Reports of the Academy of Sciences of the USSR

1966

SovietRxiv

Abstract

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Reports of the Academy of Sciences of the USSR

1966. Volume 166, No. 5

PHYSICS

A. A. IVANOV

ON THE STABILIZATION OF DRIFT INSTABILITY IN TRAPS WITH A MAGNETIC FIELD INCREASING TOWARD THE BOUNDARY

(Presented by Academician E. K. Zavoisky, 22 VI 1965)

It is known that the flute instability does not build up in traps with a magnetic field increasing toward the plasma boundary when $\beta = 8\pi P/H^2 \ll 1$, since it is energetically unfavorable for particles to pass into the region of stronger magnetic fields. It was natural to suppose that, for the same reason, a magnetic field increasing toward the plasma boundary should also stabilize the drift instability ([2]). We shall show, however, that this does not occur.

In order to isolate the effect of interest to us, let us consider the case in which the electron temperature T_e is much greater than the ion temperature T_i, since already at $T_e = T_i$ even a small shear of the lines of force stabilizes the drift instability ([3]). We choose the simplest geometry: a rarefied plasma $\beta \ll 1$ in the field of a straight infinite filament, with a dielectric cylinder serving as the outer boundary of the plasma. We shall consider oscillations whose phase velocity along the magnetic field $v_{\|\phi}$ is much smaller than the Alfvén velocity $v_A = H/\sqrt{4\pi n_0 M}$, where n_0 is the plasma density and M is the ion mass. In this case the electric field of the oscillations may be regarded as potential, $\mathbf{E} = -\nabla\varphi$ ([4]). In addition, we introduce the following simplifying assumption: $v_{\|\phi} \gg c_s$, where c_s is the ion-sound velocity. When this condition is fulfilled, the motion of ions along the field may be neglected. Finally, we shall assume that the plasma pressure is not too small, $m/M < \beta \ll 1$, where m is the electron mass and M is the ion mass.

Under the assumptions made, drift waves and flute oscillations can be obtained most simply from the simultaneous solution of the following system of equations: the equations of motion and continuity for the ion component:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = -\frac{e}{M}\nabla\varphi + \frac{e}{Mc}[\mathbf{vH}], \tag{1}$$

$$\frac{\partial n_i}{\partial t} + \operatorname{div}(n_i \mathbf{v}) = 0; \tag{2}$$

the drift kinetic equation for the electrons ([5])

$$\frac{\partial f}{\partial t} + \dot{\mathbf{R}}\frac{\partial f}{\partial r} + \frac{dv_\parallel}{dt}\frac{\partial f}{\partial v_\parallel} = 0, \tag{3}$$

$$f_0 = \frac{m}{T_\perp}\left(\frac{m}{2\pi T_\parallel}\right)^{1/2} n_0 \exp\left[\frac{m\mu H}{T_\perp} - \frac{mv_\parallel^2}{2T_\parallel}\right]$$

and the condition of quasineutrality

$$n_i = \int fH\, dv_\parallel\, d\mu. \tag{4}$$

Here $\mu = v_\perp^2/2H$ is the transverse adiabatic invariant; n_i is the ion density; $T_\perp$ and $T_\parallel$ are the transverse and longitudinal temperatures of the electro-

In our case $\operatorname{rot}\mathbf{H} = 0,\ H_z = H_r = 0,\ H_\theta \sim 1/r.$ Therefore

$$\dot{\mathbf{R}} = v_\parallel \mathbf{e}_\theta + \frac{c}{H}[\mathbf{E}\mathbf{e}_\theta] + \frac{H'}{H}\frac{1}{\omega_e}(\mu H + v_\parallel^2)\mathbf{e}_z,$$

$$\frac{dv_\parallel}{dt} = -\frac{e}{m}E_\theta - \frac{e}{m\omega_e}v_\parallel E_z\frac{H'}{H}, \tag{5}$$

where $\mathbf{e}_\theta$ and $\mathbf{e}_z$ are unit vectors along the magnetic field and along the z-axis, respectively; $\omega_e = eH/mc,\ H' = dH/dr.$

Solving equations (1) and (2) by the method of successive approximations, expanding in the small parameter $\omega/\omega_i \ll 1,\ \omega_i = eH/Mc$, and omitting nonlinear terms, we find

$$\frac{\partial n_i}{\partial t} = -\operatorname{div}\left\{n_i\frac{c}{H}[\mathbf{e}_\theta\nabla\varphi] + \frac{Mc^2}{eH^2}\left[\mathbf{e}_\theta\left[\mathbf{e}_\theta\frac{\partial\nabla\varphi}{\partial t}\right]\right]\right\}. \tag{6}$$

Let us consider the time behavior of perturbations of the stationary state of the form

$$A_{km}e^{ik_z z+im\theta-i\omega t}.$$

Then from equations (3), (4), (5), (6) we obtain the dispersion relation

$$\frac{Mc^2}{e}\frac{1}{r}\frac{d}{dr}\left(\frac{n_0 r}{H}\frac{d\varphi_{km}}{dr}\right)-k_z^2\frac{Mc^2}{eH^2}n_0\varphi_{km}+\frac{k_z}{\omega}c\frac{1}{r}\frac{d}{dr}\left(\frac{n_0 r}{H}\right)\varphi_{km}-$$

$$-\int H\frac{\frac{c}{H}\frac{\partial f_0}{\partial r}+\frac{e}{m}\frac{\partial f_0}{\partial v_\parallel}\left(k_\parallel+k_z\frac{H'}{H}\frac{1}{\omega_e}v_\parallel\right)}{\omega-k_\parallel v_\parallel-k_z\frac{1}{\omega_e}(\mu H+v_\parallel^2)\frac{H'}{H}}\varphi_{km}\,dv_\parallel\,d\mu. \tag{7}$$

The denominator of the integrand in equation (7) vanishes at two points v_1 and v_2, where $v_1\approx\omega/k_\parallel-\mu H'k_z/\omega_e k_\parallel\ll u_e$; $v_2\sim u_e^2k_\parallel/\omega\gg u_e\gg\omega/k_\parallel$; $u_e=(2T_\parallel/m)^{1/2}$. Therefore, in integrating with respect to $v_\parallel$ one may neglect the residue at the point v_2, since it is exponentially small. Finally we have

$$\rho_H^2\frac{H^2}{n_0 r}\frac{d}{dr}\left(\frac{n_0 r}{H^2}\frac{d\varphi_{km}}{dr}\right)-\rho_H^2k_z^2\varphi_{km}+\frac{k_z}{\omega}\frac{cT_\parallel}{eH}\frac{d\ln(n_0r/H)}{dr}\varphi_{km}-$$

$$-\varphi_{km}+\frac{i\sqrt{\pi}}{|k_\parallel|u_e}\left(\frac{cT_\parallel}{eH}k_z\frac{d\ln(n_0/H)}{dr}-\omega+\frac{cT_\perp}{eH}k_z\frac{d\ln H}{dr}\right)\varphi_{km}=0,$$

$$\rho_H^2=\frac{Mc^2T_\parallel}{e^2H^2}.$$

Solving this equation by the WKB method (3), i.e., assuming that $Rk_r\gg 1$ and $\frac{R}{k_r}\frac{dk_r}{dr}\ll 1$, where R is the characteristic scale of the inhomogeneity,

$$k_r=\left(-k_z^2\rho_H^2+\frac{k_z}{\omega}\frac{cT_\parallel}{eH}\frac{d\ln(n_0r/H)}{dr}-1\right)^{1/2}\Big/\rho_H,$$

we obtain, using the smallness of the quantities $\omega/k_\parallel u_e$, γ/ω:

$$\pi l=\int_a^b\frac{1}{\rho_H}\left(-k_z^2\rho_H^2+\frac{k_z}{\omega}\frac{cT_\parallel}{eH}\frac{d\ln(n_0r/H)}{dr}-1\right)^{1/2}dr,$$

$$\gamma=\frac{\sqrt{\pi}}{u_e}\omega^2\frac{\displaystyle\int_a^b\left(\frac{cT_\parallel}{eH}k_z\frac{d\ln(n_0r/H)}{dr}\frac{1}{\omega}-1+\frac{cT_\perp}{eH}\frac{H'}{H}k_z\frac{1}{\omega}\right)dr\Big/\rho_H^2k_r^2|k_\parallel|}{\displaystyle\int_a^b(1+k_\perp^2\rho_H^2)\,dr\Big/k_r^2\rho_H^2}. \tag{8}$$

Here l is an integer, $l \gg 1$; a and b are turning points determined from the condition $k_r = 0$. For sufficiently small r, when $k_z^2\rho_H^2 \ll 1$, the point a is determined from the condition

$$1 \simeq \frac{cT_\parallel}{eH}\,\frac{d\ln(n_0 r/H)}{dr}\,\frac{k_z}{\omega};$$

the second turning point lies in the region where $k_z^2\rho_H^2 \gg 1$ and is determined from the condition

$$k_z^2\rho_H^2 \simeq \frac{k_z}{\omega}\,\frac{cT_\parallel}{eH}\,\frac{d\ln(n_0 r/H)}{dr}.$$

It follows from equations (8) that $\gamma > 0$ when

$$\omega\left(\frac{cT_\parallel}{eH}\,k_z\,\frac{d\ln(n_0 r/H)}{dr} - \omega - \frac{cT_\perp}{eE}\,k_z\,\frac{H'}{H}\right) > 0. \tag{9}$$

In the case $T_\parallel = T_\perp$, the excitation condition (9) takes the form

$$\omega\left(\frac{cT_\parallel}{eH}\,k_z\,\frac{d\ln n_0}{dr} - \omega\right) > 0,$$

i.e., the same condition is obtained as for

$$\frac{d\ln P}{d\ln r}\Bigg/\frac{d\ln H}{d\ln r} = \frac{1}{\beta} \gg 1 \text{ [3]}.$$

From the first equation of the system (8) one obtains an upper estimate for the frequency

$$\omega \ll \frac{cT_\parallel}{eH}\,k_z\,\frac{d\ln(n_0 r/H)}{dr}.$$

Substituting it into the second equation, we obtain a sufficient condition for instability in the case $T_\parallel = T_\perp$ (7)

$$k_z^2\rho_H^2 > \frac{d\ln(r/H)}{d\ln n_0}. \tag{10}$$

It follows from condition (10) that short waves are excited at arbitrary magnetic-field gradients. Thus, a magnetic field increasing toward the plasma boundary does not lead to stabilization of drift waves, as in the case of flute oscillations.

To clarify the physical meaning of this difference, let us consider the work of resonant particles in changing the wave energy ε_{km} per unit time. By definition,

$$\frac{\partial\varepsilon_{km}}{\partial t} = -\frac{1}{2}\left(j_{km}E^*_{km} + \text{c.c.}\right),$$

where j_{km} is the current density of the resonant particles, and c.c. denotes the complex-conjugate quantity. Since the resonant particles are electrons moving along the field $\mathbf{H}$ with velocity

$$v_1 = \omega/k_\parallel - \mu H' k_z/\omega_e k_\parallel$$

and along the z-axis with velocity v_z, we have

$$\frac{\partial \varepsilon_{km}}{\partial t} = \frac{1}{2} e \int \left(E^*_{\parallel km} v_1 + E^*_{zkm} v_z + \text{c.c.} \right) \delta n_{km}(v_1, \mu)\, H\, d\mu, \tag{11}$$

where $H \delta n_{km}(v_1, \mu)$ is the density of resonant particles in the interval $d\mu$. The second term on the right-hand side of equality (11) must lead to damping of the perturbation, since it is associated with the work of moving the resonant particle into a region of larger magnetic field. Indeed,

$$e E^*_{zkm} v_z \tau = -e \frac{\mu H'}{\omega_e} E^*_{zkm} \tau = -m\mu H' \frac{c}{H} E^*_{zkm} \tau = m\mu \Delta H,$$

where ΔH denotes the change in the magnetic field for a displacement

$$\Delta r = -\frac{c}{H} E^*_{zkm} \tau,$$

equal to the displacement of the resonant particle in the wave field during the time of interaction of the particle with the wave, $\tau \sim 1/\gamma$. The increase in the energy of the oscillations occurs at the expense of the energy of the thermal motion of the resonant electrons along the magnetic field. In a magnetic field increasing toward the boundary, the velocity of the resonant electrons v_1 is greater than $\omega/k_\parallel$ by the amount

$$-\frac{k_z}{k_\parallel} \frac{\mu H'}{\omega_e}.$$

Therefore, the resonant particle in the wave field performs a larg–

work than in the absence of the increasing magnetic field; moreover, the excess work exactly compensates the energy expended by the wave in displacing the resonant particle into the region of the stronger magnetic field, i.e.,

$$\frac{\partial \varepsilon_{km}}{\partial t} = e \frac{\omega}{k_\parallel} \int \delta n_{km}(v_1, \mu) H\, d\mu. \tag{12}$$

From formula (12) the excitation condition (9) follows.

Thus, for $T_\parallel = T_\perp$ there will always be waves that are excited for any gradients of the magnetic field.

Let us now consider the more general instability condition (9). It follows from it that, when the equality

$$T_\parallel \frac{dn_0}{dr} = n_0 \frac{T_\perp - T_\parallel}{H} \frac{dH}{dr} \tag{13}$$

is satisfied, drift oscillations are stable, $\gamma < 0$. Since in our case $T_\parallel$ and $T_\perp$ are constant quantities, relation (13) can be rewritten in the form

$$\frac{dP_\parallel}{dr} + \frac{P_\parallel - P_\perp}{H} \frac{dH}{dr} = 0, \tag{14}$$

where $P_\parallel = n_0 T_\parallel$, $P_\perp = n_0 T_\perp$.

We have paid special attention to the stability of the plasma state described by relation (14), since in an arbitrary geometry relation (14) is one of the equilibrium conditions when the distribution function of the Larmor circles depends only on $\varepsilon = (v_\parallel^2 + v_\perp^2)/2$ and $\mu = v_\perp^2/2H$, but does not depend on which field line the particle is on, i.e. $f(\varepsilon, \mu, L) = f(\varepsilon, \mu)$ [(6)].

The stability of a plasma for such a special distribution function was considered in detail by Taylor. He showed that a plasma in a magnetic field increasing toward the boundary is stable with respect to flute perturbations. But he did not take into account the effect of resonant particles. The investigation carried out by us shows that Taylor' s conclusion may apparently be extended also to drift waves, with the essential reservation that the stability of drift waves is connected only with the special form of the distribution function. In this case the plasma proves stable with respect to the excitation of drift waves for any gradients of H.

I express my gratitude to L. I. Rudakov for formulating the problem and for supervising the work.

Received
3 VI 1965

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Note: Figure translations are in progress. See original paper for figures.