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Fig. 1

Figure 1: Fig. 1

Abstract

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ON THE DEFINITION OF A CLASS OF NONLINEAR PULSE AUTOMATIC SYSTEMS

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1. Nonlinear pulse automatic systems ([1]) with quantization in time (NPAS) constitute a broad variety of pulse systems, differing in the types of modulation, the character of the nonlinearities, and the features of the linear parts. There is no constructive definition of NPAS in the literature that would encompass all systems of this class and establish an internal connection between them.

Below, using the example of a system with one nonlinear element, a definition of NPAS is constructed that fills this gap and covers systems with all possible kinds of pulse modulation, arbitrary nonlinearities, and linear parts.

Fig. 1

2. Consider an NPAS in which a nonlinear element can be singled out (see Fig. 1). The linear part (LP) of such a system is an arbitrary linear pulse system, in particular an ordinary continuous linear system. The nonlinear element (NE) is either an arbitrary ordinary nonlinearity—backlash, saturation, dry friction, etc.—or an arbitrary nonlinear pulse modulator: amplitude, width, frequency, phase, amplitude-width, width-frequency, amplitude-width-frequency, etc. The question arises: does there exist a structure that encompasses the entire variety of systems of this class? If it exists, what form does such a structure have, what mathematical apparatus is required, and what equations correspond to it?

3. In connection with the posed problem, let us consider the structure of Fig. 2 (for simplicity of the mathematical description we shall assume that the LP is a stationary pulse system and that the external action $f(\bar{t})$ is equal

Fig. 2

Figure 2: Fig. 2

to zero). Here IPE is an ideal pulse element carrying out quantization in time, so that, in particular, $w(\bar{t}) = w(n) = v(n) = v(\bar{t})\big|_{t=n}$, $n = \ldots, -2, -1, 0, 1, 2, \ldots$; $\bar{t} = t/T_0$ is relative time; T_0 is the repetition period; ON is an ordinary nonlinearity with hysteresis, which is characterized by a nonlinear function of the form

$$v(\bar{t}) = F(x(\bar{t}),\, v(\bar{t} - 0)) \tag{1}$$

and by the initial value $v(0) = v_0$; LFE is a linear shaping element with pulse response $k(\bar{t}, \bar{\tau}) = K_i(\bar{\tau}) s(\bar{t} - \bar{\tau} - \beta(\bar{\tau}), \gamma(\bar{\tau}))$, $\gamma(\bar{\tau}) = (\gamma_1, \gamma_2, \ldots)$ is a vector, in general infinite-dimensional; LCF are linear continuous filters with pulse responses of the form $k(\bar{t}, \bar{\tau}) = k_n(\bar{t} - \bar{\tau})$; LCDF is a linear continuous-discrete filter composed in an arbitrary manner of an arbitrary number of IPEs, LCFs, and linear discrete filters with pulse responses of the form

$$k(\bar{t}, \bar{\tau}) = \sum_{m=-\infty}^{\infty} k_d(m) \delta(\bar{t} - \bar{\tau} - m).$$

For different values of the parameters, the structure in Fig. 2 represents a NIPACS with different types of pulse modulation, different nonlinearities, and different linear parts. In the case of degeneration of LPF_1 into an inertia-free amplifier or a delay element, the reduced structure depicts a system in which a sequence of discrete values is subjected to nonlinear transformation. In such a system, instead of ordinary nonlinearities ON (see Fig. 2), one should consider discrete nonlinearities, which, in contrast to (1), are characterized by relations of the form

$$v(\bar{t}) = v(n) = F(x(n), v(n-1)). \tag{2}$$

Fig. 2

In the absence of hysteresis, the formal distinction between ordinary and discrete nonlinearities is essentially erased. In this case the nonlinearity and the PIE commute with one another.

If, in the structure of Fig. 2, the repetition period of the PIE within the NE is made to tend to zero, the indicated structure represents a NIPACS with an ordinary nonlinearity. If, in addition, the repetition periods of the PIE within the LC are made to tend to zero, we arrive at the degenerate case of a NIPACS —an ordinary continuous nonlinear automatic control system.

4. Fixing the repetition periods of the PIE and assuming that the repetition period T_0 of the PIE within the NE is a multiple of the repetition periods of the PIE in the LC, the equation of the system in Fig. 2 can be written in the form

$$x(\bar{t}) = \int_{-\infty}^{\bar{t}} k(\bar{t}, \bar{\tau})(f(\bar{\tau}) - \Psi(x(\bar{\tau}), \bar{\tau}))\, d\bar{\tau}, \tag{3}$$

where

$$k(\bar{t}, \bar{\tau}) = \sum_{m, m_1 = -\infty}^{\infty} k_{\,1}(m_1 - \bar{\tau})\, k\ \ (m)\, k_{\,2}(\bar{t} - m_1 - m) \tag{4}$$

is the impulse response of the LC, and $y(\bar{t}) = \Psi(x(\bar{t}), \bar{t})$ is the corresponding equation of the NE.

The frequency equation of the system can formally be written in the form

$$X(j\bar{\omega}, \bar{t}) = K(j\bar{\omega}, \bar{t}) F\{f(\bar{t}) - \Psi(x(\bar{t}), \bar{t})\}, \tag{5}$$

where

$$K = (j\omega, \bar{t}) = \frac{1}{T_0} K_{\,2}(j\bar{\omega}) K^*(j\bar{\omega}) \sum_{l=-\infty}^{\infty} K_{\,1}(j(\bar{\omega} + l \cdot 2\pi)) e^{jl\omega\bar{t}} \tag{6}$$

is the parametric (?) frequency characteristic of the LC; $X(j\bar{\omega}, \bar{t})$ is the parametric image of $x(\bar{t})$; $F\{\ \}$ is the Fourier transform of the expression in braces; $K\,(j\omega)$, $K^*(j\omega)$ are the ordinary and impulse frequency characteristics of the corresponding elements of the LC; $\omega = \bar{\omega} T_0$ is the relative

frequency. From (5) and (6) follow the design equations of NIAS in the form

$$X(j\bar{\omega}) = K(j\bar{\omega}) F\{f(\bar{t}) - \Psi(x(\bar{t}), \bar{t})\} \tag{7}$$

with frequency characteristics of the form

$$K(j\bar{\omega}) = K_1(j\bar{\omega}) K_2^*(j\bar{\omega}). \tag{8}$$

In particular, under the condition

$$\int_{-\pi}^{\pi} |K_{\mathrm{n}1}(j\bar{\omega})|^2 d\bar{\omega} \gg \sum_{\substack{l=-\infty \\ l \neq 0}}^{\infty} \int_{-\pi}^{\pi} |K_{\mathrm{n}1}(j(\bar{\omega} + l \cdot 2\pi))|^2 d\bar{\omega} \tag{9}$$

equation (7) is an approximate notation for equation (5), in which

$$K(j\bar{\omega}) = \frac{1}{T_0} K_{\text{n}1}(j\bar{\omega}) K_{\text{n}2}(j\bar{\omega}) K^*_{\text{id}}(j\bar{\omega}). \tag{10}$$

If the LCF_1 is assigned to the NE, equation (7) represents the exact equation of the system, and

$$K(j\bar{\omega}) = K_{\text{n}2}(j\bar{\omega}) K^*_{\text{id}}(j\bar{\omega}). \tag{11}$$

When considering NIAS with discrete nonlinearities and with ordinary nonlinearities without hysteresis, there is no need to assign the LCF to the NE, since in these cases the properties of the linear part are in fact entirely taken into account by the frequency characteristic of the form (8). If, in the latter cases, one restricts consideration to systems with amplitude modulation and assigns the PFE to the linear part, one arrives at the NIAS equation in the form (7) with a frequency characteristic of the form

$$K(j\bar{\omega}) = K^*(j\bar{\omega}). \tag{12}$$

It is precisely this case (and those reducible to it) that is studied by the theory of pulse systems based on the mathematical apparatus of difference equations.

5. One of the basic relations used in frequency-domain investigations of nonlinear automatic systems is the approximate [3] condition for the existence of steady motions in these systems

$$K(j\bar{\omega}) = -1/J. \tag{13}$$

According to the foregoing, in the case of pulse systems the frequency characteristic $K(j\bar{\omega})$, standing in the left-hand side of (13), is, generally speaking, a formal product of the ordinary and pulse frequency characteristics, by which the dynamic features of the entire linear part—or of a portion of it—that are essential (from the standpoint of ordinary frequency representations) are determined, and which, owing to this, can be used *purely formally* as the design frequency characteristic of the linear part. The form of the equivalent complex gain coefficient J depends not only on the nature of the NE (see [4,5]), but, generally speaking, also on the properties of the linear continuous filter directly preceding the nonlinearity. The harmonic solution of equation (13), depending on the specific features of the problem, determines either a continuous process or a sequence of discrete values close to the corresponding (quasi-harmonic) exact solution. In certain cases (see [6,7]) the indicated sequence represents the exact solution of equation (13).

6. With the constructions developed above, which clarify the physical uniqueness of nonlinear pulse systems and the features of their mathematical description, the definition of the class of NIAS is practically exhausted. The article establishes the connection between discrete (difference) and continuous approaches to the study of nonlinear pulse systems. As is seen from

of the preceding; in the nonlinear case, unlike the linear one (see (8)), the indicated approaches, generally speaking, are not identical to one another.

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Note: Figure translations are in progress. See original paper for figures.