Corresponding Member of the Academy of Sciences of the USSR A. V. POGORELOV

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Abstract

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THEORY OF ELASTICITY

Corresponding Member of the Academy of Sciences of the USSR A. V. POGORELOV

ON THE ENERGY OF POSTCRITICAL DEFORMATION OF A THIN ELASTIC SHELL

In the author' s work ([1]) (Chapter I, § 2) it was shown that the investigation of the postcritical elastic state of a thin shell reduces to a variational problem for a certain functional $W = U - A$, defined on isometric transformations of the initial form of the middle surface. The term U of this functional is the deformation energy of the shell and is determined by the formula

$$U = \int_\gamma cE\delta^{5/2}\alpha^{5/2}\rho^{-1/2}\,ds_\gamma + \cdots, \tag{*}$$

where 2α is the angle at the edge γ of the isometric transformation, ρ is the radius of curvature of γ, δ is the thickness of the shell, E is the modulus of elasticity, and c is a constant $\simeq 0.19$. The integration is performed along the arc of the edge γ. The formula was derived under the assumption that the edge γ is located far from the boundary of the shell.

In a number of cases of shell loading, the transition to stable equilibrium after loss of stability is accompanied by the approach of the edge γ of the isometric transformation to the boundary of the shell. Naturally, for such elastic states formula $(*)$ can no longer be used. At the same time, the investigation of these states is of considerable interest, since in them one of the most important characteristics of the load-bearing capacity of the shell is determined—the lower critical load.

In this connection, in the present note we intend to determine the influence of the clamped boundary of the shell on the energy of its deformation under the assumption that the edge of the isometric transformation is sufficiently close to the boundary. As a direct application of the result obtained, we shall determine the magnitude of the lower critical pressure on a shallow spherical segment rigidly clamped along its boundary.

In the work ([1]) we obtained the formula for the deformation energy $(*)$ per unit length of the edge γ by minimizing the general expression for the deformation energy, which, after the corresponding normalization of the variables, had the form

$$\overline{U} = \frac{E\delta^{5/2}\alpha^{5/2}\rho^{-1/2}}{12^{3/4}(1-\mu^2)}\, I,$$

where

$$I = \frac{I_i + I_e}{2}, \quad I_i = I_e = \int_0^{\xi^*} (u^2 + {v'}^2)\, ds, \quad u' + v + {v'}^2/2 = 0.$$

The terms I_i and I_e correspond to deformations in the inner and outer half-neighborhoods of the edge. For sufficiently thin shells the upper limit ξ^* of the expressions I_i and I_e proves to be significant and, in view of the exponential attenuation of the deformations straightening the edge, was taken to be equal to infinity. The boundary conditions for the varied functions were suggested by the character of the deformations under consideration. Namely, it was assumed for

of both half-neighborhoods

$$u(0) = 0, \qquad v(0) = -1, \qquad u(\infty) = v(\infty) = 0. \tag{**}$$

In the case of sufficient proximity of the ridge γ to the edge of the shell, the general expression for the energy of the deformation straightening the ridge remains the same, with the sole difference that the limit of integration ε_e^*, corresponding to the outer half-neighborhood (adjacent to the edge), is equal to the dimensionless distance τ of the ridge from the edge. Thus,

$$I_i = \int_0^{\infty} (u^2 + {v'}^2)\, ds, \qquad I_e = \int_0^{\tau} (u^2 + {v'}^2)\, ds.$$

Moreover, the nearness of the edge violates the symmetry of the problem with respect to the boundary conditions at $s = 0$. Instead of the conditions $(**)$ we shall require only that

$$u_i(0) + u_e(0) = 0, \qquad \frac{1}{2}(v_i(0) + v_e(0)) = -1.$$

This expresses the condition of continuity of the radial displacement u and of the rotation v on passing through the ridge. As for the boundary conditions far from the ridge in the inner half-neighborhood and at the edge of the shell in the outer half-neighborhood, they are, naturally, assumed to be zero, i.e.

$$u_i(\infty) = v_i(\infty) = 0, \qquad u_e(\tau) = v_e(\tau) = 0.$$

Fig. 1

Fig. 1

Figure 1: Fig. 1

Applying the same considerations as in work ([1]), we arrive at the conclusion that, as the ridge γ approaches the edge of the shell, the deformation energy is determined by the same formula $(*)$, with a coefficient c depending on the parameter τ. Namely,

$$c(\tau) = \frac{1}{12^{3/4}(1-\mu^2)} \min_{(u,v)} I,$$

where the minimum of I is taken over twice differentiable functions u, v satisfying the boundary conditions

$$u_i(0) + u_e(0) = 0, \qquad \frac{1}{2}(v_i(0) + v_e(0)) = -1,$$

$$u_i(\infty) = v_i(\infty) = 0, \qquad u_e(\tau) = v_e(\tau) = 0$$

and the nonholonomic constraint

$$u' + v + v^2/2 = 0.$$

The parameter τ is the dimensionless distance of the ridge from the edge of the shell. The true distance is

$$l = \frac{1}{12^{1/4}} \sqrt{\frac{\rho\delta}{\alpha}}\, \tau.$$

A numerical solution of the problem of determining the coefficient $c(\tau)$ leads to the result shown graphically in Fig. 1.

As an example of practical use of the result obtained, let us consider the question of the magnitude of the lower critical pressure on a shell in the form of a shallow spherical segment, rigidly clamped along the edge. The deformation energy U and the work A done by the external pressure p in the snapping of the spherical segment are determined by the formulas

$$U = 2\pi c(\tau) E \left(\frac{\delta}{R}\right)^{5/2} \rho^3, \qquad A = \frac{\pi}{2} \frac{\rho^4}{R} p,$$

(see ([1]), Ch. I, § 3), where R is the radius of curvature of the segment, and ρ is the radius of the circle of buckling, i.e. the radius of the base of the mirror-reflected segment under the isometric transformation.

Fig. 2

Figure 2: Fig. 2

Let r be the radius of the base of the shell, and h its height. Then

$$r - \rho \simeq \frac{1}{12^{1/4}} \sqrt{\frac{\rho\delta}{a}}\, \tau.$$

Hence, noting that $\rho/a \simeq R$, $h \simeq r^2/2R$, we shall have

$$\rho = (1 - 0.38\tau\sqrt{\delta/h}).$$

Now, from the stationarity condition for the functional $W = U - A$ with respect to the deformation parameter τ, we find the pressure p borne by the shell as a function of τ, and its minimum value, i.e., the lower critical load,

$$p_i = \overline{p}_i E(\delta/R)^2,$$

$$\overline{p}_i = \frac{3c}{\sqrt{2}} \sqrt{\frac{\delta}{h}} \min_{(\tau)} \left(\frac{c(\tau)}{1 - 0.38\tau\sqrt{\delta/h}} - \frac{c'(\tau)}{1.14\sqrt{\delta/h}} \right). \qquad (***)$$

Fig. 2

In Fig. 2 the dependence of the dimensionless load parameter $\overline{p}_i$ on the rise of the segment h/δ is presented graphically. For segments with rise $h/\delta > 6$, this dependence is well approximated by the formula

$$\overline{p}_i = \frac{(3c/\sqrt{2})\sqrt{\delta/h}}{1 - 1.3\sqrt{\delta/h}}, \qquad c \simeq 0.19.$$

In conclusion we note that the result obtained for the value of $\overline{p}_i$ agrees well with the data of the corresponding experimental study, the results of which are given in work ([1]).

Physical-Technical Institute of Low Temperatures
Academy of Sciences of the Ukrainian SSR

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CITED LITERATURE

1. A. V. Pogorelov, *Geometrical Theory of Stability of Shells*, "Nauka," 1966.

Note: Figure translations are in progress. See original paper for figures.