respect to the group $SL(6)$: the wave equations, the S-matrix, and the unitarity condition are invariant. In passing to physical amplitudes, however, the symmetry is broken: the wave equations, the S-matrices, and the unitarity condition are not invariant. The authors of a number of papers have required strict invariance for the physical amplitude, which in the case under consideration leads to the condition $B(s,t) \equiv 0$.

It is obvious that this incorrect requirement leads to a contradiction with the unitarity condition, and there is no question of the compatibility of the group $SL(6)$ with the unitarity condition.

In conclusion, we note that for the given process of scattering of a singlet meson by a quark, the broken $SL(6)$ symmetry gives nothing new in comparison with unitary symmetry. However, for higher representations the broken $SL(6)$ symmetry gives new consequences. Thus, for example, in unitary symmetry the matrix elements of the vector and axial currents for the baryon octet depend on

12 form factors, while in the broken $SL(6)$ symmetry these 12 form factors are expressed in terms of 8 independent form factors*, if all nonregular structures are taken into account (8). Moreover, the form factors for the decuplet and the form factors of transitions between the octet and the decuplet are also expressed in terms of these 8 independent form factors.

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* In papers (6, 9), nonregular structures are not taken into account; therefore all form factors are expressed in terms of 5 independent ones.

Note: Figure translations are in progress. See original paper for figures.