PRODUCTION OF A NEUTRINO PAIR DURING THE MOTION OF AN ELECTRON IN A MAGNETIC FIELD

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Abstract

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PRODUCTION OF A NEUTRINO PAIR DURING THE MOTION OF AN ELECTRON IN A MAGNETIC FIELD

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1. During the motion of charged particles in a magnetic field, not only the emission of photons but also the production of particles may occur. In particular, during the motion of an electron in a magnetic field there may take place processes of formation of electron–positron pairs (conversion of an emitted photon into a pair), as well as processes caused by the weak interaction of the electron (emission of a neutrino pair, inverse μ-decay, etc.). These processes may be of interest, in particular, in astrophysics.*

2. We shall consider the emission of a neutrino pair during the motion of an electron in a magnetic field, under the assumption that a direct interaction of the electron and the neutrino takes place. The treatment is carried out within the framework of the universal $V-A$ variant of the theory of the weak interaction.

We write the probability of the process per unit time in the form

$$dW = \frac{1}{(2\pi)^5}\,|u_{if}|^2\delta(E-E'-k_{10}-k_{20})\frac{d^3k_1}{2k_{10}}\frac{d^3k_2}{2k_{20}}, \qquad (1)$$

where

$$u_{if} = -\frac{G}{\sqrt{2}}L_\alpha M^\alpha; \qquad (2)$$

$$L_\alpha = \int d^3x\,(\bar{\psi}_{ef}O_\alpha\psi_{ei})e^{-(\mathbf{k}_1+\mathbf{k}_2)\mathbf{r}}, \qquad M_\alpha = (\bar{u}_\nu(\mathbf{k}_1)O_\alpha\vartheta_\nu(\mathbf{k}_2)); \qquad (3)$$

$\mathbf{k}_1$ and $\mathbf{k}_2$ are the momenta of the neutrino and antineutrino; ψ_e is an orthonormal system of electron functions in the magnetic field. Performing the summation over spins and the integration over the momenta of the produced neutrino–antineutrino pair (see, for example, (2)), we obtain**

$$S_f \int M_\alpha M_\beta^* \frac{d^3k_1}{k_{10}} \frac{d^3k_2}{k_{20}} \delta(E - E' - k_{10} - k_{20}) = \frac{16\pi}{3} \int d^3q \, (q_\alpha q_\beta - q^2 g_{\alpha\beta}), \quad (4)$$

where $\mathbf{q} = \mathbf{k}_1 + \mathbf{k}_2$; $q_0 = k_{10} + k_{20} = E - E'$.

The integrals L_α (3) are known from the quantum theory of radiation by electrons in a magnetic field (see, for example, (3)); they are expressed in terms of the functions $I_{n,n'}(x)$

$$I_{n,n'}(x) = \sqrt{\frac{n'!}{n!}} \, e^{-x/2} x^{(n-n')/2} L_{n'}^{n-n'}(x), \quad (5)$$

where $x = q^2 \sin^2 \vartheta / 4\eta$. Here an assumption has been made, without loss of generality, that the vector $\mathbf{q}$ lies in the (y, z) plane; ϑ is the angle between the z axis and the vector $\mathbf{q}$. In what follows it is assumed that in the initial state $p_{zi} = 0$.

* The processes of pion emission and β-decay of the proton in a magnetic field were considered recently by Zharkov (1).

** The metric $(ab) = a_0 b_0 - \mathbf{a} \cdot \mathbf{b}$, $\hbar = c = 1$ is used.

Having carried out the averaging over the spin of the initial electron and the summation over the spin of the final one, we obtain the following expression for the probability

$$W = \frac{G^2}{3(2\pi)^4} \int d^3q \, [q_0^2 H_{00} + \mathbf{q}^2 (\sin^2 \vartheta H_{22} + \cos^2 \vartheta H_{33}) -$$

$$-2q_0 |\mathbf{q}| (H_{20} \sin \vartheta + H_{30} \cos \vartheta) + 2q^2 \cos \vartheta \sin \vartheta H_{23} -$$

$$-q^2 (H_{00} - H_{11} - H_{22} - H_{33})], \quad (6)$$

where

$$H_{\alpha\beta} = \frac{1}{4} S_{iS} f (L_\alpha L_\beta^* + L_\beta L_\alpha^*). \quad (7)$$

To obtain the total probability of neutrino-pair production it is necessary to sum over the states of the final electron and to perform the integration in formula (6). We note that the summation over the momentum component p_{zj} and the quantum number s'*, as well as the integration over the azimuthal angle of the vector $\mathbf{q}$, is carried out trivially.

In performing the summation over the quantum numbers n' of the final electron, we pass, as usual, to integration. In doing so it proves convenient to introduce a new variable (see, for example, [5])

$$n - n' = n\frac{2\alpha}{1+\alpha}\left(1 - \frac{\alpha}{2(1+\alpha)}\beta^2\sin^2\vartheta\right). \tag{8}$$

3. It is obvious (cf., for example, [6]) that the invariant characteristics of the process (for example, the radiation intensity) for a particle in an external electromagnetic field, after summation over final states and averaging over the spins of the initial state, can depend only on the field $F_{\mu\nu}$ and the vector p_μ. From these quantities one can construct the following dimensionless invariants:

$$\chi^2 = -e^2 F_{\mu\nu}F^{\mu\alpha}p^\nu p_\alpha/m^6, \qquad f^2 = e^2 F_{\mu\nu}F^{\mu\nu}/m^4, \qquad g^2 = e^2\varepsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta}F^{\gamma\delta}/m^4.$$

In an external magnetic field $\chi = \gamma\beta H/H_0$ ($p_z = 0$), $f = H/H_0$, $g = 0$, where the critical field $H_0 = m^2/e = 4.4\cdot 10^{13}$ oersted. This field exceeds by many orders of magnitude all fields known at present. We shall consider the case of ultrarelativistic electrons; then $\chi \gg f$.

Since $\gamma \gg 1$, in what follows we shall systematically expand all quantities in powers of $1/\gamma$ and retain the leading terms of the expansion. To the indicated accuracy

$$q_0 = E\beta^2\frac{\alpha}{1+\alpha}. \tag{9}$$

We also introduce a new variable ρ, related to $\mathbf{q}^2$ by the relation

$$\rho = \frac{1}{1+\alpha}\left(1 - \frac{\mathbf{q}^2}{q_0^2}\right). \tag{10}$$

We note that, with the adopted accuracy, $0 \le \alpha \le \infty$, $0 \le \rho \le 1$.

For carrying out the subsequent calculations it is convenient to express the functions $I_{n,n'}$, $I_{n-1,n'}$, $I_{n,n'-1}$, $I_{n-1,n'-1}$ entering formula (6) through $I_{n,n'}$ and $I'_{n,n'}$. We shall use the known quasiclassical asymptotic expressions for $I_{n,n'}$ and $I'_{n,n'}$ in terms of the cylindrical functions K (see, for example, [4])

$$I_{n,n'}(x) = \frac{1}{\pi\sqrt{3}}(1+\alpha)^{1/2}\tau^{1/2}K_{1/3}\left(\frac{2}{3}n\alpha\tau^{3/2}\right), \tag{11}$$

$$I'_{n,n'}(x) = \frac{1}{\pi\sqrt{3}}\frac{(1+\alpha)^{3/2}}{\alpha}\tau K_{2/3}\left(\frac{2}{3}n\alpha\tau^{3/2}\right). \tag{12}$$

* Regarding the summation over s' see, for example, [4].

This representation is valid in the essential range of variation of the argument for $n \gg 1$, $n' \gg 1$, $(1+\alpha)\tau \ll 1$, where

$$\tau = \frac{1}{\gamma^2} + \cos^2\vartheta + \rho. \tag{13}$$

The main contribution to the transition probability is given by the region $n\alpha\tau^{3/2} \lesssim 1$. If one takes into account that $n' \simeq n/(1+\alpha)^2$, it follows from this that the condition $n' \gg 1$ can be satisfied only if $f = H/H_0 \ll 1$. In this case the electron in the final state remains ultrarelativistic. Thus the expressions obtained by us will represent functions of χ. As for the dependence on $f = H/H_0$, the whole approach is valid only for $f \ll 1$, and we shall retain the zeroth term of the expansion in f. In this sense, for the given problem the range of applicability of the method used is the same as that of the Nikishov-Ritus method (6), where $f = 0$.

Analysis of the expression for the probability shows that the main contribution is given by the regions $\cos\vartheta \sim 1/\gamma$, $\rho \sim 1/\gamma^2$ for $\chi \ll 1$, and $\cos\vartheta \sim 1/\gamma\chi^{1/3}$, $\rho \sim 1/\gamma^2\chi^{2/3}$ for $\chi \gg 1$. Taking this into account, one can expand the expression for the probability in powers of $1/\gamma(\chi^{1/3}/\gamma)$; it then turns out that the terms of higher order in γ mutually compensate, so that only terms $\sim \gamma^{-4}$ remain. After the indicated transformations we obtain the following expressions for the probability:

$$W = \frac{2}{9}\frac{1}{(2\pi)^5}G^2E^6R\int_0^1 d\rho\int_{-1}^1 d\cos\vartheta\int_0^\infty \frac{\alpha^5}{(1+\alpha)^6}\left[F_{1/3}K_{1/3}^2\left(\frac{2}{3}n\alpha\tau^{3/2}\right) + F_{2/3}K_{2/3}\left(\frac{2}{3}n\alpha\tau^{3/2}\right)\tau\right]d\alpha; \tag{14}$$

$$F_{1/3} = 2(\alpha^2+2\alpha+1)\tau^2 + (\alpha^3-\alpha^2-4\alpha-2)\mu\tau +$$

$$+2(\alpha^2+2\alpha+1)\frac{\tau}{\gamma^2} - (3\alpha^2+2\alpha+1)\frac{\mu}{\gamma^2} - (\alpha^3-\alpha^2)\mu^2,$$

$$F_{2/3} = (\alpha^3+3\alpha^2+4\alpha+2)\tau - (\alpha^3+\alpha^2+4\alpha+2)\mu + (1+2\alpha-\alpha^2)\frac{1}{\gamma^2}, \tag{15}$$

$$\mu = \frac{1}{\gamma^2} + \cos^2\vartheta.$$

To calculate the integral over α in the expression for the probability (14), we use the representation (see, for example, (7))

$$\frac{1}{(1+\alpha)^m} = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} \frac{\Gamma(-s)\Gamma(s+m)}{\Gamma(m)}\alpha^s\, ds. \tag{16}$$

Then the integral over α is easily evaluated. The integrations over the variables ρ and $\cos\vartheta$ are elementary, so that only integration over s remains. This integration is easily carried out for the cases $\chi \gg 1$, $\chi \ll 1$. For $\chi \ll 1$ the contour of integration may be closed to the right, and the integral reduces to the residues at the corresponding poles; in this case we obtain an expansion in powers of χ. For $\chi \gg 1$ the contour of integration may be closed to the left and, taking the residues at the poles, we obtain an expansion in $\chi^{-1/3}$. We write out the principal terms of the expansion in both cases:

$$W = \frac{7\cdot 17\, G^2 m^5}{4\cdot 9\sqrt{3}(2\pi)^3}\chi^5 \qquad (\chi \ll 1); \tag{17}$$

$$W = \frac{2G^2m^5}{(6\pi)^3\gamma}\left(\ln\chi - C - \frac{1}{2}\ln 3 + \frac{11}{8}\right)\chi^2 \qquad (\chi \gg 1), \tag{18}$$

where C is Euler’ s constant, $C = 0.577$.

4. To calculate the expression for the intensity of neutrino-pair production it is necessary to perform a summation over the energies of the final particles with

with weight $q_0 = E\alpha/(1+\alpha)$. The method of calculation is completely analogous to that given above. Then, with the indicated accuracy, we obtain:

$$I = 2\left(\frac{5}{6\pi}\right)^3 G^2m^6\chi^6 \qquad (\chi \ll 1); \tag{19}$$

$$I = \frac{2G^2m^6}{(6\pi)^3}\left(\ln\chi - C - \frac{1}{2}\ln 3 + \frac{71}{80}\right)\chi^2 \qquad (\chi \gg 1). \tag{20}$$

(We note that in the case $\chi \ll 1$ the quantities W and I can be obtained especially simply by method II of work 3.)

In the case $\chi \ll 1$, the spectrum of radiation of neutrino pairs is the same as that of photons; the mean angle of the vector $\mathbf{q}$ with the (x, y) plane is $\sim 1/\gamma$, and the mean angle between the vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ has the same magnitude.

In the case $\chi \gg 1$, the energy of the emitted neutrinos is of the order of the electron energy; the mean angle between $\mathbf{q}$ and the (x, y) plane is of the order $1/\gamma\chi^{1/3}$, and the mean angle between the vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ is of the order $1/\gamma\chi^{1/2}$.

5. Let us compare the results obtained, (19), (20), with the intensity of photon radiation.

In the case $\chi \ll 1$,

$$\frac{I_\nu}{I_\gamma} \simeq 3\left(\frac{5}{6\pi}\right)^3 \frac{G^2 m^4}{\alpha}\chi^4 \simeq 10^{-22}\chi^4; \tag{21}$$

thus, the effect of radiation of neutrino pairs turns out to be suppressed not only because of the smallness of the constant $G^2 m^4$, but also because of the dependence on a high power of χ.

In the case $\chi \gg 1$,

$$\frac{I_\nu}{I_\gamma} \simeq \frac{1}{(4\pi)^3}\frac{3^{4/3}}{2\Gamma(2/3)}\frac{G^2 m^4}{\alpha}\chi^{4/3}\ln\chi \simeq 10^{-24}\chi^{4/3}\ln\chi; \tag{22}$$

whence it follows that the intensities of radiation of neutrino pairs and photons become comparable at $\chi \sim 10^{16}$ in a field $H \sim 10^4$ oersted; this corresponds to the monstrous electron energy $E \sim 10^{32}$ eV.

Let us note that the probability of inverse μ-decay at $\chi \ll 1$ will contain an additional exponential $\exp(-\mathrm{const}/\chi(m_\mu/m_e)^2)$, and the effect proves still smaller; at $\chi \gg 1$ the probability of the process does not exceed the probability (18).

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Note: Figure translations are in progress. See original paper for figures.