# GENERAL CRITICAL STRESSED STATE OF A STRICTLY CONVEX SHELL

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## Abstract

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# GENERAL CRITICAL STRESSED STATE OF A STRICTLY CONVEX SHELL

Let us imagine a strictly convex shallow shell that is in a basic stressed state under the action of an arbitrary load distributed along its edge. Suppose that in this state, in the middle surface of the shell, stresses $\sigma_1, \sigma_2, \tau$ arise, referred to the principal directions on the surface. We wish to determine under what conditions the stressed state characterized by the quantities $\sigma, \tau$ becomes unstable and buckling of the shell is possible.

In considering this problem, we shall assume the shell to be sufficiently shallow; we shall suppose that the principal curvatures and the principal directions on the middle surface vary little, and, finally, that the stresses $\sigma$ and $\tau$ characterizing the stressed state vary little. As for the fixing of the edge of the shell, we assume only that it excludes displacements normal to the surface of the shell.

Let the loss of stability of the shell be accompanied by the buckling of a system of small elliptic regions, on the average uniformly distributed over its surface. Under this assumption, the deformation energy associated with buckling, per unit area of the surface, is determined by the formula

$$U = \frac{2E\delta^2 2\pi\sqrt{ab}}{\sqrt{12(1-\nu^2)}}(\lambda^4 + \mu^4 + 4\lambda^2\mu^2)\sigma\gamma,$$

(see ([1]), Chap. II, § 4), where $a$ and $b$ are the principal curvatures of the shell; $\lambda, \mu, \sigma$ are parameters characterizing a separately taken region of buckling; $\gamma$ is the density of distribution of the centers of buckling.

We shall determine the work performed by the external load during deformation of the shell. In this connection we first introduce the concept of a **global** deformation of the metric of the surface under buckling of the system of regions. Let the surface $S$ be deformed with buckling over a system of small approximately identical regions, the centers of which are situated sufficiently uniformly that one may speak of the density of their distribution. Let $S'$ be the surface obtained under such deformation from $S$. Take on the surface $S$ two arbitrary points $A$ and $B$. On the surface $S'$ there corresponds to them a certain pair of

Fig. 1

Figure 1: Fig. 1

points $A'$ and $B'$. Let $d$ and $d'$ be the spatial distances between these points. The ratio

$$\varepsilon = (d' - d)/d \qquad (*)$$

in the limit as $B \to A$, and hence as $d \to 0$, gives the relative deformation of the surface $S$ in passing to $S'$. We shall call this relative deformation local, in contrast to the global relative deformation, which may be visualized as the relative deformation under a rough consideration of the form of the surface $S'$, ignoring the details connected with buckling. The exact definition of this concept is as follows.

In view of the regularity of the distribution of buckling regions under the deformation of the surface $S$ into $S'$, the ratio $(*)$ changes little for values

$d$, large in comparison with the dimensions of an individual buckling region and small in comparison with the radius of normal curvature of the surface. In this connection, for such $d$ the quantity $\varepsilon$ in a certain sense characterizes the deformation of the surface associated with buckling. We shall call the quantity $\varepsilon$ the global relative deformation of the metric of the surface. Let us give an example explaining this definition.

Let us imagine that the surface $S$ is a rectangular flat region, and that the deformation with buckling into the surface $S'$ consists in the formation of a corrugation by means of an isometric transformation (Fig. 1). Then the local relative deformation is obviously equal to zero, while the global one is equal to

**Fig. 1**

$$\varepsilon = (b' - b)/b < 0.$$

The relative global deformations may be found as functions of the parameters $\lambda, \mu, \sigma, \vartheta$, characterizing an individual buckling region (see (1), Ch. II, § 3), and of the density $\bar{\gamma}$ of the distribution of their centers. Omitting the rather complicated calculations, we give the final formulas:

$$\varepsilon_1 = 6\pi\lambda\mu(\lambda^2 + \mu^2)\sigma\bar{\gamma}\sqrt{\frac{a}{b}}\cos 2\theta,$$

$$\varepsilon_2 = -6\pi\lambda\mu(\lambda^2 + \mu^2)\sigma\bar{\gamma}\sqrt{\frac{b}{a}}\cos 2\theta,$$

$$\gamma = 12\pi\lambda\mu(\lambda^2 + \mu^2)\sigma\bar{\gamma}\sin 2\theta.$$

Here $\varepsilon_1$ and $\varepsilon_2$ are the relative deformations along the principal directions of the original surface, and $\gamma$ is the change of the angle between the principal directions.

The work done by the external load, referred to a unit area of the middle surface of the shell, is

$$A = (\varepsilon_1\sigma_1 + \varepsilon_2\sigma_2 + \tau\gamma)\delta,$$

or, substituting the values of $\varepsilon_1$, $\varepsilon_2$, and $\gamma$,

$$A = 6\pi\lambda\mu(\lambda^2 + \mu^2)\sigma\bar{\gamma}\left\{\left(\sqrt{\frac{a}{b}}\sigma_1 - \sqrt{\frac{b}{a}}\sigma_2\right)\cos 2\theta + 2\tau\sin^2\theta\right\}.$$

Now, similarly to what was done in paper (1), Ch. II, §§ 3, 4, from the stationarity of the functional $W = U - A$ we find the condition under which the stressed state characterized by the quantities $\sigma_1$, $\sigma_2$, and $\tau$ will be critical. Omitting the corresponding calculations, we give this condition:

$$\frac{E\delta}{\sqrt{3}(1-\nu^2)} \leq \left[(R_2\sigma_1 - R_1\sigma_2)^2 + 4R_1R_2\tau^2\right]^{1/2}.$$

Here $R_1, R_2$ are the radii of normal curvature along the principal directions on the surface; $\sigma_1, \sigma_2$, and $\tau$ are the stresses referred to the principal directions; $\delta$ is the thickness of the shell; $E$ is the elastic modulus; $\nu$ is Poisson' s ratio.

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## REFERENCES

1. A. V. Pogorelov, *Geometric Theory of Shell Stability*, "Nauka," 1966.

*Note: Figure translations are in progress. See original paper for figures.*