ON THE THEORY OF OPEN RESONATORS FORMED BY PARALLEL DISKS

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

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PHYSICS

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ON THE THEORY OF OPEN RESONATORS FORMED BY PARALLEL DISKS

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1. We consider resonators consisting of two disks placed opposite one another (Fig. 1), with $kl \gg 1$, where $k = \omega/c$ is the wave number. The calculation of these resonators is based on the assumption that, at their edge ($r = a$), electromagnetic waves at frequencies close to their critical frequencies are reflected with the reflection coefficient from the open end of a plane waveguide, which for waves E_{0q} and H_{0q} can be written in the form

Fig. 1

$$R_E = -e^{i(\beta' + i\beta''_E)s},$$
$$R_H = -e^{i(\beta' + i\beta''_H)s}, \tag{1}$$

where $s = \sqrt{2l/k}\,w$, and w, in our case, is the radial wave number.

The dependence of β', β''_E, β''_H on the index q is given in Fig. 2, constructed on the basis of (1). Taking into account the difference between β''_E and β''_H, which is significant for small values of q, leads to qualitatively new results.

2. We introduce the functions $f_E(r)$ and $f_H(r)$, proportional respectively to the electric and magnetic azimuthal components of the field:

$$f_E(r) = CJ_{m-1}(wr) + DJ_{m+1}(wr),$$
$$f_H(r) = CJ_{m-1}(wr) - DJ_{m+1}(wr), \tag{2}$$

where C and D are arbitrary constants, $J_m(x)$ is the Bessel function, and m is the azimuthal index.

Using expressions (1), we write the impedance boundary conditions at the edge of the resonator:

$$f_E(r) + \frac{y_E}{w}\frac{df_E(r)}{dr} = 0,$$
$$f_H(r) + \frac{y_H}{w}\frac{df_H(r)}{dr} = 0, \tag{3}$$

where (for $|s| \ll 1$)

$$y_E = \frac{\beta' + i\beta''_E}{2}s, \qquad y_H = \frac{\beta' + i\beta''_H}{2}s. \tag{4}$$

Substitution of (4) into (2) gives, in the first approximation in s, for $m = 1, 2, ...$

$$s = \frac{2\nu_{m-1,n}}{M + \beta' + i\beta''} \quad \text{or} \quad s = \frac{2\nu_{m+1,n}}{M + \beta' + i\beta''}, \tag{5}$$

where

$$M = \sqrt{2k/l}\,a = 2ka/\sqrt{\pi q}; \qquad \beta'' = (\beta''_E + \beta''_H)/2,$$

$v_{m,n}$ is the n-th zero of the Bessel function $J_m(x)$. Expressions (5) give the natural frequencies of the resonator

$$\omega = \frac{\pi c}{l}\left(\frac{q}{2} + p\right), \qquad p = \frac{s^2}{4\pi} = p' - ip''. \tag{6}$$

The ratio D/C, according to the order of the formulas in (5), is determined from

$$\frac{D}{C} = -i\,\frac{wa}{2m}\,\frac{\beta''_E - \beta''_H}{4}\,s \quad \text{or} \quad \frac{C}{D} = i\,\frac{wa}{2m}\,\frac{\beta''_E - \beta''_H}{4}\,s. \tag{7}$$

For $m = 0$, the natural frequencies are determined by the formulas

$$s = \frac{2v_{1,n}}{M + \beta' + i\beta''_E} \quad \text{or} \quad s = \frac{2v_{1,n}}{M + \beta' + i\beta''_H}. \tag{8}$$

3. For $m = 1, 2, ...$, the natural oscillations are characterized by three indices m, n, q and also by that Bessel function (J_{m-1} or J_{m+1}) whose roots give the natural frequencies and which, in the first approximation, determines the field of the natural oscillation.

Fig. 2

Therefore we shall denote these oscillations by four indices: $(m,\, m-1,\, n,\, q)$; $(m,\, m+1,\, n,\, q)$.

The derived relations are valid for $|s| \ll 1$. This means that for $q \sim 1$ one must have $ka \gg 1,\ a \gg l$; for $q \gg 1$ it may also be that $l \gg a$. Thus, no restrictions are imposed on the ratio of the diameter of the disks to the distance between them (for each such ratio there exists its own spectrum of high-Q oscillations). As $q \to \infty$, the parameters β', β''_E, β''_H coincide with $\beta = 0.824$, and our formulas pass into the formulas of paper (2).

4. When the disks have small transparency, a plane wave incident on the resonator along the z axis excites in the resonator an electromagnetic field containing only the oscillations $(1, 0, n, q)$:

$$E_x = \sum_{n,q} A_{nq} J_0\left(\frac{v_{0n} r}{a}\right) \left[e^{iv_q z} - (-1)^q e^{-iv_q z}\right], \tag{9}$$

$$H_y = \sum_{n,q} B_{nq} J_0\left(\frac{v_{0n} r}{a}\right) \left[e^{iv_q z} + (-1)^q e^{-iv_q z}\right].$$

and the components E_z and H_z, which we do not write out. Here $kl = v_q l + \pi p, \quad v_q l = \pi(q/2+\rho)$, where the parameter ρ is related to the complex reflection coefficient of the disks R by the relation

$$\rho = \frac{i}{2\pi} \ln R = \rho' - i\rho''.$$

The coefficients A_{nq} and B_{nq} are determined by the formulas

$$\begin{aligned} A_{nq} &= e^{-i(k-v_q)l}\, \frac{iTE_0}{klv_{0n} J_1(v_{0n})}\, \frac{\omega_{nq}\omega}{\omega^2 - \omega_{nq}^2}, \\ B_{nq} &= e^{-i(k-v_q)l}\, \frac{iTE_0}{klv_{0n} J_1(v_{0n})}\, \frac{\omega^2}{\omega^2 - \omega_{nq}^2}, \end{aligned} \tag{10}$$

where T is the transmission coefficient of the disks; the natural frequencies $\omega_{nq} = \omega'_{nq} - i\omega''_{nq}$ are complex; ω is the excitation frequency. The resonant values of the amplitudes (10), when diffraction losses predominate ($p'' \gg \rho''$), are inversely proportional to $v_{0n}^{5/2}$ and $q^{3/2}$, i.e., with increasing indices n and q they decrease rather rapidly. When losses in the disks predominate, the resonant values (10) are inversely proportional to $v_{0n}^{1/2}$ and do not depend on q.

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REFERENCES

1. L. A. Vainshtein, *Diffraction of Electromagnetic and Acoustic Waves at the Open End of a Waveguide*, Moscow, 1953.
2. L. A. Vainshtein, ZhETF, **44**, No. 3, 1050 (1963).

Note: Figure translations are in progress. See original paper for figures.