ON THE INFLUENCE OF EXCITATION DIFFUSION ON THE REGIME OF MULTIMODE GENERATION

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Abstract

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ON THE INFLUENCE OF EXCITATION DIFFUSION ON THE REGIME OF MULTIMODE GENERATION

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1. The spectral composition of the output radiation of quantum generators indicates the presence of a large number of axial and nonaxial modes appearing at considerable excesses of the pumping energy over the threshold. In works ([1]–[4]), the successive excitation of various types of resonator oscillations (various modes) is explained by the spatial inhomogeneity of the electromagnetic field inside the resonator.

The inhomogeneous distribution of the electromagnetic field, arising on the resonator oscillation type with the highest quality factor because of the presence of induced radiation, leads to the establishment of an inhomogeneous distribution of the inverted population due to the "depletion" of active centers at the maxima of the electric-field amplitude. As a result, more favorable conditions are created in the resonator for the generation of a neighboring oscillation type, differing in frequency, i.e., for the excitation of a neighboring mode. At the same time, diffusion of excitation inside the resonator will lead to smoothing of the inhomogeneous distribution of the inverted population and to the possibility of generation only on one (or several) oscillation types ([2]). The solution of the coupled equations for the inverted population and the electromagnetic field inside a Fabry–Perot resonator with allowance for excitation diffusion was given in works ([2], [4]). However, the results obtained there are valid only near threshold. Therefore it is of interest to solve these equations without the indicated essential restriction.

2. Let us consider the equations for the photon density and the inverted population with allowance for diffusion ([2], [4]).

$$\frac{dn}{dt} = -\frac{n-\bar{n}}{\tau} + k\frac{d^2n}{dz^2} - \sum_{i=1}^{2j+1} Dg_i n N_i \left[1-\cos\frac{2\pi m_i}{L}z\right]; \qquad (1)$$

$$\frac{dN_i}{dt} = -\gamma_i N_i + D g_{iN} i \int_0^L n \left[1 - \cos \frac{2\pi m_i}{L} z\right] dz,$$

where $n(z,t)$ is the inverted population; $N_i(t)$ is the number of photons in the i-th axial mode; m_i is the number of half-waves fitting between the resonator mirrors at a distance L; g_i is the ordinate of the normalized luminescence line shape at the frequency of the i-th axial mode; τ is the time determined by formula (18) (see below); k is the diffusion constant; γ_i is a quantity inversely proportional to the quality factor of the resonator for the i-th mode; D is a quantity proportional to the Einstein coefficient; z is the direction of the axis of a Fabry–Perot resonator of length L.

In what follows we shall speak only of axial modes, and, at a given pumping level, the number of simultaneously generating axial modes is taken equal to $(2j+1)$. The quantity $\bar{n}$ corresponds to the equilibrium value of the inverted population in the absence of induced radiation and is determined by the pump power.

Let $n(z,t)$ and $N_i(t)$ be independent of time. Then for the first equation of system (1) we obtain

$$k\tau \frac{d^2 n(z)}{dz^2} - \left[1 + \sum_{i=1}^{2j+1} Q_i\right] n(z) + \left(\sum_{i=1}^{2j+1} Q_i \cos \frac{2\pi m_i}{L} z\right) n(z) + \bar{n} = 0; \qquad (2)$$

where

$$Q_i = D\tau g_i N_i. \qquad (3)$$

We seek the solution of equation (2) by the method of successive approximations,

$$n(z) = n^{(0)} + n^{(1)}(z) + n^{(2)}(z) + \ldots$$

We shall assume that

$$\sum_{i=1}^{2j+1} Q_i \cos \frac{2\pi m_i}{L} z \ll 1 + \sum_{i=1}^{2j+1} Q_i.$$

This approximation is fulfilled quite well when a large number of modes are generated simultaneously.

Retaining all terms up to and including second order, we obtain from (3)

$$n(z) = \frac{\bar{n}}{1+\sum_{l=1}^{2j+1} Q_l} \left\{ 1 + \sum_{k=1}^{2j+1} \frac{\varepsilon_k Q_k}{1+\varepsilon_k \sum_{l=1}^{2j+1} Q_l} \cdot \left[\cos\frac{2\pi m_k}{L} z + \sum_{i=1}^{2j+1} \frac{1}{2} Q_i \right.\right.$$
$$\times \left(\frac{\cos\frac{2\pi}{L}(m_i - m_k)z}{1+\sum_{l=1}^{2j+1} Q_l + k\tau\left[\frac{2\pi}{L}(m_j - m_k)\right]^2} + \frac{\cos\frac{2\pi}{L}(m_i + m_k)z}{1+\sum_{l=1}^{2j+1} Q_l + k\tau\left[\frac{2\pi}{L}(m_i + m_k)\right]^2} \right) + \ldots \Bigg]$$
$$\varepsilon_k = \left\{ 1 + k\tau \left(\frac{2\pi m_k}{L} \right)^2 \right\}^{-1}. \tag{4}$$

Here the terms of the first approximation are proportional to the quantities Q_k, and the terms of the second approximation to $Q_{iQ}k$.

3. Substitute expression (4) into the second equation of system (1). For the stationary case we obtain

$$N_i\gamma_i \left\{ -1 + \frac{\alpha_i}{1+\sum_{l=1}^{2j+1} Q_l} \left[\left(1 - \frac{\frac{1}{2}\varepsilon_i Q_i}{1+\varepsilon_i \sum_{l=1}^{2j+1} Q_l} \right) + \sum_{k=1}^{2j+1} \frac{\frac{1}{2}\varepsilon_k Q_k}{1+\varepsilon_k \sum_{l=1}^{2j+1} Q_l} \times \left(\frac{Q_k}{1+\sum_{l=1}^{2j+1} Q_l} - \frac{\frac{1}{2}\varepsilon_i \left(Q_{k+i} + \ldots\right.}{1+\varepsilon_i \sum \ldots} \right.\right.\right. \tag{5}$$

where

$$\alpha_i = DLg_i\bar{n}/\gamma_i. \tag{6}$$

The quantity ε_i changes little within the width of the luminescence line. Therefore, in all subsequent calculations it may be regarded as constant (although, generally speaking, this is not necessary) and equal to

$$\varepsilon_i \simeq \varepsilon_k = \varepsilon = (1+d)^{-1}, \quad \text{where} \quad d = (4\pi\mu)^2 (L_{\text{dif}}/\lambda_{\text{izl}})^2; \tag{7}$$

Here μ is the refractive index of the medium inside the resonator; λ_{izl} is the wavelength of the radiation in vacuum corresponding to the center of the luminescence line; $L_{\text{dif}} = \sqrt{k\tau}$ is the diffusion length.

Restricting ourselves in (5) to the terms corresponding to the first approximation, we have

$$N_i\gamma_i \left\{ -1 + \frac{\alpha_i}{1+\sum_{l=1}^{2j+1} Q_l} \cdot \left[1 - \frac{\frac{1}{2}\varepsilon Q_i}{1+\varepsilon \sum_{l=1}^{2j+1} Q_l} \right] \right\} = 0. \tag{8}$$

According to our assumption, the number of oscillating modes is equal to $(2j+1)$. Therefore, for modes with numbers $i < 2j+1$ the number of photons in the

i-th mode N_i is different from zero, while for modes with numbers $i > 2j+1$, $N_i \equiv 0$. The mode with number $i = 2j+1$ is at threshold. Since Q_i in (3) is proportional to N_i, for each i-th mode that is in the oscillation regime $(i < 2j+1)$, the quantity $N_i \neq 0$. In this case it is not difficult to obtain from equation (8) that

$$Q_i = \frac{2}{\varepsilon}\left(1 - \frac{1}{\alpha_i}\right) + 2\left[\left(1 - \frac{1}{\alpha_i}\right) - \frac{1}{\varepsilon\alpha_i}\right] X - \frac{2}{\alpha_i} X^2 \tag{9}$$

or

$$Q_i = \frac{2}{\alpha_i}\left(X + \frac{1}{\varepsilon}\right)[(\alpha_i - 1) - X], \qquad X = \sum_{l=1}^{2j+1} Q_l. \tag{10}$$

Summing expression (9) over i in the range from 1 to $(2j+1)$, we obtain an equation for determining the quantity X

$$\left(\sum_{i=1}^{2j+1} \frac{1}{\alpha_i}\right) X^2 + \left[\frac{1}{2} + \frac{1}{\varepsilon}\left(\sum_{i=1}^{2j+1} \frac{1}{\alpha_i}\right) - \sum_{i=1}^{2j+1}\left(1 - \frac{1}{\alpha_i}\right)\right] X - \frac{1}{\varepsilon}\left[\sum_{i=1}^{2j+1}\left(1 - \frac{1}{\alpha_i}\right)\right] = 0. \tag{11}$$

After solving equation (11), explicit expressions for Q_i can be obtained with the aid of (9).

4. If the Lorentzian form of the luminescence line is adopted, then in the presence of $(2j+1)$ simultaneously oscillating modes we obtain

$$g_i = g_0/[1 + \beta(j+1-i)^2],$$

where g_0 gives the maximum value of the g-factor, corresponding to the $(j+1)$-th mode; $\beta = (\delta\nu/\Delta\nu)^2$; $2\Delta\nu$ is the half-width of the spontaneous luminescence line; $\delta\nu$ is the distance between two neighboring axial modes in the Fabry-Perot resonator.

In this case expression (6) for α_i takes the form

$$\alpha_i = \alpha/[1 + \beta(j+1-i)^2], \qquad \alpha = DL\bar{g}_0\bar{n}/\gamma. \tag{12}$$

In the last expressions it is assumed that the quantities γ_i are the same for all frequencies within the width of the luminescence line and are equal to γ.

In order to relate the total number of oscillating modes $(2j+1)$ to the pump level, determined by the quantity α, we shall assume that α corresponds to such an excitation energy at which the $(2j+1)$-th mode is at threshold. This means that $Q_{2j+1} = 0$. In this case, for $i < 2j+1$, $Q_i > 0$, while for $i > 2j+1$ all $Q_i \equiv 0$.

By virtue of (10), the condition $Q_{2j+1} = 0$ means

$$X = \alpha_{2j+1} - 1 = \alpha/(1+\beta j^2) - 1.$$

On the other hand, from equation (11), with the aid of (12), we have

$$X = \left[\frac{\alpha(4j+1)/4(2j+1)}{1+\beta j(j+1)/3} - \frac{1+1/\varepsilon}{2}\right] +$$

$$+ \left\{\left[\frac{\alpha(4j+1)/4(2j+1)}{1+\beta j(j+1)/3} - \frac{1+1/\varepsilon}{2}\right]^2 + \frac{1}{\varepsilon}\left[\frac{\alpha}{1+\beta j(j+1)/3} - 1\right]\right\}^{1/2}.$$

Hence it is not difficult to obtain that

$$\alpha = \frac{(1+\beta j^2)^2}{1-\beta j(8j^2-3j-2)/3}\left[1 + \frac{2j}{3}\frac{4j^2-1}{1+\beta j^2} d\beta\right]. \tag{13}$$

In the case when $j \gg 2$, equation (15) gives

$$j \simeq \sqrt[3]{\frac{3}{8\beta}}\sqrt[3]{\frac{\alpha-1}{\alpha+d}}. \tag{14}$$

Here the quantity d (see formula (7)) determines the diffusion. As $d \to 0$, formula (16) coincides with the corresponding result of work [5], and for $(\alpha - 1) \ll 1$ with the results of work [2]. As $d \to \infty$ in the genera-

only one mode is produced. Using (10), (12), and (13), one can also obtain an expression for Q_i

$$Q_i = \frac{2\beta(1+d)}{1-\beta j(8j^2-3j-2)/3}(i-1)[(2j+1)-i] \quad \text{for } i < 2j+1, \tag{15}$$

$$Q_i = 0 \quad \text{for } i > 2j+1.$$

Expression (15), with the aid of (3), makes it possible to determine the number of photons N_i in a given mode in the presence of $(2j+1)$ simultaneously oscillating modes.

5. From expression (14) it is seen that, at large pump energies $a \gg 1$, there is a limiting number of oscillating modes in the resonator, namely

$$j_{\max} \simeq \sqrt[3]{\frac{3}{8}\left(\frac{\Delta\nu}{\delta\nu}\right)^2}.$$

At pump energies close to threshold ($a \simeq 1$), the results obtained by us are analogous to the results of paper ([2]).

In paper ([2]) it was also shown that excitation diffusion in ruby has an insignificant influence on the oscillation regime. In semiconductors, however, owing to the high mobility of carriers, a stronger excitation diffusion occurs. Thus, for example, for heavily doped gallium arsenide at liquid-nitrogen temperature ([4]), the quantity $d \simeq 500$–700, whereas for ruby at liquid-nitrogen temperature $d \ll 1$ ([2]).

For this reason it is of interest to compare the theory with experimental results. For example, in paper ([4]) it was shown that for gallium arsenide at liquid-nitrogen temperature the theoretical value of the quantity a_2 is 3.92, whereas the experimental value of this quantity is 2.1. Here a_2 corresponds to that injection current through the p–n junction at which the second mode began to oscillate. In comparison with experiment, the quantity a in paper ([4]) was determined by the formula

$$a = (E \quad - E \quad)/(E \quad - E \quad), \tag{16}$$

where E is the pump power; E is the pump power corresponding to the oscillation threshold; and E is the pump power corresponding to equalization of the populations of the upper and lower levels of the two-level system. However, in paper ([5]) it was shown that for a two-level system the quantity a must be determined by the formula

$$a = \frac{E \quad - E}{E \quad + E}\,\frac{E \quad + E}{E \quad - E} \tag{17}$$

and, moreover, the time τ becomes dependent on the pump power,

$$\tau = \frac{\tau_0}{1 + E \quad /E}, \tag{18}$$

where τ_0 is the spontaneous decay time.

At a pump power close to threshold ($a \simeq 1$), expression (17) reduces to (16). However, in the example given above the quantity $a - 1 \simeq 1$, and therefore formula (17) should be used.

If one now uses (17) and (18), then with the aid of formula (13) one can obtain $a_2 \simeq 2.46$. The remaining discrepancy may be due to heating of the diode during the oscillation pulse, as well as to possible experimental errors in determining the quantities E and E .

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CITED LITERATURE

1. C. L. Tang, H. Statz, G. A. de Mars, *J. Appl. Phys.*, **34**, 2289 (1963).
2. B. L. Lifshits, V. N. Tsikunov, *Dokl. Akad. Nauk SSSR*, **162**, No. 2 (1965).
3. H. Statz, C. L. Tang, *J. Appl. Phys.*, **35**, 1377 (1964).
4. H. Statz, C. L. Tang, J. M. Lavine, *J. Appl. Phys.*, **35**, 2581 (1964).
5. B. L. Lifshits, V. N. Tsikunov, *ZhETF*, **49**, No. 12, 1843 (1965).

Note: Figure translations are in progress. See original paper for figures.